Teaching Two-Way Frequency Tables Through Misconceptions

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Abstract: *During my first year of teaching, I thought I was alone in suffering through awkward moments as I made content mistakes in front of my students. The more I experienced these instances, the more I was able to strengthen my understanding of content and pedagogy. Why not provide my students with similar experiences to deepen their understanding of mathematics? In this article, I discuss how I used a misconception I had about two-way frequency tables to help students achieve a better understanding of how the variables in two-way frequency tables are dependent on each other.*

Keywords: *statistics, two-way frequency tables, misconceptions*

1 Introduction

The dreaded moment . . . you know the one I am talking about. You are at the front of the room excited about a problem that you are working through with your students, and then seconds before your students point it out, you realize that you have done something *majorly* wrong. During my first year of teaching, I thought I was alone in suffering through these awkward moments. The more I experienced these instances, the more I was able to learn about various mathematical concepts and about teaching. One day I decided that a misconception that I had about creating two-way frequency tables could be turned into a great lesson. I found a way to put my students in the same spot I was in when I froze for a second at the front of the classroom as I realized I had done the problem incorrectly. My goal was to use this process to help students achieve a better understanding of how the variables in two-way frequency tables are dependent on each other. After teaching this lesson, I shared it with colleagues in the masters program that we are enrolled in. We then collaborated to turn a lesson driven by misconception into one that places a large focus on students articulating their mathematical thought processes and critiquing the processes of others.

2 Context

This past spring, I spent just over a week teaching a probability unit in my Integrated II classes. Probability was the last unit covered before the state test, and as a result, my students were brain-fried, tired, and done learning new content. With 33 ornery boys and 25 girls focused on prom dresses, convincing them to engage in the lesson to help improve the school's 43% pass rate on the math End-of-Course (EOC) exam was not an easy task. In an effort to change the pace of our everyday routine, I intentionally set my students up to fail. My classes of tenth graders had learned how to create two-way frequency tables the previous year in math. Given the time of year and the repetition of material, I was hoping that my students would come into the lesson knowing how to handle the material. I was thrilled to see that many of them did and pleasantly surprised to see some of them immediately start thinking beyond what they had learned the year before.

3 The Problem

Students were asked to first find a partner and work together to create two survey questions. These questions would be used to survey the class and then create a two-way frequency table.

ME : Could I ask "What is your favorite college football team?" for one of my questions?
LUKE : No.
ME : Well, why not Luke?
LUKE : Because obviously Alabama is the only answer, so that would be boring. Me: While that is your opinion, couldn't I argue that OSU or Michigan is better?
GRACIE : You could! Would you have to pick two teams for them to choose between?
ME : Yes, you would! Make sure that both questions that you create for your data collection have two answer choices for your classmates to choose from.

After students created their questions, their task was to interview 15 other classmates and record their data (see Student Handout here: `https://tinyurl.com/two-way-handout`). Once students had their data, they were to create a two-way frequency table to showcase their results. With these directions, my classroom turned into a crazy whirlwind of surveying that resulted in a very noisy ten minutes. After the two-way frequency tables were created in their entirety, partners then began to create their own probability questions using their data before answering questions that I created for them about their results.

This lesson was an introductory lesson to the probability unit in an Integrated II curriculum. It was designed to address Common Core State Standards (CCSS) connected to conditional probabilities and two-way frequency tables (i.e., S.CP.4). In addition to addressing the CCSS, the lesson had a large focus on the construction of viable arguments and critique of others (Standards for Mathematical Practice (SMP) MP.3). Students later explored *and*, *or*, and conditional probabilities using two-way frequency tables as a tool to organize data. The skills that students learn while productively struggling through the activity help to show them that it is all right to fail. This is the same lesson that my students taught me. My tenth graders helped me to see that learning from the mistakes you make is what matters. Allowing my students to fail provides my students with a meaningful way to explore familiar topics. My tenth graders saw that making a mistake does not mean a problem cannot be solved. Instead, it means that a different method could make a huge difference.

4 Student Work

Students had a variety of methods for collecting data addressing their two survey questions. Some approaches proved to be more advantageous than others, and many others led to very interesting conversations.

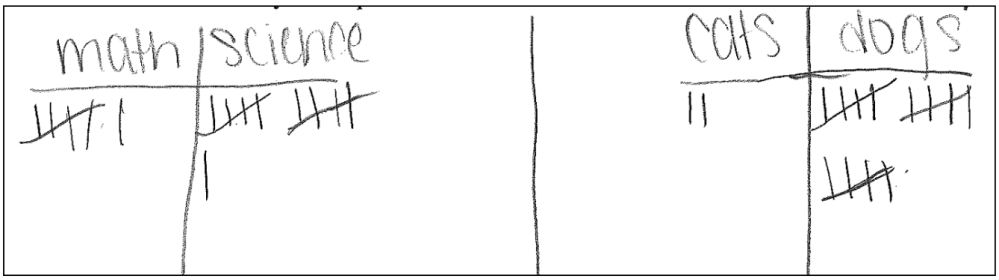

Fig. 1: *Sample collected data.*

Figure 1 shows the most common method that my students used across the three classes that I taught. You may be wondering what is wrong with this approach. I personally wondered the same thing when I collected data using this method in a whole class activity previously in the year with my Integrated I students. Figure 2 showcases how students were able to use the data collection above to complete their two-way frequency table.

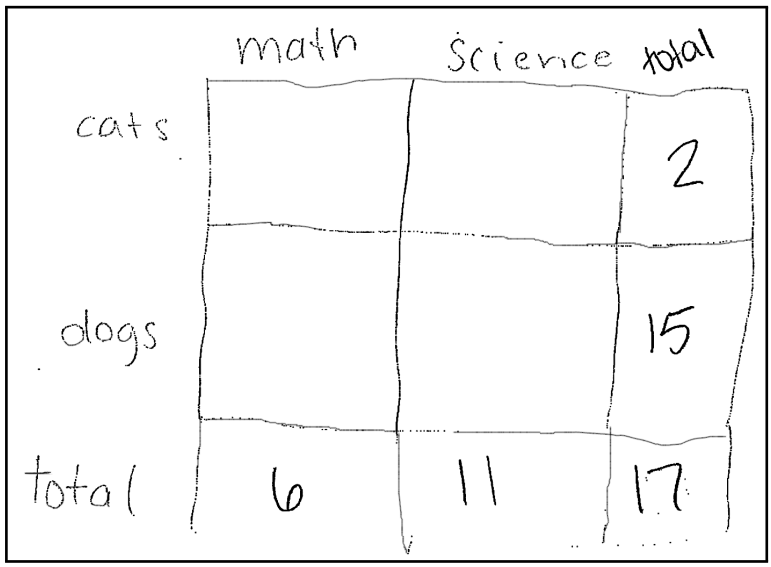

Fig. 2: *Incomplete two-way frequency table.*

Upon seeing this, the ah-ha moment finally happened for many of my students.

SARAH : Um . . . Mrs. Jakes . . . Something isn't right here.
ME : What do you mean, Sarah? It looks like you have a pretty good two-way frequency table there!
SARAH : Well I can't fill out the middle.
ME : Why not?
SARAH : I don't know! I asked the two questions just like you told us to but that only gave me the totals. I don't know who liked cats and math or who liked dogs and science. Wait . . . I should have asked a person both questions at once and written down both of their answers, shouldn't I??
ME : What do you mean?
SARAH : When you said you like dogs and math I should have written down "Mrs. Jakes—math/dogs" as one answer instead of tallying them separately. Then I could fill out the middle!
ME : That does sound like a better plan. What do you think you need to do now?
SARAH : Aw, come on. Do I really have to go re-ask *everyone*?
ME : You sure do! See how important it is to collect data based on what you plan to do with it?

This conversation was repeated many times throughout the day. Each time students surprised me as they came to the relationship between the variables on their own. The mathematical discourse partners had as their work went from Figure 1 to Figure 2 was amazing!

I had several students come up with unique and creative ways to collect their data to ensure they only had to ask their classmates their survey questions once.

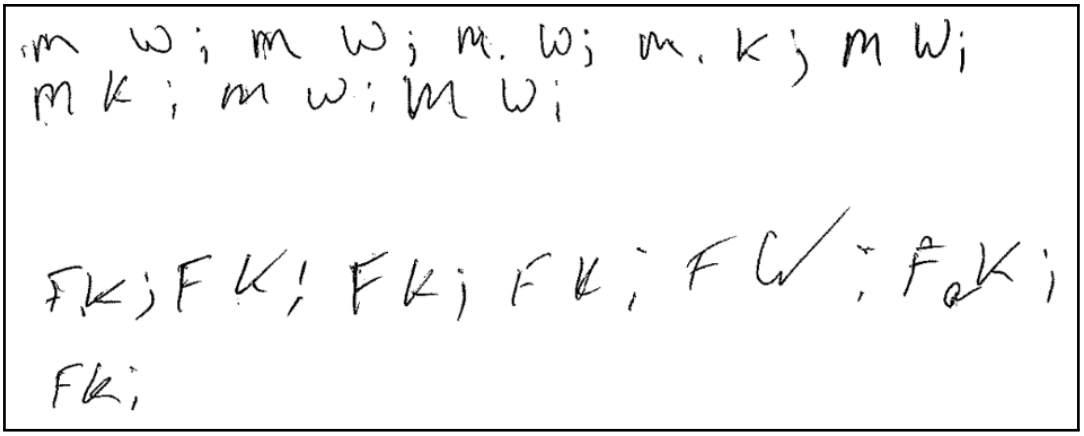

Fig. 3: *Inventive data recording method proposed by one group of students.*

In Figure 3, a group was asking peers if they preferred Kroger or Walmart and also recording gender as their other variable. An entry of "FW" signified a female student preferring Walmart, while an "MK" stood for a male answering Kroger. When these students created their table, they counted how many MW's appeared in their data collection and then recorded that number (6) in the corresponding position for males that preferred Walmart in their table, as seen in Figure 4.

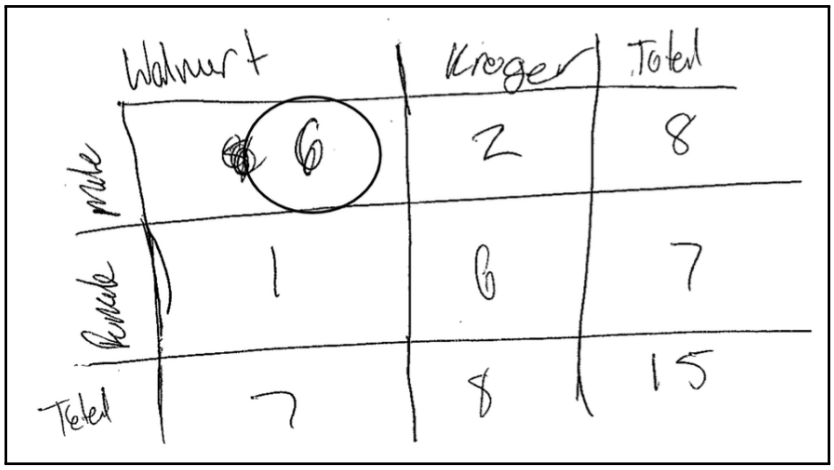

Fig. 4: *Two-way frequency table corresponding to data collected in Figure 3.*

The two gentlemen that collected this data explained to me that it "just made sense" for them to write down the responses together. When pressed for more details, they shared that each person they surveyed needed to be a single entry into their table, so they thought it would be easier to treat their response as one entity.

The most common form of data collection by the end of the lesson became a tally mark system included inside the two-way frequency table as seen in Figure 5.

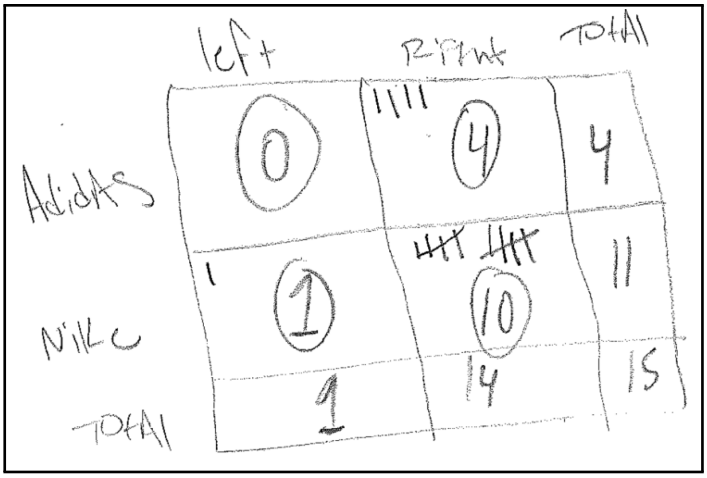

Fig. 5: *Tally mark system within two-way frequency table.*

Several groups used this approach at the beginning of the lesson. These students admitted that they chose this approach to save time. By collecting data in this way, they only had to draw one table on their worksheet rather than two. A handful of students used other methods that resulted in successful data collection the first time they surveyed their peers. A summary of methods is provided in Figure 6.

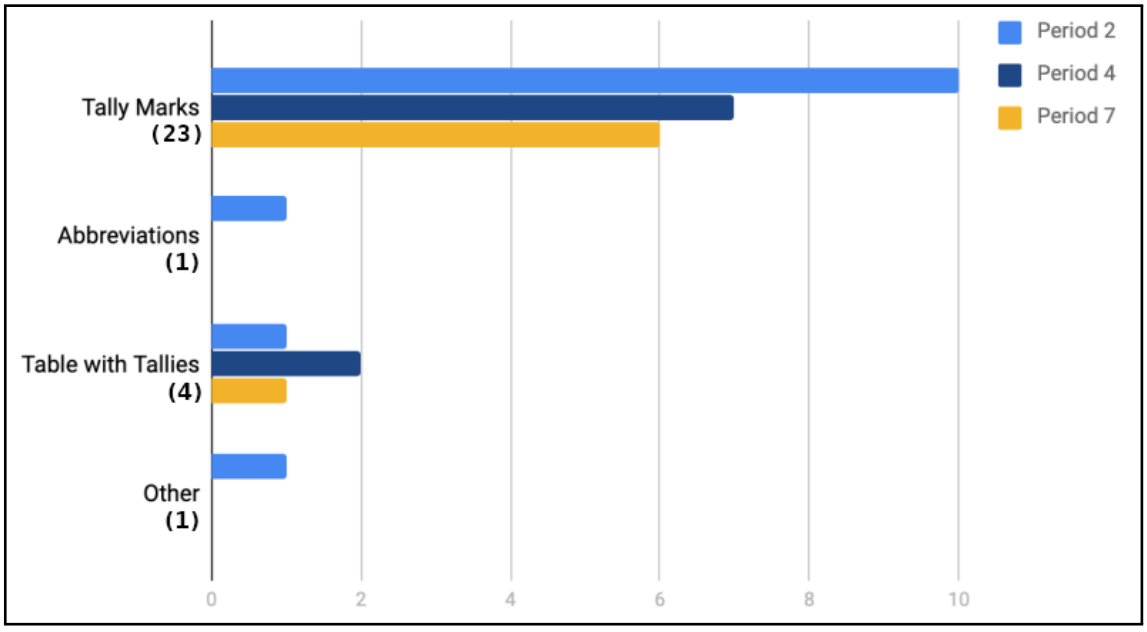

Fig. 6: *Frequency of student data recording methods.*

During the second round of data collection that many groups participated in, filling a two-way frequency table with tally marks became the optimal method. Students realized that the data recollection process was easier than they thought. When data was collected using a more advantageous method, constructing the table was much easier than anticipated. This presented the class with a brief discussion on the importance of choosing the best tool for the problem.

5 Lesson Revisions

5.1 Incorporating More Choice

In addition to learning from my students during the lesson, I received more insight on ways to improve my lesson when I shared my findings with teaching colleagues. We spent considerable time discussing (and debating) student-centered instruction. The American Institute for Research (AIR) defines student centered learning as "an approach to learning in which learners choose not only what to study but also how and why that topic might be of interest" (TEAL Center Staff, 2010). This particular quote promoted a lively debate that ended with a colleague making a very interesting observation—namely, that there are different levels and interpretations of student-centered learning. All of them prioritize the interests of students and their freedom to choose. Granting students the option to construct their own survey questions provided an element of choice based on their own interests. When students collected data the second time, they did not have a similar mathematical choice. I only exposed them to the tally marks within the two-way table as an alternative method. Rather than doing this, my peers suggested that I allow students to provide their peers with options. In my revised lesson plan, students record various methods for collecting data on a whiteboard after the first data collection. This provides students with an opportunity to compare approaches as they discuss the relative merits of various table production methods. The addition of this discussion makes the lesson more student-centered. Students choose to implement a data recording method of their choice, as long as the technique yields a complete two-way frequency table. The lesson provides an engaging way for students to interact both with each other and the material. Most notably, the lesson allows for both personal and mathematical choice to drive what students delve into along with how they collect their data (TEAL Center Staff, 2010).

5.2 Capturing Student Reflections

Much of the rich discourse that was happening around my classroom as students began to see the flaws in their data collection went uncaptured in the writing prompts I provided to students. When asked to reflect on the lesson towards the end of class, students had all but forgotten how exciting it was to learn something new (or were unwilling to commit their memories to paper). Asking students to write in the mathematics curriculum can be a challenge that I am sure all teachers face. To work around this struggle, a fellow teacher suggested the use of voice typing in Google Docs. This tool can be found in the Tools menu in a Google Doc or can be accessed on a Mac by using the shortcut `Command+Shift+S`. When the tool is enabled, a red speaker appears on the left side of the Docs window. As soon as you begin to speak, words appear in your document. To turn the tool off, simply click the red button again. We all know that students do not seem to have a problem talking in class. To help my students capture their thought process, my revised lesson has them use the voice typing tool to record their conversations as they complete their student handout. A link to a revised version of the lesson is provided here: `https://tinyurl.com/two-way-revised-lesson`. Once students have finished the activity, they turn off the tool and read through their script. At this point they will be able to highlight and delete portions of the script that are not related to the following questions they are asked to consider.

1. What conversation did you and your partner have as you were attempting to create your two-way frequency table? Include the script of your conversation below.
2. Why did you choose the two survey questions listed on your worksheet?
3. Do you think there is a relationship between your two questions? (Is an Ohio State fan more likely to choose chocolate ice cream?)
4. If we surveyed the entire school using your questions, do you think there would be a relationship? Why or why not?

I anticipate that voice typing will encourage students to answer my prompts more completely and will provide me with greater insight into each student's understanding. The addition of questions that encourage my students to consider relationships between variables will challenge them to shift their thinking from a quantitative approach to more abstract reasoning (Shore, 2017).

6 Final Thoughts

An early mistake on my part has ultimately yielded an engaging lesson for my students. Setting my students up to fail in the same way that I did provided the with an "ah-ha" moment that stuck with them. Weeks later, on our EOC exam, I saw many students scribbling rough two-way frequency tables as they attempted to make sense of various statistics questions. This alone was enough to convince me that using mistakes as a way of instruction was beneficial. The research and wisdom of my peers allowed me to take this lesson and enhance it in more ways than I could have ever thought of alone. Having students share their approaches to the task will create a wonderful dialogue in my classroom next year. The amazing voice typing tool will be a huge asset in my classroom as well. I challenge you to go through a similar process with a lesson that you feel has the potential to be great. Take the time to research how fellow educators teach the topic. The bigger challenge is to find a colleague or a group of colleagues to share your lesson with. Take the step to see how great your lessons could become if you collaborate.

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