Rewriting the Script

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Abstract: *This paper outlines a Math Design Collaborative (MDC) lesson with a focus on conditional probabilities attempted in my classroom. The paper then summarizes the analysis of the outcomes of the lesson from my colleagues and the subsequent revisions made to the original lesson plan.*

Keywords: *conditional probabilities, Math Design Collaborative*

1 Introduction

As an educator, the following scenario is a tale as old as standardized testing: I found myself in the midst of end-of-course tests with large block classes, low student engagement, and a serious need to revisit topics that had only briefly been covered before spring break. I needed an activity that would entice my tired honors geometry students to engage in a remediation of their formulaic understanding of conditional probabilities.

In an effort to satisfy district curriculum requirements, as well as to fulfill my students' needs, I planned a Math Design Collaborative (MDC) lesson (*editor's note*: a familiar source for MDC lessons is the Mathematics Assessment Project, `https://www.map.mathshell.org/lessons.php`). For those unfamiliar, an MDC lesson can be described as a scripted activity in which students collaborate with their peers to analyze sample student works and come to a group understanding of the given topic. The teacher's role in an MDC lesson is to facilitate learning, not to pass on knowledge. To implement an MDC lesson with fidelity, teachers answer students' questions with thought-provoking questions—never the answer.

My students had been challenged by MDC lessons previously; however, up until this point, we had not attempted one as a source of review. With a total of 62 students, including six who were identified as gifted in mathematics, I embarked on a three-day journey toward student discovery.

2 The Task

I chose to implement *Representing Probabilities: Medical Testing*, which is in the MDC category of "problem solving." In a typical problem-solving MDC, students take a pre-assessment, revise their answers with a partner (chosen by their common misconceptions on the pre-assessment), go through sample student responses with their partner, and then bring their ideas to a whole class discussion. The original lesson is available at the Mathematics Assessment Project website (`https://www.map.mathshell.org/lessons.php?unit=9405&collection=8`) (MARS, 2015).

Given the experience I had with the length of MDC lessons, I cut this activity down to a pre-assessment, partner revision, and a sample response analysis. My version of the lesson plan can be found at `http://bit.ly/medicalprob`. For the pre-assessment, students were given the background information shown below in Figure 1.

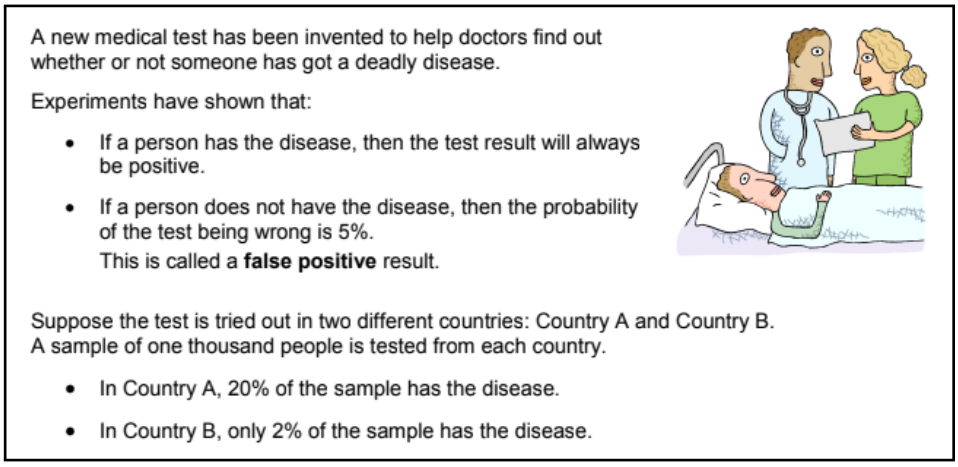


Fig. 1: *Background of medical scenario provided to students on pre-assessment worksheet (MARS, 2015).*

Figure 2 illustrates three student prompts that follow the presentation of background information on the worksheet.

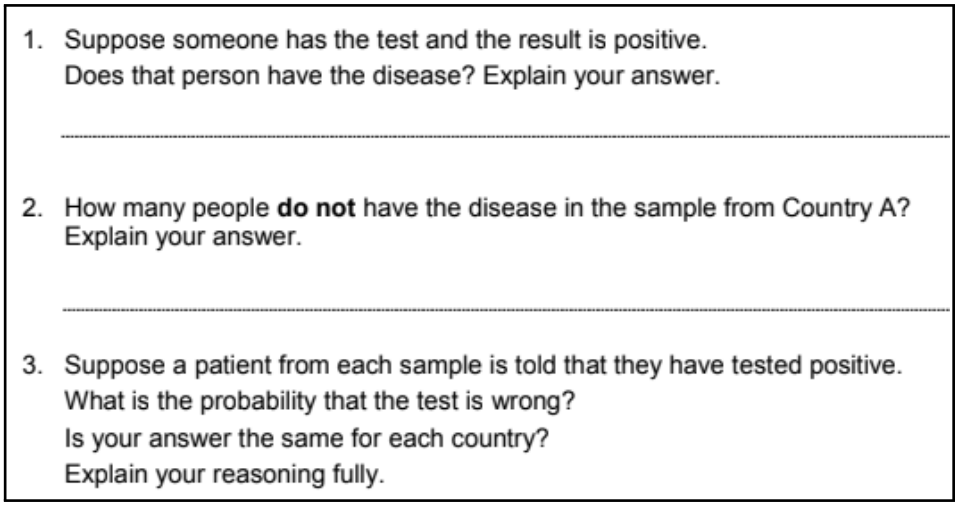


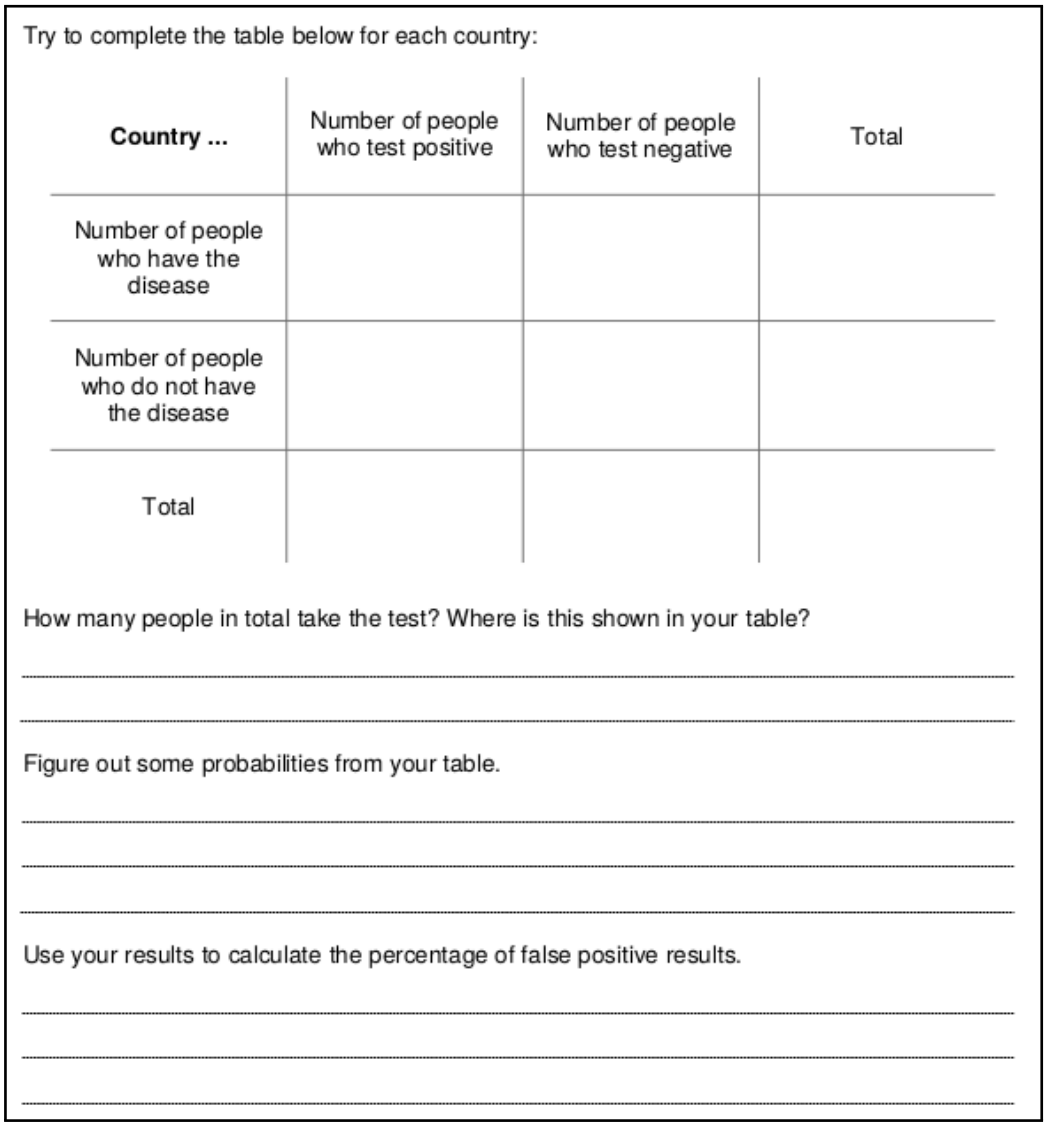
Fig. 2: *Student pre-assessment prompts (MARS, 2015).*

2.1 Day 0: Individual Pre-Assessment

Initially, I had students attempt the Medical Testing Task on their own as a pre-assessment. Students were not able to discuss the task with their peers, nor were they able to ask questions of me (as the teacher). The students were given 30 minutes to complete the pre-assessment. After completion, I took the pre-assessments home and gave feedback according to the lesson's feedback chart. Students were paired by common misconception based on this feedback.

2.2 Day 1: Revising Solutions Collaboratively

For Day 1 of the task, I returned pre-assessments to students with feedback and provided them 10 minutes to modify their original answers. After the 10 minutes, students were placed into pairs based on common misconceptions that I uncovered in their initial work. Pairs were given 30 minutes to compare answers and to try and come up with a better answer to the task. As students were collaborating, I walked around and observed their discussions. If a pair was struggling, I gave them the hint sheet provided in the Mathematics Assessment Project student materials (reproduced in Figure 3).



Try to complete the table below for each country:

Country ...	Number of people who test positive	Number of people who test negative	Total
Number of people who have the disease			
Number of people who do not have the disease			
Total			

How many people in total take the test? Where is this shown in your table?

Figure out some probabilities from your table.

Use your results to calculate the percentage of false positive results.

Fig. 3: *Student hint sheet (MARS, 2015).*

If a pair was exceeding expectations, I gave them an extension question. At the end of Day 1, students handed in their materials without confirmation of whether their answer was correct or not.

2.3 Day 2: Analyzing Sample Responses

At the beginning of Day 2, students were given back their original answer sheets. Pairs were given several sample student responses highlighting common misconceptions with the three pre-assessment prompts. An example is reproduced in Figure 4.

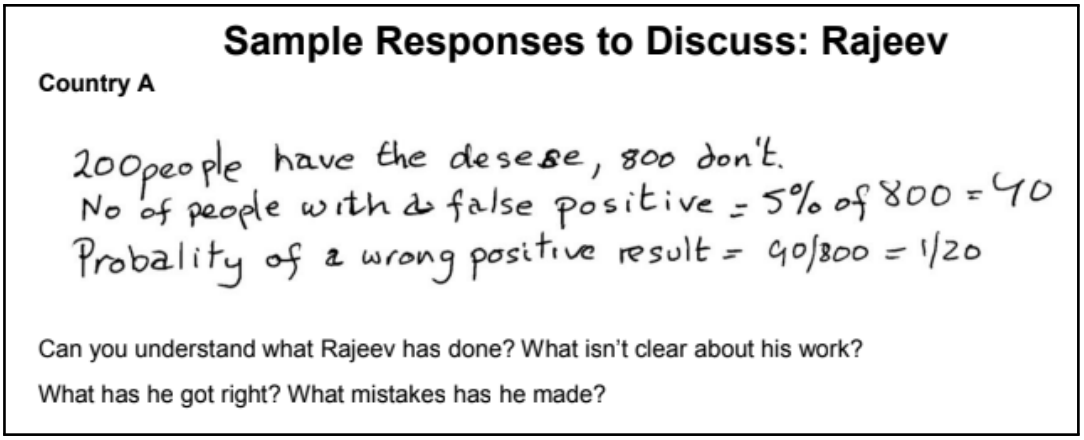


Fig. 4: *Sample student response (MARS, 2015).*

(Editor's note: All sample responses are accessible in the Materials section of the original lesson plan (`https://www.map.mathshell.org/lessons.php?unit=9405&collection=8`) (MARS, 2015)). Students were also provided with an analysis sheet (i.e., blank worksheet with spaces) for each sample response.

For 40 minutes, students worked through the sample responses and wrote down their thoughts on the analysis sheet. It was okay if a pair did not get through all 4 sample responses. I again walked around and observed discussions. At no point during Day 2 were students told if they were correct or not. At the end of Day 2, I collected their work and gave feedback. If an answer was correct, the student pair was told. If an answer was partially correct or not correct, I facilitated a discussion with them on Day 3.

3 Student Data and Work Analysis

While giving feedback on the students' pre-assessments, I noticed a few recurring misconceptions.

3.1 Misconception 1: Overlooking False Positives

The first, shown in Figure 5, was a common answer to question 1.

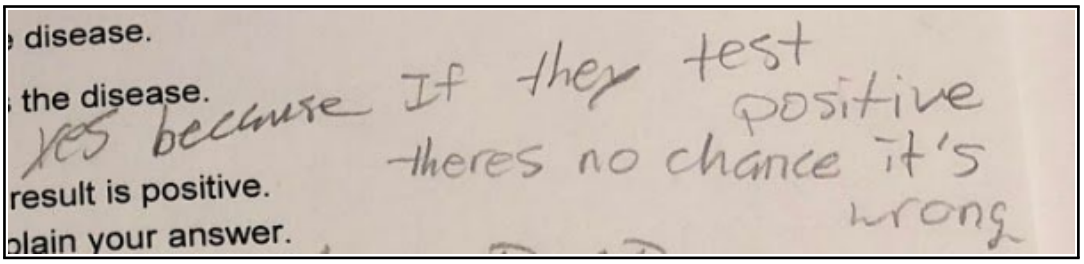


Fig. 5: *An example of student work that doesn't take the possibility of a false positive into account.*

The students who incorrectly answered this question either did not fully grasp the concept of a false positive result or forgot about the possibility of one altogether. This misconception was given the feedback of an arrow pointing to the definition of a false positive in the background information.

3.2 Misconception 2: Overlooking Conditional Probabilities

The second misconception was one of two common answers to pre-assessment question 3. In Figure 6, we see one of many students who mistook the probability of a false positive with the probability that the test is wrong. These students included the number of subjects who tested negative for the disease in their calculations, *despite the fact that those who have the disease will always test positive* (hence they will not contribute to any wrong test outcomes). Feedback given to this misconception was in the form of the question, "What does the phrase *the probability that a test is wrong* mean?"

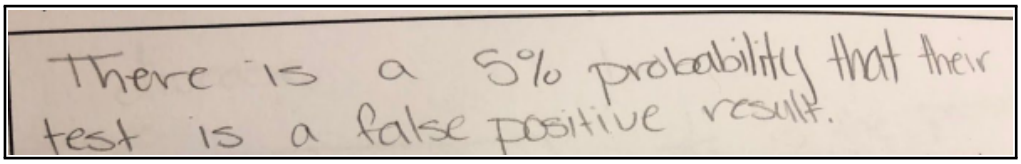


Fig. 6: *Mistaking probability of false positives with probability the test is wrong.*

The other common answer to question 3 was the student finding the percent of the population that does not have the disease (the probability that the test is negative) and thinking they had the probability of the test is "wrong." This type of thinking is illustrated by the example in Figure 7.

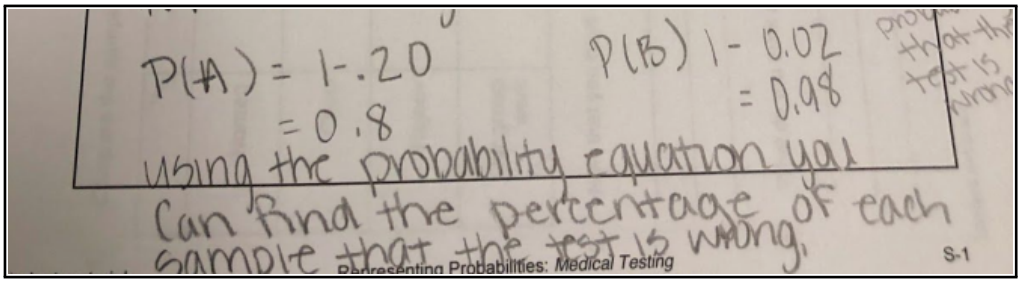


Fig. 7: *Confusing the probability that the test is negative with the probability that the test is wrong.*

3.3 Correct solution with Tables

Out of the 62 students, only 5 of them correctly answered question 3. One such response is provided in Figure 8 (left). Notice that the strategy that the student implemented in creating the table is similar to that highlighted in a sample response from Day 2 (right).

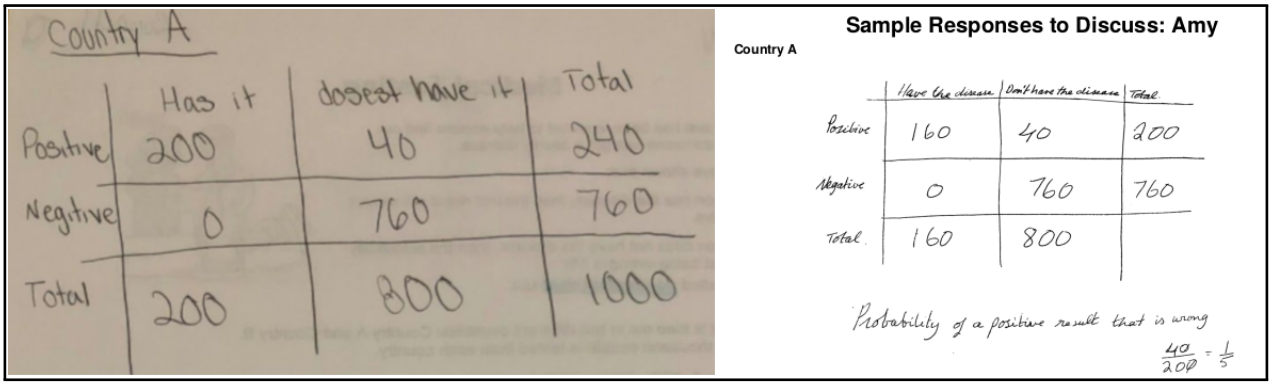


Fig. 8: *(Left) Correct solution submitted by student; (Right) Sample response item from Day 2 of lesson.*

3.4 Collected Data

Table 1 summarizes student performance on pre-assessment questions 1–3 at the beginning of the lesson (Day 0) and after revisions (Day 3).

Table 1: *Student performance on Questions 1–3.*

	Number of Correct Responses	
	Pre-Assessment (Day 0)	**Post-Assessment (Day 3)**
Question 1	34	62
Question 2	49	62
Question 3	5	62

As Table 1 indicates, by the end of Day 3, all 62 students had shown mastery of the task either through conversations with me or through their turned in analysis of the sample responses.

4 Revisions

I had the unique opportunity to share this lesson and its outcomes with 12 colleagues during a summer master's course. Each of us had implemented some sort of open math task in our classrooms prior to the start of the class. We each gave a presentation showing our lesson, student videos, samples of student work, and collected data. After each of our presentations, we studied relevant literature and helped devise revision ideas for each lesson.

After the presentation of my lesson, we read "Making Room for Inspecting Mistakes" (Lischka et al., 2018) and "Conditional Probability: Its Place in the Mathematics Curriculum" (Watson, 1995). Lischka et al. (2018) reiterated the importance of error analysis in helping students fully grasp concepts for me and my colleagues. Much of the learning that took place in the original lesson happened during the critique of sample student work in Day 2. My students formulated better answers to question 3 after they worked through and made sense of the thought processes of other students. Lischka et al. (2018) prompted me to include work samples of my own students in Day 2 along side canned examples provided by the authors of the Mathematics Assessment Project lesson.

Watson (1995) encouraged me and my colleagues to use scenarios beyond medical testing to solidify the concept of conditional probability for my students. Misconceptions can be avoided by swapping out the medical testing scenario with topics that are more relevant to my students lives.

4.1 Resources

A list of resources—including all versions of the conditional probability lesson plan—are provided below.

- Representing Probabilities: Medical Testing (MDC Official Lesson Plan, includes necessary student materials), `https://www.map.mathshell.org/lessons.php?unit=9405&collection=8`
- Initial Lesson Plan (My lesson plan, adapted from the MDC Official Lesson Plan), `http://bit.ly/medicalprob`
- Revised Lesson Plan (My lesson plan, revised from the ideas discussed with my colleagues during the summer course), `http://bit.ly/medicalprobrevised`

4.2 Revision Descriptions and Rationale

Table 2 lists proposed revisions to the official MDC lesson plan inspired by conversations with colleagues in our summer class along with a rationale for each.

Table 2: *Proposed revisions with rationale for each.*

#	Description of Revision	Rationale for Revision
1	Make it very clear in the background information that the 20% from Country A and the 2% from Country B are results from another, more accurate, test; that those people without a doubt have the disease in question.	This revision came about due to several students (and a few of my colleagues) incorrectly assuming those percentages included the false positives.
2	Grouping students by mixed misconceptions and differing levels of achievement.	Giving students the ability to collaborate with their peers who have different ways of thinking about the problem would increase student understanding. It would also give an opportunity for peer editing.
3	Rather than just using the sample student responses provided in the original task, gather a sampling of student work from the pre-assessment and use this sampling in addition to the error analysis portion of Day 2.	Students would see that their peers made similar mistakes as they did, or different mistakes. This would lead to rich conversation about the different ways in which to solve these problems.
4	Have a whole group discussion at the end of the error analysis portion.	This would allow students to discuss their thoughts and discoveries about the errors in the sample student responses.
5	Adding in a few extensions for students who correctly answer the task.	Students could be presented with the scenario of a false negative and have to discuss the implications to the answer. Students could be presented with another scenario (something closer to their interests than medical testing)

5 Reflection

Discussing this lesson and its outcomes with my colleagues was a refreshing way to look at my own teaching practices. Not only was I able to discover nuances in a scripted plan that warranted revision, I was able to revise the plan according to the research and the misconceptions of my students. Next year when I feel the standardized testing month pressure, I will look at these revisions and perhaps I can modify another activity that will peak my students' interest.

References

Mathematics Assessment Resource Service (MARS) (2015). CLASSROOM CHALLENGES A Formative Assessment Lesson—*Representing Probabilities: Medical Testing*. Available on-line at `https://www.map.mathshell.org/lessons.php?unit=9405&collection=8`

Lischka, A.E., Gerstenschlager, N.E., Stephens D.C., Barlow, A., Strayer, J.F. (2018). Making Room for Inspecting Mistakes. *Mathematics Teacher, 111*(**6**), 432–439. Retrieved from `http://www.jstor.org/stable/10.5951/mathteacher.111.6.0432`

Watson, J. (1995). Conditional Probability: Its Place in the Mathematics Curriculum. *Mathematics Teacher, 88*(**1**), 12–17. Retrieved from `http://www.jstor.org/stable/27969160`

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