Stella and the Stunners: The Mathematical Legacy of Rudd Crawford

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1 Introduction

1.1 Stella and the Stunners

This is a story inspired by a remarkable student Solomon, that led to a collaboration of two unforgettable characters—Rudd Crawford, a high school teacher; and a Dutch baroness, Stella—and their ongoing commitment to mathematics learning.

Rudd's part of the tale begins in Concord, MA, in the 1960s where in his first teaching assignment he befriended John, a pleasant, easy-going kid who didn't really get the hang of the math done in class, largely because he couldn't concentrate on it. John would have been diagnosed with attention deficit disorder by today's standards.

One day, Rudd gave John a piece of chalk, an eraser, and a problem to work on at the blackboard. *Plant ten trees, in five rows, with four trees in each row.* Once John understood that a tree could be in more than one row, he went to work. He was persistent, which was a contrast to his usual lack of effort. Eventually he became frustrated, scribbled something on the board, went back to his seat, and sat down. Rudd took a look at what John had scribbled and said, "John, come over here and look at what you did." He came over and saw a sketch similar to the one depicted in Figure 1. His face lit up: he'd solved it without knowing he'd done it. Each side of the star represents a row of trees!

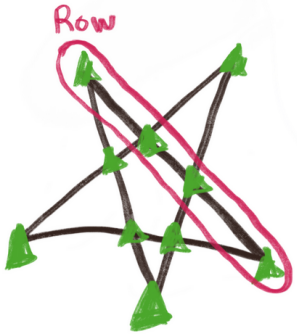

Fig. 1: *John's solution to the Trees Task.*

This was Rudd's first foray into nonconventional teaching . . . and with a nonconventional student, no less! Rudd had always had a penchant for brain-teaser type problems, which he solved and kept as a small collection on filing cards. While working towards his Ed.D. at Harvard, he would pull out and solve problems on his bus ride home. The following task was one of Rudd's favorites.

Martha the Grocer

Martha has a 2-pan scale and a set of 4 shiny brass weights. She claims that she can weigh out any whole number of ounces up to and including 40 using just these weights. And she's right! What is the weight of each of her 4 weights? (Note that you can put weights on either pan of the scale. Thus either addition or subtraction of the weights is possible.)

What Rudd began to realize was that these problems had an allure that traditional coursework couldn't match. They required a different type of thinking and a different mindset. Also, they often beget new questions. For example, when solving the Martha the Grocer task, Rudd's students soon realized that the solution involved powers of three. This observation lead Rudd and his students to a new problem—namely, *why powers of three?* A new idea for teaching began to germinate.

2 Building a Course around Brain Teasers and Recreational Mathematics

2.1 Teaching at Oberlin

Now we fast-forward to the mid-1970s. Rudd is now married to Lisa, a world-class harpsichordist. They find their way to Oberlin, OH where Lisa teaches in the Oberlin Conservatory and Rudd will teach for four years at Oberlin Junior H.S. until a slot opens up at the high school. Early on, he was given a pre-algebra class to teach to a group of very bright 7th and 8th graders. He quickly realized (in his words), "Sheesh. Learning about parentheses and My Dear Aunt Sally. The students can learn that in ten minutes." So he reworked the course to focus largely on the brain-teaser problems that he continued to collect.

It was a great year—"I just kept throwing problems at the kids; we'd do some in class and some were homework. And the students kept coming back for more." Rudd says of that experience, "That was another big moment for me, realizing the way these problems could reach into the back alleyways of the brain and help students to do more than either they or I thought they could accomplish." Eventually, he moved to teach at the high school, and a year or two after that, his former pre-algebra students followed, all grown up, taking Rudd's senior math course. They started right in on him: "We haven't done anything hard in math since middle school—give us some more of those problems."

2.2 An Inspiration for Mathematical Problem Solving

"Hmmm, what to do?" By now, the students were doing high school level contest problems, and some of them were really hard—for Rudd, too. But because he didn't know where to find the right material for these students, he wound up waffling on it—ducked the whole thing.

Then along came Solomon, a senior with a luminous mind, brilliant in the way he could visualize abstract concepts. He and Rudd did a lot of work after school, as they explored the edges of what high schoolers typically learn. Solomon had little patience for the routine tasks of the Senior Math class. Nevertheless, Rudd was a real fan of his, and was horrified when Solomon died in

a bicycle accident before his first week of college classes. "He and I were close—I spoke at his memorial service, and his family gave me his backpack."

"I thought about Solomon a lot. I mused on the way we teachers always think we're giving our students valuable material to help them in their future lives. I mused on how Solomon wasn't going to have a future life. And then the whole thing flipped around. The point wasn't what I couldn't do for Solomon's life, but what his life could do for mine. I realized what Solomon could do for me was to goad me into giving students high school, contest-level brain teasers."

2.3 Rudd and Stella

The last character in our story is Ecaterina Elizabeth van Heemsvloet tot Schattenberg. Or as her friends know her, Stella. Stella was a Dutch baroness, born in Amsterdam in 1926. She studied piano from an early age, showing considerable talent. She was 14 when the war came to Holland, and she spent five difficult and dangerous years as a message courier in the Dutch resistance. In her university years after the war, she discovered a love for mathematical puzzle problems and would carry a problem around in her purse until she solved it.

In the early 1950s, Stella went to Paris to study piano and music theory with the famous Nadia Boulanger. She studied there for four years but has thrown a curtain over her life in the later years of that decade. In 1961 she traveled to Venice, where she learned to ride a motorbike and had the misfortune of driving it into the Grand Canal.

She was rescued by the handsome, aristocratic Graziano Pelligrini, who had seen the accident from the window of his palazzo. The two embarked on a 47-year marriage thereafter. It was during her marriage that Stella resurrected her love of mathematics, as her collection of non-routine problems grew steadily through the years.

Rudd first met Stella and her husband in 1965 at a party given by mutual Dutch friends in Amsterdam. Rudd found her charming, though as he confesses, "somewhat formidable." When Stella learned that Rudd was a mathematics teacher who liked non-routine problems, a bond was established. She and he began to trade problems by mail, each trying to stump the other. Stella's talent lay in maintaining intense concentration on whatever she was doing at the moment. Even when she was simply peeling a carrot, one felt that the whole world fell away except for Stella, the knife, and the carrot.

In the early 1980s, Stella began asking Rudd questions about what kind of mathematical work he was giving his students at Oberlin High School. This occasion happened to coincide with Rudd's resolve to enrich his students' learning with thought-provoking, non-routine problems. When Stella offered to help him start his collection of more challenging problems, he could not resist. The mathematics department ended up appointing Stella as a volunteer overseas consultant.

2.4 Incorporating Non-Routine Problems into the Existing Curriculum

Their first step toward instituting an ongoing strand to students' normal coursework in Rudd's traditional classes was to assemble and organize a large collection of problems and to think very carefully about what was to be achieved through this complementary experience.

Rudd and Stella agreed that their problems should ...

- Exhibit good mathematics and reward sound thinking—no tricks should be required to generate a correct solution.

- Should be posed in a novel, quirky, and motivating context.
- Can be solved using the mathematics students already know and understand.
- Require students to apply their knowledge to an unpracticed situation.

Without the context of the just-taught new material of a lesson, students would enter the problem space at an earlier point than for the recently taught schemas of a time-rate-distance situation, for instance. Problem situations would require exploration before starting a formal solution. Being puzzled at the outset was the expectation. Only through investigation and longer and deeper thinking should one expect to come upon something recognizable that could provide a starting point for solution.

2.5 Two Example Problems from Rudd and Stella

2.5.1 The Situps Problem

This problem is an example of an easier candidate for students to explore, with the possibilities of more than one approach for solution:

The Situps Problem

To make the team, you are going to have to do 89 sit-ups for the coach a week from today. You decide to work up to it. You will start by doing 3 sit-ups today (no sense rushing into things) and end on the 8th day with 89. You don't know how many you will do tomorrow, but you decide that from the 3rd day on, the number of sit-ups you do will be the sum of what you did on the two preceding days. That is, the number you do on Wednesday will be the sum of the number you did on Monday and the number you did on Tuesday; the number you do on Thursday will be the sum of what you did on Tuesday and Wednesday, and so on.

Your task: Find out how many sit-ups you should do tomorrow to make this work, so that you come out with 89 a week from today.

Solution: One possible solution to the Situps Task is provided in Figure 2.

Your sit-up regime will be 3, x, __, __, __, __, __, 89.

Filling in the blanks we'll have 3, x, $3 + x$, $3 + 2x$, $6 + 3x$, $9 + 5x$, $15 + 8x$, 89.

And $89 = 24 + 13x \rightarrow 13x = 65 \rightarrow x = 5$.

So you'll do 3, 5, 8, 13, 21, 34, 55, 89.

Fig. 2: *Possible solution to the Situps Task.*

2.5.2 The Coffee Break Task

Other problems might be more difficult, involving perhaps an extension of ideas recently taught. As an illustration, when students are taught how to solve systems of equations, their focus (under-

standably, since this is new learning) is upon the mechanics of solution – eliminating variables until the value of each can be found, and then answering the question. It is impressed upon them that in order 'to solve a system,' one must have as many independent equations as variables.

But what happens when one does not? Can a selected question be answered without knowing the specific value for each variable? Consider this question from the Rudd-Stella collection:

The Coffee Break Task

Your boss took everyone for a coffee break. A look at the check told you that she paid \$15.75 for three sandwiches, seven cups of coffee, and one piece of pie. Next time, the check was exactly \$21 for four sandwiches, ten cups of coffee, and one piece of pie. If you go by yourself tomorrow and have a sandwich, a cup of coffee, and a piece of pie, how much will you pay?

The solution to this problem relies upon this lesser known property of variables: Letter is not used directly (i.e., it need not be evaluated). For example, If $A + B = 43$ then $A + B + 2 =?$

As the growing collection of problems was being assembled, it became apparent for the need to classify each problem according to different tabs to make them easier to find for specific uses. For example, one tab was content: Introductory; Visual; Logic; Arithmetic: $\mathbb{Z}, \mathbb{Q},$ and $\mathbb{R}$; Algebra Prep; Algebra; Symbol-Pushing; Geometry: Informal, Euclidean, Analytical; Trigonometry; Pre-Calculus; Calculator. Another tab was level of difficulty. A third was by course. This categorization eventually evolved into the Stella Decimal System. This became a crucially important tool for keeping track, locating, and selecting of problems as the collection grew.

3 A Promise Fulfilled: Implementing Stella's Stunners

The next step was deciding how to implement non-routine problem solving as a separate but parallel experience to traditional coursework. Though this process would evolve over time, Rudd and Stella thought deeply about several considerations before starting out.

3.1 Different 'Rules of Engagement'

Unlike a traditional lesson, the purpose of Stellas is not to learn something new and then to automate that new knowledge through repetitive practice. Stellas present situations more like the world outside the classroom in which there is no guidance, no lesson, only a need to accomplish a task using what you know or can readily learn. Any valid method that solves the problem (on your own) is acceptable.

You start out being puzzled and not knowing what to do. You must do something. What can you try? There are actions called heuristics like *Look for a Pattern* or *Try Specific Cases* that are sometimes useful, though their application guarantees no success. Heuristics enable you to poke around with the given information of a problem to try various ideas and try to gain a foothold of where to start. (Heuristics will not be taught, but they will be mentioned from time to time as 'useful things to try.')

Like the work of a mathematician, you face a situation that presents a quandary. Something requiring more investigation and discovery—and time—until you happen upon a useful idea. You have to think longer and harder than traditional study to find anything which may prove useful. Sometimes you may succeed. And sometimes you may not.

Another consideration is that some students or parents may think this kind of task is unfair. "You are not teaching them what they need to know." A counter question is whether it is unfair not to give students some experience with how to perform a mathematical task without the direct guidance of their teacher. These are all examples of how emotions and expectations regarding the Stella work will arise and must be attended to by the classroom teacher.

3.2 Classroom Management and Assessment

A second important consideration was that the Stella strand take little time away from normal coursework, but that it must engage every student in a significant way. Problem sets of 8 problems were given to students on a Friday, discussed on Monday as to questions, and due to be written up in individual notebooks turned in the following Friday. Notebooks from the previous problem set were graded over each weekend.

The problems were of varying levels of difficulty to accommodate student differences, consisting of a combination of visual, logic, arithmetic, algebra, geometry, calculator, and an occasional advanced math problem. The first couple were usually gimmies; then increasing in difficulty to the final two which might be quite challenging.

Problems 'came' from Stella. It was explained that Stunners were more difficult in the sense that there was nothing to identify which ideas might work. For that reason, Stunners were to be considered as challenges, requiring more time and more thinking through. Unlike exercises, students were not as likely to solve each one. But they should write up their efforts, including the amount of time spent working on each. They were also encouraged to express any thoughts or emotions in their notebooks.

Stellas were introduced gradually to acclimate students to the differing expectations and conditions, starting with the most capable class first. Rudd learned what worked along with his students. An average performance on a problem set was customarily about half the problems solved. A graduated evaluation scheme was devised to reflect this increased difficulty. Notebooks were read closely and responded to individually. Getting half the problems solved placed the score in the 70s. An overall notebook score for each grading period counted as one test score among the other tests. But it was used only if that score would help the student's average.

Over time other classes were added to the Stella experiment. Eventually students from the other two Oberlin High math teachers clamored for Stellas. At the height of the Stella enterprise, Rudd was producing weekly problem sets (excepting test and other special weeks) for ten different courses. Rudd continued using Stellas with each of his classes until he retired in 2005.

4 Impact of the Stunners

4.1 Impact on Students

From the beginning, Stella's Stunners were a sensation. They were different and outrageous in how they were presented. Some were easier than others, but mostly students took them on as intriguing challenges. Students met frustration (which they wrote out to Stella in their notebooks): "I hope you die, Stella." They also experienced the elation of success over a difficult task: "I FINALLY got 3 even after I didn't think I could!"

At the low performance end, students who had the most trouble with Stella's Stunners also had the most trouble with traditional work. At the high end, two male students—David and Joey—became

so enthralled with Stella's Stunners that they took over Rudd's file drawer of problems and did every single problem together. They continued to solve contest problems at higher and higher levels until they wound up on the U.S. Math Olympiad team. But it was the students in the middle who seemed to benefit the most. They met frustration and overcame it with perseverance. They learned how to work harder and longer on a mathematics task. They learned how to read each situation more carefully, how to experiment with the given information through specific examples, how to try different approaches. They became more flexible in their thinking and more resilient when they did not achieve success. Mostly, they learned more about their own thinking, and in the process discovered how to become better students.

4.1.1 In Students' Own Words

The impact of Stellas is best told in students' own words. Following a suggestion after retirement, Rudd solicited from former students through *Facebook* what the Stella experience had meant to them. This is a small sample of what he received back:

M1 "Stella made the SAT's easy. In fact, Stella got me into law school!"

N "I remembered doing math problems that were more of a challenging puzzle and fun than a monotonous stream of repetition that begins to feel like a broken record 3 problems in [I'm looking at you arithmetic *scowl*.... f**king 100 problems in a row that all feel like the same thing over and over and over..er..er..er...fzzzzt...er...]."

M2 "They really irritated me because it was always one little thing that I was missing when trying to solve them. They irritated my dad even more because I'd always take them home for his help."

J "I'm a better person for having known Stella. She challenged my thinking Mr. C."

A "Btw, I have loved them and they were a big part in helping me learn to love math and recognizing my potential. Maybe that was you . . . anyhow. Stella too lol. I usually never figured them out as a high schooler . . . but after you showed us it wasn't as hard as it looked, it helped (me) learn to think outside the box in everyday life as an adult and not be afraid to tackle something, even if it's big and monstrous looking. Just break it into smaller parts till you figure it out. Thank you for that . . . math has become my favorite thing now."

Parent "One of the great things about Stella's Stunners is that there are no right ways to solve the problems; they allow students with different strategies for problem-solving to be equally productive. One year we had two daughters in the same high school mathematics class doing Stella's Stunners. There was one plane geometric problem that I remember them both solving. One daughter had (and still has) a bent for theoretical analysis, and she solved the problem using geometric theory. The other daughter was (and still is) a practical, hands-on person, and she solved the problem by constructing geometrical figures on paper, cutting them out, and weighing them. They both arrived at the same answer; they were both pleased with themselves at having solved the problem; and they both were amused at the glee their parents took at seeing two entirely different ways of thinking at work productively on the same problem."

For most of these self-selected former students (and parents), Stellas were an enjoyable experience; they relished the mental challenge. Stellas provided a value-added experience to their mathematics studies that went beyond customary lessons. In looking back, students realized that Stellas taught them something about how to think, how to learn, and how to persist.

4.1.2 A Rudd Retrospective

Asked for his own assessment of the Stella experiment, looking back from retirement, Rudd said: "I found it enormously satisfying to have the written back-and-forth with each student (notebook), keeping in closer touch with both their minds and their feelings than I could have done otherwise. I felt as if I was really being their teacher in a more individual way."

Asked for a favorite problem from his large collection, and why it was a favorite, elicited this response of **Aggie's eggs**:

> Aggie's Eggs
>
> Aggie sells eggs. Last Saturday she sold Beneva half her eggs plus half an egg. Then she sold Calvin half of her remaining eggs plus half an egg. Then she sold David half of her remaining eggs plus half an egg. Then she sold the remaining 27 eggs to you. How many eggs did she start with?

Rudd described his students' fascination with this problem, "It starts with a seeming impossibility—half an egg. There's discussion: Yes, these are fresh raw eggs, not hard-boiled, no, no eggs got broken, etc. If necessary, I would give just a hint—she obviously has a lot of eggs to begin with, but what if she starts with just a few eggs? Try some easy small numbers—what do you see?" Wait for the light bulb to light up: Oh. an odd number of eggs works. "There's no obvious way to 'plug in an equation'; one just has to think hard. (Answer: 223, by the way.) Then, once it's done, there are lots of ways to play with it. For example, what about starting Aggie with n eggs and tracking the story algebraically?"

4.2 Stella's Stunners Go Online!

A year or two following retirement in 2005, Rudd was getting around to doing something about those old teaching files and thought about the valuable resource he had assembled in the form of the 1000+ problems. Over the years through workshops and institutes, teachers from other districts had shown great interest in the sample problems presented. Eventually, Rudd was hooked up with the Ohio Resource Center (ORC) where an offer was made. Working with the ORC staff, Mathematics Content Specialist Dr. Sigrid Wagner, and Dr. Margaret (Peggy) Kasten, former Director, Rudd would rewrite about 350 of his problems and provide sundry supporting information for their use on this best practices website with Stellas having their own special designation.

Stella's Stunners enjoyed continuing acclaim as individual problems and problem sets until funding support for the ORC eventually ended. Up until that time, it was one of the most frequently visited areas of the mathematics resources. (*Editors' note:* Old website materials have a website graveyard they go to when the original funding and host site vanish. This is an archive with this link to take readers directly to the original ORC Stella collection: `https://web.archive.org/web/20121011232704/http://ohiorc.org/for/math/stella/`)

But Stella wasn't done yet. In 2016, she and Rudd, now teamed with retired Oberlin math professor Dr. Michael Henle, pooled their considerable intellectual talents and began working with Oberlin College computer science students to rebuild a new and even larger Stella web resource. That site

is nearing completion and now houses over 700 problems, problem sets, and heretofore unseen additional components on how to use Stellas with high school students. This new site will be accessible soon; watch for the announcement.

5 Stella Coda

It is a frequent narrative of the teacher who made a difference in the life of his or her student. Not so well known, outside the circle of education, is the equally observable inspiration that some students bring to the lives of their teachers and the amazing consequences, in this case, for the community of Ohio mathematics educators. This has been one of those stories.

Thank you, Solomon. Well-done, Rudd. Stella, stunning as always.

References

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Steve Meiring began his education career in Indiana in 1965, teaching junior high school (all subjects) between earning his bachelor's and master's degrees. After four years, he came to Ohio to serve 26 years as a mathematics consultant for the Ohio Department of Education. During that tenure, he served on the prestigious Mathematical Sciences Education Board of the NRC and the American Junior High School Examination committee of the Mathematical Association of America.

Editors' Note: A number of Stella's Stunners from the original Ohio Resource Center site have been reorganized and catalogued at The Ohio Digital Mathematics Project (ODMP) at `https://u.osu.edu/odmp/`. The site is maintained by a team of graduate students at the College of Education and Human Ecology at The Ohio State University.