The Trouble with KISSing and Other Mnemonic Devices

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Abstract: *In this article, the authors share research on the use of mnemonic devices in mathematics instruction. In particular, they address the following questions:* What are mnemonic devices? What are their benefits? What should teachers be cautious about? How should mnemonic devices be used?

Keywords: *Problem solving strategies, mnemonic devices, instruction*

1 Introduction

Many middle school students say subtraction of negative numbers is easy; you just need to KISS. For the first number, you keep it (KI) the same, then you switch (S) the subtraction to addition, and lastly switch (S) the second number to its opposite. For example, $8-(-2)$ becomes $8+2$. Similarly, when dividing fractions, keep the first number the same, switch the operation to multiplication, and switch the second number to its reciprocal. So, $\frac{1}{2} \div \frac{1}{4}$ becomes $\frac{1}{2} \times \frac{4}{1}$. Unfortunately, problems arise when students overgeneralize and apply this procedure to other calculations. For example, students may apply this mnemonic to multiplication and falsely conclude that $8 \times (-2)$ is equivalent to $8 \div 2$. Instances of this type of overgeneralization were observed when we interviewed 20 seventh grade students about their understandings of integers and computations involving integers (Graybeal, Strickland, Rodriguez, & Guerra, 2013).

In this article, we share research on the use of mnemonic devices in mathematics instruction. What are mnemonic devices? What are their benefits? What should teachers be cautious about? How should mnemonic devices be used?

2 What are mnemonic devices?

The word mnemonic is derived from the Ancient Greek word for *memory*. Mnemonic devices are "structured ways to help people remember and recall information" (Brigham & Brigham, 2001, p. 1) that take many forms and have many purposes. The most common form in mathematics instruction is the first letter mnemonic or acronym. In this form, the first letter of each word that is to be remembered is used to generate a new word or phrase. KISS is such an example. Acrostics are sentences formed by words which share the same first letters as the words to be remembered. For example, Please Excuse My Dear Aunt Sally is an acrostic for the Order of Operations: Parentheses, Exponents, Multiplication/Division, and Addition/Subtraction. Songs and jingles can serve as music mnemonics. For example, many students learn the quadratic formula through song. Lastly, ode mnemonics are short poems that communicate ideas such as “Ours is not to reason why; just invert and multiply.”

The purposes of mnemonic devices also vary. They can be used to recall mathematical conventions, definitions, facts, computational procedures, and general problem solving heuristics. Examples of some common mathematical mnemonics organized by form and purpose are listed in Table 1.

Table 1: Common Mathematical Mnemonics Organized by Form and Purpose

	Computational Procedures (useful after the development of conceptual understanding)	**Conventions, Definitions, and Facts**	**Problem Solving Heuristics**
First Letter Mnemonics or Acronyms	FOIL for binomial multiplication KISS for integer subtraction	SOH, CAH, TOA for trigonometric ratios; PEMDAS for Order of Operations	STAR for *Search*, *Translate*, *Answer*, *Review*
Acrostics	*Does McDonalds Serve Cheese Burgers?* for long division	*Please Excuse My Dear Aunt Sally* for Order of Operations; *King Hector Doesn't Usually Drink Cold Milk* for prefixes of metric units	
Music and Ode Mnemonics	*Ours is not to reason why; just invert and multiply* for fraction division	Song for Quadratic Formula	

3 Benefits of Mnemonic Devices

The use of mnemonic devices is supported by research in special education and is “validated for students with high-incidence disabilities, particularly students with learning disabilities, as well as for students at all levels of education” (Brigham & Brigham, 2001, p. 1). Our research also supports the efficacy of mnemonics in special circumstances. We individually interviewed 20 seventh grade students in three different middle schools about their knowledge of integer operations. An outline of the interview protocol is included in Table 2. The results were fairly consistent among the 20 students. Most were able to answer the purely computational questions correctly and explain how they arrived at their answers using a rule (such as KISS).

Table 2: Interview Protocol

Protocol

- What do you know about integers?
- What is a real life example of integers?
- If I give you a set of numbers, can you place them on a number line for me?
  -5, 1, 7, -2, 4, 0
- Which is the smallest number? Which is the biggest number?
- Now we are going to ask you to solve some arithmetic problems.

Sample arithmetic problems		**Sample Prompts to Facilitate Explanations**
$8+2$	8×2	*How do you know this?*
$-8+2$	-8×2	*Draw a picture.*
$-8+(-2)$	$-8\times(-2)$	*What is a situation that represents this?*
$8-2$	$8\div 2$	*How do you think about problems like these?*
$-8-2$	$-8\div 2$	*Prove that this is correct.*
$-8-(-2)$	$-8\div -2$	*Explain how you got this answer.*
		Use a number line to explain this.

Table 3 shows the results of our interviews focusing on the computations involving negative numbers. Overall, the percentage of correct answers was 81.3%, and the percentage of correct explanations was 73.6%, but the percentage of explanations which demonstrated conceptual understanding was only 23.6%. For example, an explanation that -8×2 equals -16 because, "A negative times a positive is a negative" was counted as a correct computational answer and a correct explanation; we did not, however, see this explanation as demonstrative of conceptual understanding. An explanation demonstrating conceptual understanding might include an observation that "this is two groups of -8." One student who used such an approach accompanied her written work with a drawing of two groups of eight circles, noting that each circle represented negative one. Such work provided evidence that students' use of mnemonics such as KISS helped them compute accurate answers.

4 Cautions about Mnemonic Devices

Although the 7th graders we interviewed were able to find accurate answers to computation, most of them relied on mnemonic devices to determine these answers. The majority knew their answers were correct only because the rule said it worked. For example one student said, "I can't explain KISS. It's just how we were taught."

Without a solid conceptual understanding, the students in our study used a variety of rules to compute with integers. The majority used KISS for subtracting integers with many overgeneralizing this rule. For example, when computing $8\times(-2)$, one student stated that "you would have to divide by two; you have to do the opposite of what you were given." This student kept 8, switched the operation to division, and switched the (-2) to 2 and found the answer to $8\div 2$. We found that students who were taught mnemonics for integer computations often misused these rules, mixed together rules, and confused themselves to the point where they didn't know when to use particular rules.

Undoubtedly, mnemonic devices can help students memorize conventions, definitions, facts, and computational procedures. For example, mathematical definitions such as the names and meanings

Table 3: Students' Answers and Explanations

	Addition $-8+2$ $-8+(-2)$	Subtraction $8-(-2)$ $-8-(-2)$	Multiplication -8×2 $(-8)\times(-2)$	Division $8\div(-2)$ $(-8)\div(-2)$	Overall
Questions posed	$n=40$	$n=40$	$n=37$	$n=27$	$n=144$
Correct computational answers	$n=35$ (87.5%)	$n=27$ (67.5%)	$n=32$ (86.5%)	$n=23$ (85.2%)	$n=117$ (81.3%)
Correct explanations	$n=34$ (85%)	$n=24$ (60%)	$n=29$ (78.4%)	$n=19$ (70.4%)	$n=106$ (73.6%)
Explanations that were correct and demonstrated conceptual understanding	$n=24$ (60%)	$n=3$ (7.5%)	$n=6$ (16.2%)	$n=1$ (3.7%)	$n=34$ (23.6%)

of the trigonometric ratios must be taught to students so that they can communicate precisely with others. The mnemonic SOH CAH TOA seems to be useful for this purpose. But, when it comes to procedures, as teachers we must ask ourselves why we are asking students to memorize specific steps which may not generalize to more complex problems. In particular, caution should be heeded when considering the use of mnemonics for computational procedures. For example, the acronym FOIL can be useful in helping students remember to correctly multiply two binomials such as $(x+2)(2x-3)$. By multiplying the first terms, outer terms, inner terms, and last terms, all terms have been properly distributed. But, what happens when a student is asked to multiply two expressions that don't fit this mold? For example, what would a student who relies on FOIL do when presented with a computation such as $(x^2+3x-4)(2x-3)$? Students who rely on FOIL would not have an approach that is flexible enough to work in this situation.

The teaching of procedural mnemonics also sends a very narrow message about mathematics. It tells students that their job is to memorize and follow procedures without stopping to think about *why* they are doing *what* they are doing.

5 Recommendations

When considering the use of a mnemonic device, first ask yourself if the information to be learned is of factual or procedural nature. Mnemonics that help students remember conventions, definitions, and facts may be useful, especially for students with disabilities. But, mnemonics for procedures can be dangerous because students often overgeneralize and misapply them. Instead of teaching procedural mnemonics, focus on helping students develop generalizable strategies. For example, instead of FOIL, teach students to think of multiplication of two expressions as finding an area. If young students are taught to approach 573×89 by sketching an array, then later finding the product of $(x^2+3x-4)(2x-3)$ is a natural extension.

573×89

	500	+70	+3
80	40,000	5,600	2,400
+9	4,500	630	27

So, $573 \times 89 = 40,000 + 5,600 + 240 + 4500 + 630 + 27 = 50,997$

$(x^2 + 3x - 4)(2x - 3)$

	x^2	$+3x$	-4
$2x$	$2x^3$	$6x^2$	$-8x$
-3	$-3x^2$	$-9x$	12

So, $(x^2 + 3x - 4)(2x - 3) = 2x^3 + 6x^2 - 8x - 3x^2 - 9x + 12 = 2x^3 + 3x^2 - 17x + 12$

The most useful mnemonic devices focus on the process of solving a mathematical problem, not an isolated mathematical skill or procedure. These mnemonics are generalizable to a wide variety of situations. The STAR Strategy is a particularly effective mnemonic device. In the STAR strategy, students:

- *Search* the word problem,
- *Translate* the word problem into an equation in picture form,
- *Answer* the problem, and
- *Review* the solution.

Strickland and Maccini (2010) found STAR to be effective in supporting the mathematical learning of students. Students should be taught to work through the steps and to use the STAR Strategy Checklist to ensure all steps are followed. There are many resources available to help teachers support students' understanding of difficult mathematical concepts. For example, much has been written about effective ways to teach the Order of Operations without PEMDAS (Dupree, 2016), fraction computation (Cardone, 2015) and integer operations (Gregg & Gregg, 2007). If students have a strong conceptual understanding of the procedures, mnemonics are not necessary.

Table 4: STAR Strategy Checklist (Adapted from Strickland & Maccini, 2010)

Check off each step as you work	STAR Strategy Steps
	Search the word problem •Read the problem •Ask yourself, "What facts do I know?" "What do I need to find out?" •Write down facts
	Translate the words •Use manipulatives, drawings, or symbols
	Answer the problem
	Review the question •Re-read the problem •Ask, "Does the answer make sense? Why?" •Check answer

In summary, mnemonics can be an effective instructional practice when teaching students mathematical problem solving process, conventions, definitions, and facts. Caution should be heeded when considering mnemonics for procedures. When helping students develop mathematical procedures, be sure to help students realize that mathematics makes sense!

References

Brigham, R., & Brigham, M. (2001). *Mnemonic instruction. Current Practice Alerts of the Council for Exceptional Children*. Retrieved from http://teachingld.net/ld_resources/alerts/5.htm

Cardone, T. (2015). *Reason why when you invert and multiply*. Retrieved from http://www.nctm.org/publications/mathematics-teaching-in-middle-school/blog/reason-why-when-you-invert-and-multiply/

Dupree, K. M. (2016). Questioning the order of operations. *Mathematics Teaching in the Middle School, 22*(**3**), 153–159.

Graybeal, C.D., Strickland, T.K., Rodriguez, M., & Guerra, N. (2013). Conceptual understanding of integer operations: A study of middle school students and their thought processes. *The Banneker Banner, 28*(**1**), 4–9.

Gregg, J., & Gregg, D. U. (2007). A context for integer computation. *Mathematics Teaching in the Middle School, 13*(**1**), 46–50.

Strickland, T. K., & Maccini, P. (2010). Strategies for teaching algebra to students with learning disabilities: Making research to practice connections. *Intervention for School and Clinic, 46*(**1**), 38–45.

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