# Four Corners in Learning Mathematics with Technology

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***Abstract:*** *The authors describe the application of the four corners activity in a classroom of future mathematics teachers. Teaching candidates took one of four positions (strongly agree, agree, disagree, strongly disagree) with respect to each of three statements emphazing connections between communication and mathematics. Insights regarding the approach are shared along with suggestions for use of the method in various instructional settings.*

***Keywords:*** *Teaching methods, communication, dialog*

## 1 Introduction

Student communication plays a central role in mathematics learning. The Common Core State Standards highlight student communication in two of the Standards of Mathematical Practice. In the standard *Construct viable arguments and critique the reasoning of others*, one important aspect is that students "justify their conclusions, communicate them to others, and respond to the arguments of others." Communication also plays an important role in the standard *Attend to precision*: "Mathematically proficient students try to communicate precisely to others" (Council of Chief State School Officers & National Governors Association Center for Best Practices, 2010). *Communication* is one of the Process Standards (National Council of Teachers of Mathematics, 2000), and states that students should be conversant in four categories of communication. Namely, they should be able to:

- Organize and consolidate their mathematical thinking through communication;
- Communicate their mathematical thinking coherently and clearly to peers, teachers, and others;
- Analyze and evaluate the mathematical thinking and strategies of others;
- Use the language of mathematics to express mathematical ideas precisely.

In addition, communication is also part of the *Representation* standard. Students need to be able to "Create and use representations to organize, record, and communicate mathematical ideas." Developing such communication skills is a long process. Therefore, practices that allow for stronger student-student and student-teacher communication are paramount in the mathematics classroom. Prospective mathematics teachers who participate in these practices in their preparation are more likely to conduct them with their own students in the future. In this article, we chronicle our use of a particular activity, Four Corners, which we used to engage future mathematics teachers in connections between communication and mathematics to make their beliefs about technology more

explicit. Mathematics teachers in schools can adapt the activity for their own students by changing our discussion prompts.

Communication in mathematics deals with some of the same issues as communication in other settings. Wolfe (2010) explains:

> Rhetoric and composition, literacy studies, and quantitative literacy [. . .] have all embraced the influential ideals of John Dewey, who persuasively argued that civic participation in a democratic society requires a liberating literacy that prepares citizens to think for themselves. They all emphasize communication and reasoning, not as they occur in isolated academic settings but in complex, real-world contexts where individuals must reason through a sea of often contradictory information in order to come to an informed opinion. (Wolfe, 2010, p. 454)

Wolfe's point about preparing students to engage with communication and reasoning is important for their meaningful participation in society. Similarly, future mathematics teachers benefit from explicit instruction and critical thinking about the connections that exist between teaching, their own personal philosophies about teaching, and communication.

Communication in mathematics also has some unique characteristics that need to be addressed. For instance, when using writing to communicate mathematics, mathematical concepts and ideas are conveyed through words, numbers, specialized symbols, and graphs or diagrams. Furthermore, making connections between these different modes of representations of mathematical ideas is crucial for developing understanding. Some future teachers misinterpret the term "writing" as "composing paragraphs of text," and do not associate writing with mathematics. It is important that future mathematics teachers are aware of their own beliefs and possible misconceptions about communication.

## 2 Rationale and Context

Four Corners, an interactive debate activity, requires students to reveal their position on a specific statement by standing in a particular corner of the classroom. This activity elicits the participation of all students by requiring that everyone take a position (i.e., strongly agree, agree, disagree, or strongly disagree) with respect to a statement. Then, students are asked to explain their position using examples, evidence from the class, or their own experience. The Four Corners debate has been used in other content areas such as History and English (Facing History and Ourselves, 2016) and was adapted by the first author to be used in a class for future mathematics teachers. The Four Corners debate format can also be used with students in schools (Hopkins, 2003).

This activity was conducted in the Fall of 2015 in the course *Learning Mathematics with Technology* for future secondary mathematics teachers in their first year. The course emphasized verbal and written communication, inquiry, and cooperation (Flores, 2014). The Four Corners activity was provided during the sixth week of the semester at a time when students were familiar with student speaking and writing as key components for the course.

## 3 Procedure for Four Corners

Below, we provide a step-by-step procedure for teachers to consider when engaging the Four Corners activity. Then, we share the results of our engagement with the activity in the course.

### 3.1 Step one: Preparation

The instructor begins by labeling four corners of the classroom using the designations strongly agree, agree, disagree, and strongly disagree. Then, the instructor constructs prompts to students that require a response. Different statements may be composed for the specific needs of the course. The statements we included in the activity for our course included the following:

- Teaching with technology is important for me to be able to communicate important concepts to students.
- My understanding of mathematical concepts and language is less important than understanding how to complete the activity.
- Writing in mathematics is less important for me to communicate concepts to the students.

We created these statements for our activity to foster a deeper consideration of the goals of the course and to promote personal reflection about mathematics and communication from a teacher perspective. In other words, the intent of these statements was to help push teacher candidates to make connections between what they were learning in our university classroom and how they might communicate that knowledge to their own students in writing or through oral communication. The prompts were constructed to stimulate critical thinking, rhetorical awareness, and use of sources in meaningful and useful ways.

### 3.2 Step two: Introduce Statements

Next, the instructor distributes statements to candidates—either in class or as a homework activity—and gives them the opportunity to construct written responses. We provided our students with a worksheet that required them to mark their opinion (strongly agree, agree, disagree, or strongly disagree), and then provide a brief explanation.

Providing time to write responses encourages candidates to make critical connections between the prompt and specific experiences with mathematics, opening the door for reflection and engagement with mathematical activities and communication.

### 3.3 Step three: Four Corners Discussion

Once candidates have written personal responses to the statements, the instructor reads one of the statements aloud in class, asking candidates to move to the corner of the room that best represents their opinions. Once students are in their places, the instructor asks for volunteers to justify their positions, referring to evidence or specific reasons, especially from material they learned in the class, as well as other relevant information from their own experiences. The practice should be repeated until all statements have been discussed.

### 3.4 Step four: Reflection

There are several ways teachers can engage students in reflection after the prompts have been discussed. For example, the instructor may have students write a brief reflection at the end of class to articulate how the activity made them reconsider their beliefs about teaching. The instructor may also ask students to talk with one another about the most compelling portion of the activity. Both of these options encourage candidates to further articulate what they learned from the activity, further supporting communication and reasoning.

# 4 Our Results

We present sample student responses and rationales for each of our three aforementioned statements.

## 4.1 Importance of technology to communicate concepts

After the students wrote their answers on their worksheet, the first statement was read aloud:

- Teaching with technology is important for me to be able to communicate important concepts to students.

Students were asked to move to the corner of the room corresponding to their position, in this case distributing themselves equally among the groups corresponding to strongly agree, agree, and disagree. We asked students to explain their position. Students who strongly agreed gave as evidence activities they had done in class, such as using a motion detector to capture data from the fall of a stuffed toy in a parachute. In this activity, students place a motion detector above the parachute before they release it. Before the parachute opens, the toy falls with increasing velocity. As the parachute opens, the acceleration decreases until the toy reaches terminal velocity. The data collected by the motion detector every 0.03 seconds are graphed with a program in the graphing calculator (Figure 1). The units along the horizontal and vertical axes are time and distance from the motion detector, respectively. Students are asked to plot their graph on grid paper and describe the graph. Students answer questions such as *How do features on the graph connect to events of the fall? Can you see in the graph when the parachute opened completely?*

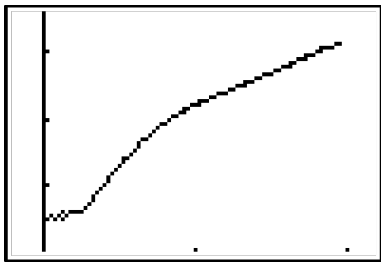

**Fig. 1:** *Distance vs. time in parachute fall.*

When justifying why she strongly agreed with the statement, one student noted that:

> In the activity with the parachute we used technology to measure and graph the fall of the stuffed animal. Had we not had the technology, more of the time would have been spent figuring [out the fall] instead of making connections with the data. Also, technology is so relevant to students in the 21st century that it only makes sense to connect with them on their level.

The student explained that the technology allowed students to focus on the analysis of the graph rather than on the task of collecting data. Students who disagreed with the statement also explained their position based on their preferences or well thought rationales. One student remarked that he still prefers to have mathematics being developed at the board, step by step, and use paper and pencil to work out mathematical problems and tasks. Another student pointed to the fact that in none of her mathematics classes in college were they allowed to use calculators. These students expressed the importance of explicit instruction using basic technological tools (pen, paper, board) so that students may understand the fullness of a given mathematical situation. Although students did not connect these "low brow" technologies to an engagement with modern day technology,

their responses tell us all something about how instruction in a specific way, with use of pencil and paper, is important to basic instruction for some students. By asking teacher candidates to make connections between communication and technology in mathematics, we help them to consider and describe the role of technology in their students' learning in their future classrooms.

### 4.2 Understanding mathematical concepts and understanding how to complete an activity

After students explained their positions, the second statement was read aloud:

- My understanding of mathematical concepts and language is less important than understanding how to complete the activity.

For this prompt, students divided themselves into two groups—strongly disagree and disagree. Some of the reasons expressed by future teachers were that just being able to complete a mathematical activity without really understanding the underlying ideas was not meaningful and that having a thorough understanding of the concepts and ideas does provide, in many cases, ways to complete the activity. Specifically, one student noted the following:

> [I] strongly disagree because understanding math concepts and language will help to actually understand the activity as opposed to just completing it. For example, if I were to just try to complete an activity on GeoGebra, I would just move the sliders until I get the graph I want. However, if I take the time to understand the activity, then following activities will be easier, and I will be able to explain the activity better to others.

This student models the importance of understanding the material and the connections that exist between mathematical concepts and language.

Another student who strongly disagreed noted that both mathematical conceptual understanding and language use are "equally important. Comparing the two sinusoidal functions would not have been possible had we not had understanding of both concepts and completing the activity." This student was able to use a specific instance during the class to illustrate the connections between mathematical concepts and effective use of language.

One student discussed how language and mathematical concepts helped to clarify ideas related to graphing noting that, "one must know the concepts in order to apply them. For example, in order to plot a quadratic equation ($y = Ax^2 + Bx + C$), the student must be able to understand how each piece $(A, B, C)$ affects the graph." By mentioning the importance and influence of the parameters, this student comments on how language is connected to modeling understanding.

Students who simply disagreed with the statement were also specific about their responses. One student noted the responsibility of the teacher to be familiar with concepts taught in the course:

> A teacher should understand the basic concepts and language because if a student were to not know how to complete an activity he/she could ask the teacher about a concept, and the teacher would be able to explain about it. I feel if a teacher knows the concepts and language they will be able to complete activities after using their knowledge.

### 4.3 Importance of writing in mathematics to communicate concepts

The third statement was also read aloud. Students engaged this statement:

- Writing in mathematics is less important for me to communicate concepts to the students.

One candidate who agreed with this statement explained that he had interpreted writing as paragraph writing, and that in mathematics symbols and graphs are used heavily. This student commented that

> while there are some instances where writing can be effective in displaying mathematical concepts, those concepts can be more accurately displayed only with visual representation. For example, to explain the concepts of FOIL in multiplying binomials, it is easier to show the student and explain it orally rather than through paragraphs.

The student's sentiment tells us something about how writing is viewed by this future teacher and how, in many ways, students fail to connect writing and communication. Students who strongly disagreed with the statement mentioned that writing was very important in their proof-based courses. They said that being able to write a proof clearly and in logical sequence was paramount. Another student noted that "there will be students who learn better through sentence explanations than simply just seeing numbers." This teacher candidate connected writing to sentence level instruction.

Students who disagreed with the statement said that writing did not play a major role in high school mathematics or the calculus sequence in college, and that students in these courses had to present mathematical computations step by step using symbols rather than using words. One student said that "although verbal justification is important & appropriate—I need to be able to explain and justify my thinking to demonstrate my understanding." Another student spoke specifically about how report writing helped to clarify ideas: "being able to write and explain what is going on is important—it allows for others to understand your work. This is why we have been writing reports about all of the different projects." This student had an understanding of the function of report writing for the course and was able to articulate just why report writing was connected to specific activities in the course.

## 5 Concluding remarks

Students—and teacher candidates, in particular—gain much through explicit conversations focusing on connections between mathematics and communication (written, oral, and visual). In our Four Corners activity, we encouraged future teachers to connect written and technology-based communication to their future practice as classroom teachers. The activity also encouraged them to further develop their language skills while encouraging them to begin to think of themselves as teachers who should be able to communicate ideas to others in specific and clear ways. The lesson helped our future mathematics teachers prepare and develop their own pedagogical stances by thinking about communication in explicit ways.

However, we are aware that there are several limitations in this study. First, the activity happened in a single course. In order to develop prospective teachers' competency in addressing communication for their future students, communication needs to be highlighted in other courses in their teacher preparation program. Second, it would be important to describe how their thoughts about communication evolve, and what beliefs remain the same. It would be interesting to ask candidates follow-up questions, perhaps during student teaching or once they are in the classroom. Unfortunately these data have not been collected and thus fall outside the scope of the present article.

Teachers in schools may use the Four Corners activity to emphasize the importance of communication in the learning of mathematics for their own students. The second of the statements used above can be used with students. Here are some statements to start discussions about communication and mathematics.

- Writing in mathematics is important for me to convey my understanding of concepts;
- Writing in mathematics is important for me to develop my understanding;
- Learning with interactive technology is important for me to understand concepts;
- Using technology is important for me to communicate my understanding of mathematics;
- Explaining mathematical ideas using my own words helps me understand them;
- Explaining my strategies to solve a problem is as important as finding a solution.

Of course, if the instructor wants to emphasize other aspects of learning mathematics, the questions can be changed accordingly.

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