Proof Without Words: Arithmetic Mean / Geometric Mean Inequality

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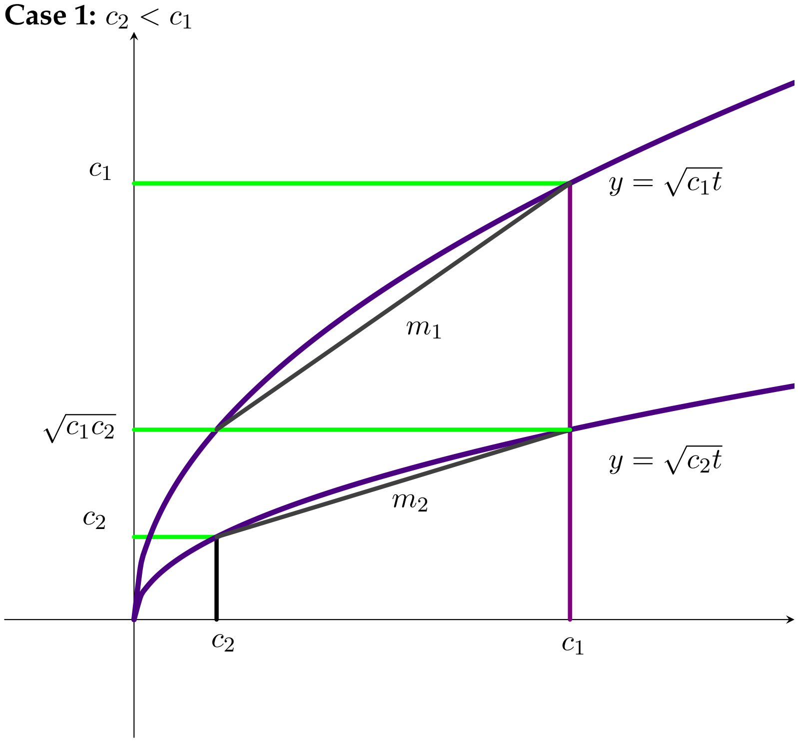
In this proof without words, we prove almost wordlessly the following inequality, if $c_1, c_2 \geq 0$, then $\frac{c_1+c_2}{2} \geq \sqrt{c_1 c_2}$ (AM–GM inequality).

Case 1: $c_2 < c_1$



$$m_1 > m_2 \tag{1}$$
$$\Rightarrow \frac{c_1 - \sqrt{c_1c_2}}{c_1 - c_2} > \frac{\sqrt{c_1c_2} - c_2}{c_1 - c_2} \tag{2}$$
$$\Rightarrow c_1 - \sqrt{c_1c_2} > \sqrt{c_1c_2} - c_2 \tag{3}$$
$$\Rightarrow c_1 + c_2 > 2\sqrt{c_1c_2} \tag{4}$$
$$\Rightarrow \frac{c_1 + c_2}{2} > \sqrt{c_1c_2} \tag{5}$$

Case 2: $c_2 = c_1$

Note that $c_2 = c_1 \implies \frac{c_1+c_2}{2} = \sqrt{c_1c_2}$.

Considering **Case 1** and **Case 2** together, we conclude that $\frac{c_1+c_2}{2} \geq \sqrt{c_1c_2}$.

References

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