McCain's Messages: Matrices and Mathematical Practices

Valerie N. Long and Brian Sharp

Abstract: *The authors discuss what happened when twenty-two middle school students were introduced to the structure of matrices as they studied how John McCain and other prisoners of the Vietnam War used the tools to communicate. In addition to sharing the results of the historically based lesson, this article describes how the students engaged in many of the Mathematical Practices from the Common Core State Standards for Mathematics.*

Keywords: *matrices, Common Core, mathematical practices*

1 Introduction

In October 1967, while serving as a naval aviator in Vietnam, Lieutenant Colonel (LTC) John McCain was shot down and became a prisoner of war (POW). He was held captive at the infamous North Vietnamese prison known as the Hanoi Hilton. LTC McCain remained a prisoner of war for five and a half years, much of the time in solitary confinement. He refused to be freed until after the rest of his fellow countrymen were released (McCain & Salter, 1999).

Being a POW takes a toll on prisoners, both physically and mentally. They were not allowed to talk or have paper or pencils, yet for POWs to survive it is crucial for them to be able to communicate with one another. Connecting with other prisoners makes it possible to maintain the chain of command, develop a sense of fraternity, keep up morale, and exchange information about the health and welfare of each other. The very act of communication between POWS during the Vietnam War strengthened their resolve and drastically improved their survival rate when compared to that of American POWs during the Korean War, who had little communication (Corcoran, 1991).

The purpose of this article is to document the results of a lesson that engaged and motivated students based on how POWs communicated during their time in the prison. Twenty-two middle school students in grades 5 through 8 participated in a summer math camp where they completed this lesson. The lesson was taught by mathematics educators, Mr. Wayne and Ms. Lane. The objectives of the lesson were two-fold. First, students learned about the structure of matrices, including how each element in a matrix is assigned a unique cell address. Second, students applied their knowledge of matrices to compose, send, and decode secret messages using the Tap Code matrix, the method used to communicate by POWs.

Making real world connections to mathematics content is vital (National Research Council (NRC), 1990 & 1998; Drier, Dawson, & Garofalo, 1999), and the role of students is to be actively involved in making sense of mathematical tasks by making connections to prior knowledge or familiar contexts and experiences (National Council of Teachers of Mathematics (NCTM), 2014). However, many teachers feel hindered by a lack of resources, ideas, or training for making connections (Gainsburg,

2008). Within this article, the authors provide teachers with a ready-made resource that enables students to connect mathematical concepts to a real-world scenario, in this case a lesson on McCain's communication techniques as a POW. Furthermore, this lesson is consistent with goals advocated by the NCTM and Common Core State Standards for Mathematics (CCSS-M). NCTM's *Principles to Actions* states that a curriculum should make meaningful connections between mathematics and real-world contexts (NCTM, 2014). This lesson uses a real-world historical event to introduce students to matrices.

The most valuable aspect of this lesson is the engagement of the students using the *Mathematical Practices* (MP) from the CCSS-M (National Governors Association Center for Best Practic-es & Council of Chief State School Officers (NGA Center and CCSSO), 2010). Among the eight *Mathematical Practices*, students engaged in four. They made sense of problems and persevere in solving them (MP1), constructed viable arguments and critiqued the reasoning of others (MP3), attended to precision (MP6), and looked for and made use of structure (MP7). Additionally, students were expected to communicate and defend their answers as the lesson developed.

Further research on how POWs communicated indicated around 1965 Captain Carlyle "Smitty" Harris, an American POW at the Hanoi Hilton, started covertly training POWs to use the Tap Code as a method of communication (Borling, 2013). John McCain was one of the POWs who became proficient using this form of communication. The Tap Code originally came from the fictional novel *Darkness at Noon* (Koestler, 1941). Captain Harris learned the Tap Code from one of his survival instructors prior to being deployed to Vietnam. To watch a video of Captain Harris explaining the Tap Code, go to `https://www.youtube.com/watch?v=3zUImnnjCtI&t=1s`, referred later as the Harris video (Shea, 2016).

In *Darkness at Noon*, the main character was a political prisoner named Rubashov (Koestler, 1941). To communicate with the prisoner in an adjacent cell, Rubashov used a code based on a 5 by 5 matrix. The entries of the 5 by 5 matrix consisted of letters of the alphabet. Since C and K often have a similar sound, they shared the same address (i.e. cell location) in the matrix. This allowed all 26 letters to fit into the 5 by 5 grid. See Figure 1 for a representation of the Tap Code matrix, also known as the Knocking Code matrix. LTC McCain wrote a book, *Faith of My Fathers* (1999), where

	1	2	3	4	5
1	A	B	C/K	D	E
2	F	G	H	I	J
3	L	M	N	O	P
4	Q	R	S	T	U
5	V	W	X	Y	Z

Table 1: *The Tap Code matrix used by POWs.*

he described many instances when he used the Tap Code to communicate with other POWs. By stringing letters together, McCain and his fellow POWs sent each other messages. McCain tapped the row number of the letter, paused, then tapped its column number. For example, to send the message `TC`, shorthand for "Take care," McCain represented the letter `T` with four taps, a pause, and then four more taps (• ••••). To represent the next letter, `C`, McCain made one tap followed by a pause, then three more taps (• •••).

McCain became very proficient using the Tap Code. In a 2013 interview about his captivity, McCain stated, "After a while, you tap with the same person for months on end you can tap as rapidly as we are talking now — like the old telegraph operators in the old west" (McCain,

2018). To see John McCain describe how he used the Tap Code to discuss poetry, go to `https://time.com/5219605/john-mccain-pow-poem/`, referred to later as the McCain video (Waxman, 2018).

Fig. 1: *McCain Video.*

Armed with knowledge of John McCain's background as a POW and the mechanics of how the Tap Code works, teachers could develop an interesting lesson that allows students to discover the structure of matrices. This next section describes how such an activity occurred with a group of 22 middle school students.

2 The Lesson

Since the students participating in the activity were middle school age, very few of them knew much about the Vietnam War. After a short introduction, the students watched the Harris and McCain videos, which motivated them to learn how to use the Tap Code.

After watching the videos, Mr. Wayne displayed the Tap Code matrix on the smart board, see Figure 1. He then asked Ms. Lane to tap out her favorite letter. She responded by tapping (•• ••), which corresponded to `G`. When Ms. Lane finished tapping, Mr. Wayne asked the class to guess which letter was Ms. Lane's favorite. Students wrote their answers on white boards and held them up when asked. Since the tapping sequence was purposefully chosen to be a "double," the instructors anticipated there was a greater chance for students to correctly determine Ms. Lane's favorite letter. However, many students answered with a letter other than `G`. The instructors were surprised at the variety of strategies students used to arrive at their answer. One student, Amy, choose `D` as her answer. When asked why, Amy said, "Because she tapped four times." Amy used a method the instructors identified as the *total taps* strategy. In this case, there were a total of four taps in the message. Amy counted over four letters in the top row, from the left, to arrive at `D`.

A different student, Billy, guessed `N`. Billy used a method the instructors called the *game board* strategy. He moved his finger about the matrix in a manner similar to how pieces are moved on a game board. He started in the bottom left corner at `V`, counted two spaces to the right, then two spaces up to land on `N`.

Another student in the class, Collin, used a strategy similar to Billy's, but guessed the letter M. The difference in Collin's strategy was he started counting from the numeral 1 in the top left corner of the grid instead of at `V`. In explaining his reasoning, Collin stated, "Because I went over two to `B`. Then down two to `M`."

Debbie and Eddie guessed L and V, respectively. These guesses were not based on any mathematical convention, but because L is the first letter in Ms. Lane's last name and V is the first letter in her first name. Thirteen students correctly guessed G. Frank reasoned that G was the correct answer because "Go down two, a pause, then over two."

2.0.1 Additional Questions

At this point, the instructors did not want to reveal the correct answer, but wanted to ask students additional questions that would allow them to evaluate their strategies and make adjustments on what made sense to them. Therefore, Mr. Wayne asked, "Ms. Lane, what grade did you get in Spanish in college?" The instructors purposefully asked this question because the answer was a letter grade with a limited number of logical answers. In answering the question, Ms. Lane tapped (• •••), which corresponded to D.

Again, we were surprised with the student responses. Abby, who also used the *total taps* strategy for the first question, shifted her approach for this task to what the instructors called the *graphing* strategy. The *total taps* strategy would have given Abby the answer "E," which is not a typical letter grade. It seemed Abby realized that E was not a logical answer, so she changed to the *graphing* strategy. Her graphing approach was similar to the method used for graphing points in the Cartesian Plane. She started at the bottom left corner of the matrix, went right one space and up four spaces to arrive at F. Abby justified her answer by stating, "Oh, that makes sense. We worked on graphs. It all lines up because the y- and x-axis is kind of what it [the matrix] looks like." Gary agreed, "I think it's a graph because the coordinates are on a graph." Hank chimed in, "Matrix is another name for graph." Ivan added, "It's like a graph, but the bottom [of the matrix] should be $1, 2, 3, 4, 5$. So, I got F." Even though these students initially had a high degree of confidence in their strategy, they were not sure that Ms. Lane would have actually received an F in Spanish. This may have caused them to question the validity of their *graphing* strategy.

2.0.2 Encouraging Valid Strategies

Up to this point, students could use incorrect strategies to arrive at plausible answers. Since the instructors did not provide the correct answers, students became increasingly motivated to learn whether they were employing appropriate strategies. With the increase in motivation levels, the instructors decided to ask a question where the student would be required to use a valid strategy to arrive at the correct answer. Hence, Ms. Lane asked Mr. Wayne "Where did you go to college?" Mr. Wayne directed, "I'll tap three letters. First letter." Then Mr. Wayne tapped (•••• ••••••). Next, he stated, "Second letter" and tapped (••••• •). Finally, he said, "Third letter" and tapped (• •). While Mr. Wayne tapped his answer, Ms. Lane recorded the tapping sequence as dots, with commas for pauses, on the white board.

Students placed the letters they thought Mr. Wayne communicated on their individual white boards. Abby seemed confused with the answers she was getting, which may have been due in part to the letters she was getting when using her graphing strategy. This strategy would have led her to the letters D Z V, which is an unusual abbreviation for a university. At this point, she realized her strategy might not be valid, so she asked, "Is the first four taps the x or the y?" Mr. Wayne responded, "That is what you have to figure out." Once Mr. Wayne made this statement, the students began vigorously discussing amongst themselves how the order of the taps related to the matrix. Gary came to the board and explained, "You go four down and five over so the first letter is a U. Then, five down and one so V. The first one is A. So UVA." As Gary explained his answer, Mr. Wayne wrote $(4, 5)$ next to the U, $(5, 1)$ next to the V, and $(1, 1)$ next to the A. Once Gary finished his explanation, Janet exclaimed, "Oh, you go down first" realizing that matrix cell locations were

written differently than graphical coordinates. Abby had an epiphany and proclaimed, "This is like a graph, but the y comes before the x." The instructors felt that after Gary's explanation and the ensuing comments there was a consensus among the students as to how cell locations in matrices are labeled using an ordered pair. They realized that in the ordered pair, the first number identifies the row and the second number identifies the column.

2.0.3 Assessing Student Learning

To assess their understanding of cell locations, the instructors decided to have students send each other a message using the Tap Code. Students wrote the tapping sequence of their message on their white boards. Then the instructors called on volunteers to tap the message for the class to decode. Karl tapped (•• •••), (•• ••••). The class quickly decoded the message as "Hi." Amy tapped (• ••), (•• •), (• •). The class correctly interpreted this message as BFA which stands for 'best friend always.' Lara tapped (•• ••), (••• ••), (••• ••), which the class determined was GMM. The instructors had no clue what Lara meant by GMM. The class explained that GMM was the acronym for the YouTube show "Good Mythical Morning." After these examples, Mr. Wayne decided to wrap up the lesson with an exit ticket. Before giving the students their exit ticket, Mr. Wayne told the class he was going to tap a message that John McCain often communicated to his fellow POWs before bedtime. He tapped (•• ••), (••• •••). The class responded by saying "Good night."

3 Data from Exit Ticket

At the conclusion of the lesson, the instructors provided the students with an exit ticket (see Appendix A). The purpose of the exit ticket was to assess the students' knowledge of how cell addresses are assigned in matrices (Questions 4 and 5) and to determine their perceptions about the overall lesson (Questions 1, 2, and 3).

Question 4 on the exit ticket asked students to determine the location of the number 8 in the given matrix. The results showed that 20 out of 22 students correctly identified the cell location as $(2, 3)$. The remaining two students answered the question with the dot representation used during the lesson to identify a tapping sequence. Specifically they wrote ".. | ..." as their answer. For Question 5, the students were asked to identify the number at cell location (1, 3). Twenty-one out of 22 students accurately stated the number was "7". The other student answered "$(2, 1)$ and $(1, 1)$" which corresponds to the cell locations in the matrix for the "1" and "3" listed in the ordered pair $(1, 3)$. The number of correct responses for these two items on the exit ticket show an overwhelming majority of the students mastered the primary objectives of the lesson.

The data from Questions 4 and 5 indicate students learned about the structure of matrices. However, students' responses to Question 1 were surprising. Question 1 asked "What did you learn?" All student responses focused on the context of the lesson and none mentioned content related to matrices. Twenty-one students indicated something similar to "I learned the Tap Code" or "How to tap out secret messages." The other student stated that she learned about "new abbreviations." The data from Question 1 reflect the high level of motivation students had towards the context of the lesson. Math content was learned, but the context was the driving force that enabled students to make connections. The marrying of context and content resulted in a vibrant, successful lesson.

Question 2 asked students to identify what they found most interesting in the lesson. For half of the students, the historical context was what they found most interesting. Mary found it interesting that "Soldiers used this *language* in war." Abby answered, "That while POWs were in their cells they used this code to communicate and pass time." Other items of interest for the students included

the use of abbreviations and acronyms to shorten messages, the reason C and K occupied the same cell, and the overall process for decoding Tap Code messages. For Question 3, students identified what they were still unsure about. Most of the responses related to the historical context of the lesson. Students wanted to know how POWs were able to memorize the matrix and quickly send and decode messages. None of the students indicated that they were unsure about the structure of matrices or the naming of cell locations.

4 Conclusion

The results of our exit slip showed students met the desired objectives in which they learned about the structure of matrices. One reason this lesson was successful was that students were motivated by the historical context. The results of the exit slip indicated that learning about John McCain and the history of the Tap Code was the most interesting part of the lesson. The connection to historical contexts directly corresponds to recommendations that mathematics should be applied to outside contexts and connections should be made between mathematics and the real world (NRC, 1990; 1998; Drier, Dawson, & Garofalo, 1999; NCTM, 2014).

Throughout instruction students engaged with many of the *CCSS Mathematical Practices* (NGA Center and CCSSO, 2010). Students constructed and presented viable arguments when explaining their answers (MP3). Amy used a compelling argument to explain her *total taps* strategy. Similarly, Abby's explanation of her *graphing* method seemed perfectly logical. Both of these approaches, although invalid, led to plausible answers. The instructors purposefully presented questions where multiple solution methods may have seemed appropriate.

Eventually, the instructors presented questions that challenged the validity of students' incorrect solution methods. When students arrived at incorrect answers, they were forced to assess their progress and make changes in their approach (MP1). When Abby tried to determine where Mr. Wayne attended college, her graphing strategy led to the answer DZV. Abby questioned whether the answer made sense in the context of the problem. She asked whether the first set of taps corresponded to the x or y coordinate. Gary helped his classmates understand the pattern and structure for decoding messages sent using the tap sequence (MP7). He explained how the first set of taps indicated the row number of the matrix, while the second set of taps pointed to the column number. The precision with which Gary explained his strategy (MP6) enabled others to see the structure associated with naming cell locations in a matrix.

The combination of historical context and mathematical content made for a valuable lesson where students met the goals that were established. Students enjoyed learning how John McCain and other POWs used the Tap Code to communicate with each other. In the process, the students learned about the structure of matrices. This was all possible because the classroom environment encouraged students to engage in a variety of *CCSS Mathematical Practices* (NGA Center and CCSSO, 2010). The success of this teaching episode demonstrates mathematical concepts do not necessarily have to be the primary focus of a lesson. Often teachers feel the emphasis of the lesson needs to be on the content, while superficially mentioning real-world connections (Gainsburg, 2008). This teaching episode shows it is perfectly acceptable to make *context* the heart of instruction when creative connections are combined with relevant content and good teaching fundamentals like the *CCSS Mathematical Practices* (NGA Center and CCSSO, 2010).

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APPENDIX A: Exit Ticket

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Name _____

JOURNAL. *Think about what we did in today's lesson.*

1. What did you learn?

2. What did you find most interesting?

3. What are you still unsure about?

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4. What is the location of the number 8?

5. What number is in cell location (1, 3)?