Digital Resources of "Floor to Ceiling" Tasks

Debora Kuchey, *Xavier University*
Michael Flick, *Xavier University*

Abstract: *The authors elaborate upon the National Council for Teachers of Mathematics' standards and describe how mathematical rich tasks found on various websites can help support those standards. The authors describe many of these websites and what they have to offer to teachers and students.*

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"Imagine a classroom, a school, or a school district where all students have access to high quality, engaging mathematics instruction... Where students confidently engage in complex mathematical tasks chosen carefully by teachers. They draw on knowledge from a wide variety of mathematical topics, sometimes approaching the same problem from different mathematical perspectives or representing the mathematics in different ways, until they find methods that enable them to make progress... Orally and in writing, students communicate their ideas and results effectively. They value mathematics and engage actively in learning it." (NCTM, 2000)

This was the vision of the National Council of Teachers of Mathematics, published in 2000. It is nearly 20 years later, and the question remains, is this vision being realized in your math classroom? On your math team?

The Revised Ohio Learning Standards for Mathematics, 2017 include the Eight Standards for Mathematical Practices (ODE, p. 3):

> The Standards for Mathematical Practice describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments, and professional development should all attend to the need to connect the mathematical practices to mathematical content in mathematics instruction that specifies how students should engage in mathematics.

These standards ensure that mathematics is more of a challenge to the minds of students than the memories of how they were told to do math.

The Eight Ohio Standards for Mathematical Practices are:

- SMP 1: Make Sense of Problems and Persevere in Solving Them
- SMP 2: Reason Abstractly and Quantitatively

- SMP 3: Construct Viable Arguments and Critique the Reasoning of Others
- SMP 4: Model with Mathematics
- SMP 5: Use Appropriate Tools Strategically
- SMP 6: Attend to Precision
- SMP 7: Look for and Make Use of Structure
- SMP 8: Look for and Express Regularity in Repeated Reasoning

How do you ensure that your students are engaging in mathematics using these practice standards? Can teachers rely on traditional texts to present mathematics using an inquiry method? Can a text engage students in rich mathematical discourse? Can online programs provide students with multiple ways in which their peers approach a given problem? If not, why are you still relying on these teaching methods? How can we make mathematics meaningful to students? How can we turn students on to mathematics?

The National Council of Teachers of Mathematics published Principles to Action: Ensuring Mathematical Success for All Students in 2014. They developed eight Mathematics Teaching Practices that should be the foundation for mathematics instruction and learning. This framework was informed by over twenty years of research. If teachers are guided by this framework, they can move "toward improved instructional practice" and support "one another in becoming skilled at teaching in ways that matter for ensuring successful mathematics learning for all students" (NCTM, 2014, p. 12).

NCTM's eight mathematics teaching practices are:

1. Establish mathematics goals to focus learning.
   Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.

2. Implement tasks that promote reasoning and problem solving.
   Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.

3. Use and connect mathematical representations.
   Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.

4. Facilitate meaningful mathematical discourse.
   Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.

5. Pose purposeful questions.
   Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.

6. Build procedural fluency from conceptual understanding.
   Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.

7. Support productive struggle in learning mathematics.
   Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.

8. Elicit and use evidence of student thinking.
   Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

Lastly, it is important that teachers consider the task and prepare for each of the Practices for Orchestrating Productive Mathematics Discourse (Smith & Stein, 2018). These practices ensure that the teacher considers important aspects of the task that will assist them in engaging students in productive discussions.

First. the teacher must plan clear learning goals. Secondly, the teacher needs to anticipate how the students might tackle the problem, what strategies they might use, and how this will connect with the goals of the lesson. It is important that the teacher monitor the students as they progress through the different parts of the task. If the teacher makes a table of possible strategies used during the anticipation stage, this table can be used to document the strategies students use as they tackle the problem. Then the teacher should strategically sequence who will share their work and when they will share it. It is important that different students are involved in sharing out, so the teacher must keep track of who shares and when. It is important to begin with the most commonly used strategies, followed by the most concrete approaches to the most abstract approaches. The goal should be for the different presentations to build on one another and develop powerful mathematical ideas.

Selecting and sequencing student work with cognitively demanding tasks in a group environment can teach important mathematical ideas (Nabb, Hofacker, Ernie, & Ahrendt, 2018). The following websites provide rich mathematical tasks that ensure all students are actively engaged in learning mathematics. These tasks can be referred to as "floor to ceiling" tasks, as they have multiple entry points and are accessible to all students. These tasks can be used in any math classroom as well as preparing individual and teams of students for mathematics competitions.

When Math Happens (Dan Meyer) - 3 Act Task: https://whenmathhappens.com/3-act-math/. This site can be used to access rich mathematical tasks designed by Dan Meyer. Links are included to numerous other tasks designed by other educators. These tasks in their totality span grades K-12 and beyond.

> A great task that connects to the current sales of Girl Scout Cookies can be found on the following website: https://blog.mrmeyer.com/2016/3acts-nissan-girl-scout-cookies/. Students seek to determine how many boxes of Girl Scout Cookies are needed to fill the back of a Nissan Rouge trunk.

Open Middle-Challenging Math Problems Worth Solving (Nanatte Johnson, Robert Kaplinsky, Bryan Anderson, Dan Luevanos and Zac Miller) - Open Middle Task: http://www.openmiddle.com/. This site contains tasks from kindergarten through high school mathematics. Everyone starts with the same problem and comes to the same conclusion. The difference lies in the middle, in how students solve the problem.

> A fun order of operation task that allows students to fill in the boxes with the digits 0 - 9 to make the largest or smallest possible solution can be found at the following link: http://www.openmiddle.com/order-of-operations-2/

Which One Doesn't Belong (website created by Mary Bourassa): http://wodb.ca/index.html. The problems presented on this website are unique. The uniqueness lies in the fact that each of the

four items could be chosen as the one that does not belong. It is important for students to share the reasoning for which the item was chosen as not belonging with the other items. The tasks are arranged according to standards and grade level.

Would You Rather: http://www.wouldyourathermath.com/. Prompts are provided and students are asked to choose between two options. Students are required to justify their choice. These prompts and the justifications encourage rich discourse.

Estimation 180: http://www.estimation180.com/. The tasks on this website includes many visuals which lead students to use their prior knowledge to come to an estimation that is greater than their lowest estimate and less than their greatest estimate. Justification for estimations are required. These estimation tasks encourage students to partake in rich mathematical discourse.

You Cubed: https://www.youcubed.org/tasks. This website is manned by a small group of people who are working to get as many free and inspiring math ideas out to teachers and students as possible. Dr. Jo Boaler of Stanford University is the university director of the site. The site includes lesson plans, tasks, videos and Week of Inspiration plans for all grade levels.

Math Pickle: Put your students in a pickle! http://mathpickle.com/. Math tasks and puzzles can be found in the videos on this site that challenge students of various ages.

If you are not familiar with the sites, it is well worth your time to investigate each one. Xavier University hosted a Math Camp this past summer for students entering grades three through six. Graduate students seeking Elementary Math Specialist Endorsements engaged third through sixth grade students in tasks that would allow students to experience the joy of math and build their confidence, instead of focusing on remedial activities and continued drill. Within a day, students became eager to begin the afternoon math session of camp, despite previous hesitation in "having to attend" math camp.

It is time for the 2000 NCTM vision of mathematics to be realized in every classroom. It is time for all students to be given the opportunity to explore mathematics using rich mathematical tasks and engaging in critical discourse. These practices are for all students, not just students talented in math and students who compete on competition teams. Train all of your students to be competition-ready by allowing all students to make sense of mathematics through rich, engaging mathematical tasks, followed by scaffolded explanations and discourse.

Below are three thought provoking questions from the 2019 State Tournament of Mathematics that was held on February 23rd at test centers across Ohio. These problems can be solved in a variety of ways and could be used with teams in the classroom. Answers will follow in the Fall 2019 issue of the Ohio Journal of School Mathematics.

1. Find the sum of all possible values of $\frac{x}{y}$ if $20x^2 - 19xy = y^2$.

2. How many ordered pairs (x,y) of positive integers satisfy 20x + 19y = 2019?

3. The expression $2 + \frac{1}{0+\frac{1}{1+\frac{1}{9}}}$ can be written as the reduced fraction $\frac{a}{b}$. Find a + b.

Fig. 1: *Selected questions from the 2019 State Tournament of Mathematics.*

References

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Debora Kuchey, Ed.D., served as a Teacher Leader in the Kentucky Middle Grades Mathematics Teacher Network for several years. She is currently the College Representative for the Greater Cincinnati Council of Teachers of Mathematics. Dr. Kuchey is an Associate Professor in Early and Middle Childhood Mathematics Education at Xavier University.

Michael Flick, Ph.D., has served the Ohio Council of Teachers of Mathematics as State Contest Coordinator for over 30 years. He has received numerous teaching awards and honors. Dr. Flick is Professor and Executive Director of the Education Centers at Xavier University.