Think Like An Egyptian

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Abstract: *We share a lesson designed to motivate an understanding of unit fractions, equivalent fractions, and equal shares via "thinking" like an Ancient Egyptian. The context of Ancient Egyptians provided novel context to help students make sense of fractions. Through their work with fraction tiles and by writing fractions in hieroglyphs, students grappled with multiple representations.*

Keywords: *mathematics history, fractions, multiple representations*

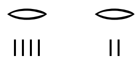
1 Introduction

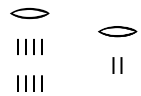
> Once upon a time there were eight hungry Egyptians. Food was scarce, and they only had enough ingredients to make five loaves of bread. How could the Egyptians divide the bread so that each Egyptian got an equal amount?

In fifth grade, students are required to "interpret a fraction as division of the numerator by the denominator ($\frac{a}{b} = a \div b$)" and to "solve word problems ... by using visual fraction models or equations to represent the problem." (CCSSM 5.NF.B.3). For the above problem, students should interpret and visually represent $\frac{5}{8}$ as the result of dividing 5 by 8 and recognizing that when 5 wholes are shared equally among 8 people, each person has a share of size $\frac{5}{8}$. Thus, for the story above, we say that each Egyptian will get $\frac{5}{8}$ of a loaf of bread. Since work with unit fractions—those with numerators equal to 1—begins in third grade, students often perceive the $\frac{5}{8}$ portion that each Egyptian receives as $\frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8} + \frac{1}{8}$ (CCSSM, 2010), with each Egyptian getting an eighth size slice from each of the five loaves.

In this article, we share a lesson we explored with fifth graders focusing on ancient Egyptians' solutions to the above problem. (The lesson was taught primarily by a preservice teacher with assistance from her mathematics education professor). The Egyptians determined equal shares, but in such a way that no one received multiple slices of the same size. Instead, they divided four loaves in half, then divided the remaining bread into eight pieces, giving each Egyptian $\frac{1}{2}$ of a loaf plus $\frac{1}{8}$ of a loaf as their equal share. As students explored this method and worked to solve similar problems, they made connections between unit fractions, equivalent fractions and equal shares—all important concepts in the fifth grade curriculum (Cramer, Wyberg & Leavitt, 2008; Wilson, Edgington, Nguyen, Pescosolido & Confrey, 2011).

2 Ancient Egyptians' Conceptions of Fractions

Ancient hieroglyphs suggest that Egyptians viewed fractions as composed of distinct unit fractions. To represent unit fractions, they used the hieroglyph for "mouth" which had the meaning "part." This hieroglyph was placed over the denominator of a fraction (See Figure 1) (Katz, 2009). When the Egyptians worked with a fraction that was an iteration of a unit fraction, say $\frac{3}{4}$, they did not think of it as three $\frac{1}{4}$ size pieces. Rather, they decomposed the fraction $\frac{3}{4}$ into distinct unit fractions, so that $\frac{3}{4}$ was considered as $\frac{1}{4}+\frac{1}{2}$. The Egyptians wrote smaller fractional pieces on the left, thus $\frac{3}{4}$ looked like  (Gillings, 1972). Returning to our story problem, the Egyptians would have written $\frac{5}{8}$ in the following manner:



These hieroglyphs represent different-sized slices that each person received to have equal shares of the bread. Exploring mathematical tasks within a historical context motivates students (Fauvel, 1991; Mclean, 2002; Percival, 2003), while supporting them as they make sense of problems and persevere in solving them (CCSSM SMP #1).

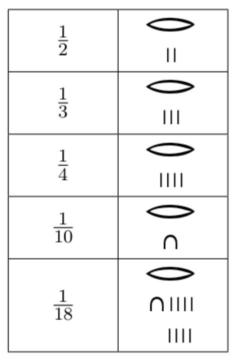


Fig. 1: Ancient Egyptian fraction hieroglyphs.

3 The Lesson: Part 1—Equivalent Fractions with Fraction Tiles

The first part of our lesson focused on students' conceptual understanding of unit fractions and was designed to increase their fluency with equivalent fractions. The lesson began with a discussion of equivalent fractions. When asked to describe an equivalent fraction, a student responded, "it's one-half and two-fourths." Another student clarified, "It means like the fractions are the exact same amount." Students revealed their understanding of fractions as iterations of unit fractions. For instance, a student came to the board and drew a picture to show that $\frac{3}{4}$ is the same as three $\frac{1}{4}$ size pieces. The discussion ended with the students producing multiple representations of wholes, dividing circles into n number of equal-sized pieces and shading in all of the n same-sized pieces to make up a whole.

For the next segment, students were introduced to fraction tiles (they had never seen them before). Since the students had never considered equivalent fractions as sums of different size pieces, we encouraged them to model fractions in as many ways as they could using the fraction tiles. Students traced their constructions and wrote corresponding number sentences on a worksheet. Students worked with $\frac{2}{3}$, $\frac{3}{10}$, $\frac{5}{8}$ and $\frac{11}{12}$. Figure 2 highlights student work for $\frac{2}{3}$.

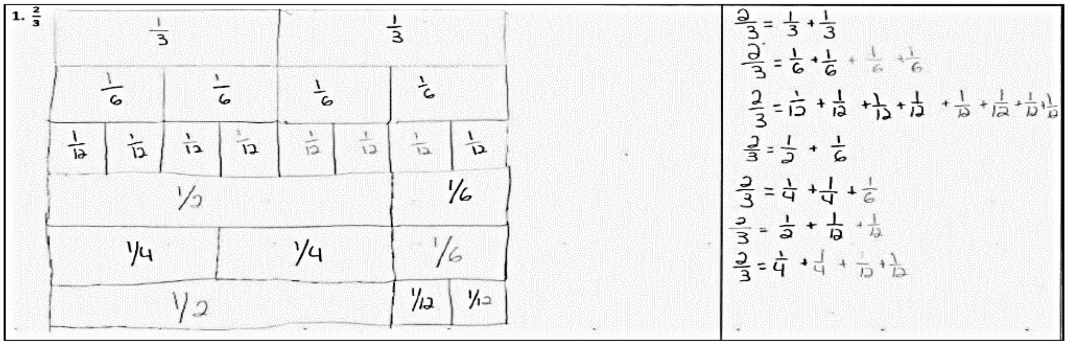


Fig. 2: Student work ($\frac{2}{3}$ example).

As the work illustrates, the student first broke up the $\frac{1}{3}$ size pieces into two $\frac{1}{6}$ size pieces, showing that $\frac{2}{3}$ is equivalent to $\frac{1}{6}+\frac{1}{6}+\frac{1}{6}+\frac{1}{6}$. Then as she looked at the pieces, she realized that two $\frac{1}{12}$ size pieces are the same as one $\frac{1}{6}$ size piece and that a $\frac{1}{2}$ size piece could be interchanged for 6 of the $\frac{1}{12}$ size pieces. This realization led her to continue decomposing $\frac{1}{2}$.

As students worked, we walked around purposefully questioning them (PtA, 2014), probing their understanding of equivalent fractions and the different-sized pieces. During these conversations, students expressed surprise that an equivalent fraction could be constructed without multiplying the numerator and denominator by the same amount. Yes, $\frac{2}{3}$ equals $\frac{4}{6}$, but $\frac{2}{3}$ also equals $\frac{1}{2}+\frac{1}{6}$; as mentioned, this was a critical understanding we wanted to develop in the students prior to their work with Egyptian fractions.

To conclude the first part of the lesson, Smith & Stein's (2011) "select and sequence" instructional strategy was used to choose and order student presentations (See preservice teacher reflection on p. 6). Student 1 (Figure 2) described her $\frac{2}{3}$ example first to help move students from decomposing fractions into progressively smaller equivalent unit fractions to making larger size pieces (e.g., in the fourth row of Figure 2, the student swaps 6 $\frac{1}{12}$ pieces for a single $\frac{1}{2}$ piece). In this vein, another student was asked to discuss the example $\frac{3}{10}$, because the only equivalent fraction that could be made with the fraction tiles used a larger size piece, with $\frac{3}{10}$ equaling $\frac{1}{5}+\frac{1}{10}$. Through discussion of these and other examples, the students showed an understanding of equal shares being composed of different-sized pieces—a prerequisite for the next component of the lesson. At this point, it was finally time to work with Egyptian fractions!

4 The Lesson: Part Two—Egyptian Fractions & Equal Shares

The second portion of the lesson began with a reading from *Ancient Egyptian Hieroglyphs* (Allen, 2012) highlighting the role of scribes in Egyptian culture. After reading, the bread story problem from the beginning of this article was introduced. Using the document camera and cut outs of Egyptians and loaves of bread (Figure 3), the instructor modeled the scenario with the help of students. She made connections to equivalent fraction work with tiles from the previous day. The class noted that they had initially thought of decomposing $\frac{5}{8}$ into equal-sized unit fractions, giving each of the eight Egyptians five $\frac{1}{8}$ sized slices of bread.

After briefly discussing this method, the students were told that Egyptian's worked to make as few cuts in each loaf as possible. The students realized that less cutting would mean the Egyptians would get different-sized slices of bread; no one could receive multiple slices of the same size. Cutouts were used to model dividing four of the loaves in half, giving $\frac{1}{2}$ of a loaf to each Egyptian with a full loaf remaining to be cut into 8 equal sized pieces, one for each Egyptian. Thus, each Egyptian had one $\frac{1}{2}$ size slice and one $\frac{1}{8}$ size slice which is equivalent to $\frac{5}{8}$ of a loaf. As the class worked to model this scenario, students made connections to the previous day's work. For example, a student indicated, "Four-eighths make half, and then you just need to add the extra one-eighth."

Fig. 3: Ancient Egyptian cutouts.

Overnight we replicated the students' different visual representations and equations for each fraction on large poster paper. These posters were distributed to small groups for further analysis. Each group was tasked with identifying which of the representations were Egyptian fractions. They recognized that different denominators correlated with different-sized slices, relating this back to the Egyptians' use of distinct unit fractions [note: According to Gillings (1972), the Egyptians sought "the 'simplest' value available" (p. 49), meaning they preferred to use the most efficient set of distinct unit fractions. However we chose to recognize all of the students' combinations of distinct unit fractions even if they were not the most efficient. For example, for $\frac{5}{8}$ we allowed students to identify both $\frac{1}{2}+\frac{1}{8}$ and $\frac{1}{3}+\frac{1}{6}+\frac{1}{8}$ as Egyptian fractions]. After analyzing their compiled work, groups shared the Egyptian fractions associated with their example. Conversations such as the following were typical.

TEACHER Why is this one an Egyptian fraction?

STUDENT Because they all have different denominators.

TEACHER And what do the different denominators mean?

STUDENTS (*in unison*) They're different-sized pieces.

As the final component to the lesson, students made posters with hieroglyphs and equations of their Egyptian fractions (Figure 4). We did not teach the students how to order the hieroglyphs from smallest to largest as the Egyptians did, but instead let them depict them using their own number sentences.

5 Discussion

The lesson presented in this paper was designed and taught by a preservice elementary teacher in her third year of preservice coursework, prior to her semester-long student teaching experience. She designed the lesson for a specific group of students as suggested by a district supervisor. Because lesson decisions are context specific, it's important to share some of her experience and insights regarding the process. We also share her thoughts regarding limitations of the lesson and manipulative use.

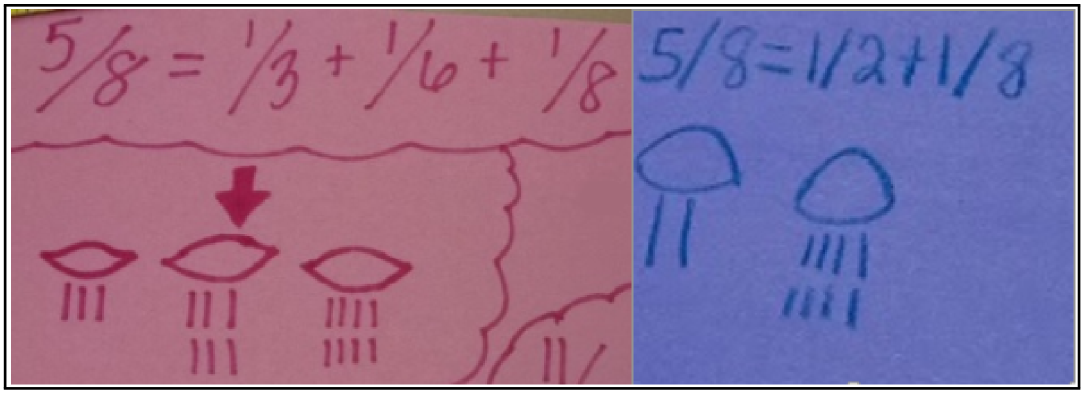


Fig. 4: Student-constructed hieroglyphs.

5.1 Preservice Teacher's Personal Reflection

Prior to teaching this lesson in the fifth grade classroom, I taught it to my preservice teacher peers to gain practice and receive feedback. My biggest takeaway was the importance of selecting and sequencing student work in the classroom (Smith & Stein, 2011) in order to facilitate meaningful mathematics discourse (PtA, 2014). I had read Smith and Stein's book in my preservice coursework, but it was not until I experienced the pitfalls of relying on volunteers that I was able to see firsthand how not selecting and sequencing removed my control over the ensuing conversations. When I taught this lesson in the fifth grade classroom, I made sure to monitor students' work not only for correctness, but also so I could select which examples I wanted the class to see and in which order. This allowed me to provide a logical progression for the students to see how different equivalent fractions were made. For example, I made sure that in the $\frac{5}{8}$ example, $\frac{1}{2}+\frac{1}{8}$ was shown before $\frac{1}{5}+\frac{1}{5}+\frac{1}{10}+\frac{1}{8}$ so that students could visually see and we could discuss how the $\frac{1}{5}+\frac{1}{5}+\frac{1}{10}$ was a way to decompose the $\frac{1}{2}$ size piece.

5.2 Limitations

5.2.1 Lesson

The lesson outline we presented worked for this particular group of fifth grade students who had not yet been exposed to fraction tiles nor ideas of creating equivalent fractions by adding different fractions together. For these students, the use of fraction tiles provided a visual to assist them as they worked with unit fractions for the first time. For students who are more proficient with the ideas of unit fractions and equivalent fractions, the lesson could be modified to focus solely on the initial story problem. We suggest providing students with cutouts of Egyptians and full loaves of bread (Figure 3), encouraging students to solve the problem in as many ways as possible. Similar to Fosnot's (2008) work with a sharing subs problem, we anticipate students coming up with solutions that would mirror Egyptian fractions. The focus would then be on comparing students work between five $\frac{1}{8}$ sized slices and $\frac{1}{8}+\frac{1}{2}$ size slices.

5.2.2 Manipulative Use

Although fraction tiles are a wonderful manipulative for children, one problem for some students was visually differentiating between the $\frac{1}{10}$ and $\frac{1}{12}$ size pieces. When analyzing student work, common misconceptions stemmed from utilizing the $\frac{1}{10}$ size piece when the $\frac{1}{12}$ size piece was needed. These pieces are physically very close in size, due to the nature of the fractions themselves and the structure of the manipulative. While a full understanding of equivalent fractions with respect to common denominators would have mediated this problem, the classroom teacher indicated her students were not yet proficient with this topic. This limitation illustrates the need to use mathematical tools appropriately (CCSSM SMP #5).

6 Conclusion

Through this lesson—one in which students learned how to think like Egyptians—we've shown what research tells us time and again; namely, that adding historical cultural context to mathematics motivates students to engage in the material and take it beyond the intended scope of the lesson (Fauvel, 1991; Mclean, 2002; Percival, 2003). After the lesson, a student asked for help looking up Egypt on a map. After the classroom teacher pointed out world maps in the back of the students' planners, the kids rushed to be the first to find Egypt. A few weeks later when visiting this classroom again, several students shared how they showed their parents how to make Egyptian fractions. The students raved about how cool and "clever" it was for the Egyptians to think of fractions as equal shares of different-sized pieces.

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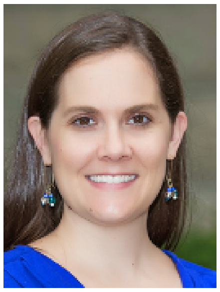

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