Shifting Perspectives about the Standards for Mathematical Practice

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Abstract: *The Standards for Mathematical Practice (SMPs) describe mathematical behaviors and habits that K-12 students should engage in during math instruction. The authors explore the process of shifting teachers' perspectives about the their role in promoting the SMPs through the use of a brief protocol-centered professional development (PD) experience. Results suggests a brief PD experience may initiate a shift in perspective about the teachers' role in promoting the SMPs and support teachers' desire to attend future PD experiences on this topic.*

Keywords: *professional development, standards for mathematical practice, reform*

1 A Context for Protocol-Centered Professional Development

The Common Core State Standards for Mathematics, which were largely adopted as Ohio's Learning Standards for Mathematics, describe eight Standards for Mathematical Practice (SMPs). These standards are listed below.

SMP1 Make sense of problems and persevere in solving them.

SMP2 Reason abstractly and quantitatively.

SMP3 Construct viable arguments and critique the reasoning of others.

SMP4 Model with mathematics.

SMP5 Use appropriate tools strategically.

SMP6 Attend to precision.

SMP7 Look for and make use of structure.

SMP8 Look for regularity in repeated reasoning.

The SMPs are the foundation of what it means to do math and should therefore play an integral role in K-12 math instruction (CCSSI, 2010; Koestler, Felton, Bieda, & Otten, 2013). Research suggests preservice teachers (PSTs) and inservice teachers (ISTs) may gain insight into students' perspectives of engaging in the SMPs through such examination (Bleiler et al., 2015; Bostic & Matney, 2014; Graybeal, 2013; Johns, 2016; Kolb, 2015; Peretin, 2014; Stohlmann et al., 2015). PD that revolves

around math tasks leads ISTs to understand students' engagement in the SMPs (Olson, Olson, & Capen, 2014). Similarly, studies of PST programs have explored the student's role in enacting the SMPs while using a professional development (PD) design that involves observation and recording student engagement in the SMPs (Courtney, 2014; Graybeal, 2013). At present, there is a lack of literature describing PD related to the teacher's role of promoting the SMPs into daily classroom instruction (Olson et al., 2014).

Courtney (2014) raises a concerning question for schools and school districts to consider: "How can PSTs help to develop reasoning, sense making, and the [SMPs] in their (future) students, when a focus on meaning, reasoning, and ways of thinking are not part of how they themselves operate?"(p. 13). Courtney goes on to discuss the need for opportunities for PSTs (and ISTs) to further develop their math thinking and reasoning related to the SMPs. He calls for further research to find ways to support PSTs in "developing such ways of thinking" (Courtney, 2014, p. 14). Research by Graybeal (2013) suggests that coursework and professional development of PSTs (and ISTs) with a look-for protocol, in tandem with designing and implementing their classroom instruction, is a successful approach to helping teachers understand the SMPs. Drawing upon these two studies, we believe that a protocol-centered PD in which PSTs and ISTs consider the teacher's perspective for promoting the SMPs might help them think more about fostering the SMPs in their daily classroom instruction. A brief PD experience that includes examining a protocol describing the SMPs in specific detail has potential to be a catalyst for creating productive conversations and support desire for follow-up learning experiences.

The present study aims to explore a protocol-centered PD for grades 4-12 ISTs and PSTs. Through this lens, we investigate teachers' perspective of their role in promoting the SMPs during classroom instruction and their interest in learning about the SMPs. The *Revised SMPs Look-for Protocol* (see Appendix A, Bostic & Matney, 2016; Bostic, Matney, & Sondergeld, in press) was used as the foundation of the protocol-centered PD because it focuses on instructional behaviors that might promote the SMPs during classroom instruction. Our research question is as follows. *How does a brief, two hour, PD experience about the SMPs affect PSTs' and ISTs' perspectives about them?*

2 Setting the Stage for Professional Development

We designed a two-hour PD session for math PSTs and ISTs from grades 4-12. The 4-12 grade band was chosen since the SMPs can look quite different in the K-3 grades and our experience lends us to better facilitating conversation during the middle and high school grades. The PSTs' session was separate from the ISTs' session due to constraints with PD location; however, the sessions had the same structure. The PD was organized through a math education organization for PSTs and ISTs at a Midwest university. A total of 50 PSTs and 29 ISTs participated in the PD. All names in this manuscript are pseudonyms. Prior to the PD experience, participants were provided an entry survey that asked the following questions: (1) What math education program are you currently in, if applicable? (2) How many years have you been in the math education program, if applicable? (3) What are the eight math practices described by the Standards for Mathematical Practices? (4) How would you describe one observable indicator for a teacher fostering each SMP? IST participants completed a nearly identical survey except (1) and (2) were replaced with the question: How many years of math teaching experience do you have? The surveys were analyzed using inductive thematic analysis, which consisted of reading and re-reading survey responses and grouping similar responses (Hatch, 2002). The groups were synthesized to reach a small number of broad over-arching ideas, or themes, which contained abundant evidence and little counterevidence, if any (Hatch, 2002).

The responses from the pre-PD survey indicated that PSTs and ISTs lacked familiarity with the SMPs and were not able to share math teaching behaviors that promote the SMPs for multiple grade-levels and content areas. Most participants were unable to identify keywords associated for each eight SMPs. Additionally, PSTs and ISTs were unable to identify any teacher-focused observable indicators suggesting promotion of the SMPs. Drawing upon these pre-PD data, we constructed a PD experience that encouraged participants to analyze a protocol that identified observable indicators descriptive of teachers' promotion of the SMPs. The intent was to begin a conversation about how to encourage a shift in perspective about the teacher's role in promoting the SMPs as well as support their interest to learn more about the SMPs. Using this lens, we enacted a protocol-centered PD aiming to support teachers' interest in learning about the SMPs.

3 Teacher-Focused Protocol-Centered Professional Development

Participants attended a two-hour PD that began with an overview of the SMPs, led by the authors. We described how the SMPs are built upon past frameworks for mathematical processes (mathematical proficiency [Kilpatrick, Swafford, & Findell, 2001]; National Council of Teachers of Mathematics' process standards (NCTM, 2000) and the difference between the SMPs and the Standards for Mathematical Content (CCSSI, 2010). Then, participants were assigned groups to explore a particular SMP. Each group was given the CCSSM description for their one SMP, a list of indicators gathered from their pre-PD survey, and vignettes from *Connecting the NCTM Process Standards and the CCSSM Practices* (Koestler et al., 2013). These vignettes served as examples of students and teachers engaging in the SMPs as part of classroom instruction. After reviewing the materials, SMP groups engaged in discussions with a goal of creating two observable indicators of teachers fostering their assigned SMP. No two groups had the same SMP. SMP groups presented their indicators to the whole group for feedback. Next, we introduced the *Revised SMPs Look-for Protocol*. We held a brief discussion about each indicator for each SMP. These discussions focused on having a shared understanding of the language within the indicators, making connections to the groups' indicators, and providing example(s) of indicators during classroom instruction.

After the session, PSTs and ISTs completed a post-PD survey asking the following questions: (1) To what degree was the session useful to you? (2) To what degree was the session relevant to you? (3) Please name one idea that you took away from this session. (4) Please name one question related to the topic of the session that you still have. (5) If there was a second session on this topic to follow up the first session, would you attend? Questions (1) and (2) used a four-point Likert scale, questions (3) and (4) were open-ended, and question (5) was dichotomous.

4 Seeing a Benefit of Professional Development

We analyzed the post-PD survey using inductive thematic analysis (Hatch, 2002), in the same manner as the pre-PD survey data. Results from the post-PD survey items (1) and (2) indicated PSTs and ISTs found the session to be useful and relevant. Additionally, after analyzing item (3) participants indicated taking away a better sense of the role in which teachers should take in fostering the SMPs. The benefits of this protocol-centered PD are centered around three main themes: the participants underwent a shift from focusing on students' actions to teachers' actions when engaging in the SMPs, the protocol assisted in developing a better perspective of the SMPs, and there were areas of confusion that require additional PD.

4.1 Focusing on Teacher's Actions

The first benefit of our PD experience is both PSTs and ISTs became more aware of the role teachers take when promoting the SMPs. One idea that an IST, Brooke, took away from the session is: "It [PD session] changed my mindset of always thinking about what the students should be doing [to] what the teachers should be doing when incorporating SMPs." She expressed a change of mindset from the student to the teacher perspective through the discussions centered around a teacher-focused protocol. Similarly, Maria, a PST, shared: "Although the SMPs are standards of what students should be doing, it is up to the teacher to provide guidance and resources to encourage students to demonstrate these behaviors outlined by the SMPs." The SMPs, as written, describe student behaviors (including ways of thinking and reasoning) but the teacher plays an important role in assisting students to engage in these behaviors. Such an idea represents the recognition of the teacher's role in promoting the SMPs as a result of PD that uses a protocol focused on math teaching behaviors. Mike, an IST, wrote that "Teachers have to know how to plan lessons with the SMPs [in mind] if their students are going to be taking part in them." We agree with Mike, teachers must know how to incorporate the SMPs into their lesson planning in order for students to engage in them. Many indicators found in the *Revised SMPs Look-for Protocol* require a teacher's consideration while lesson planning. These statements from the PSTs and ISTs indicates the shift in perspective that resulted from the PD.

4.2 Benefits of the Protocol

The second take away idea from the PD is the critical role the teacher-focused protocol played in facilitating meaningful and productive conversations. Another PST, Beth, expressed a need for a protocol that described observable math teaching behaviors linked with the SMPs. "The SMPs were written for the students, but we must determine indicators of what teachers do to engage students in these practices." Beth recognized the need for a protocol that focuses on the teacher-perspective of fostering the SMPs and relatedly, a learning opportunity to develop her perspective through this protocol-centered PD. Synthesizing across these PSTs' statements, PSTs experienced a shift in perspective after engaging in a brief protocol-centered PD. Similar shifts in perspectives were noted with the take away ideas made by ISTs.

Ja'Quan, an IST, shared that this protocol-centered PD enabled him in "focus[ing] on what each SMP looks like from the teacher rather than the student [perspective]." Therefore, Ja'Quan, like participating PSTs, experienced a shift in perspective as a result of the PD experience. Josephine, an IST, wrote that the PD experience resulted in an overall better perspective of the SMPs because she left the PD thinking about "what [the] SMP's look like from the student and teacher perspective." She gained a better perspective of the student role in engaging in the SMPs as well as the teachers' role in promoting the SMPs. Although a better understanding of the student perspective might come from reading and discussing the SMP descriptions, the newly obtained ideas of the teachers' perspective seems to stem from the discussion of the math teaching behaviors found in the *Revised SMPs Look-for Protocol*. A PST, Matthew, expressed that while the descriptions of the SMPs found in the CCSSM and Ohio Learning Standards are focused on students, there is a need for discussing the teacher perspective: "The idea of observable indicators of the SMPs [for teachers] are needed." Matthew recognized the need for thinking about what teacher behaviors allow for students to engage in the SMPs. The PD experience influenced his perspective on the value for considering ways teachers might promote the SMPs. Similarly, Sarah, an IST, added a more explicit take on the PD by saying she developed a stronger perspective about "how to encourage my students to show evidence of their thinking." This statement describes a specific math teaching behavior that stems from examining the observable indicators expressed in the *Revised SMPs Look-for Protocol*. Sarah, like her peers, recognized the importance of examining math teaching behaviors as a result of the

PD experience. In sum, the ISTs and PSTs expressed similar ideas indicating a shift in perspective as a result of analyzing the teacher-focused protocol.

4.3 Additional Questions for future Professional Development

The final theme suggests that our PD experience resulted in generating a series of additional questions about the SMPs as well as supported teachers' interest to attend future PD experiences. Results from item four of the post-PD survey indicated that while participants experienced a shift in perspective about the SMPs, there were still many questions about what the SMPs mean and how teachers can foster them. For example, Jordan, a PST, expressed concern about differentiating between SMP7 and SMP8, a common concern that was brought up during the PD. An IST, Charlie, indicated a concern about mathematical modeling (SMP4) in the classroom. Modeling was another important topic discussed during the PD experience. However, since the PD was limited to two hours, concerns such as those were not thoroughly addressed, indicating a need for future PD experiences. Results from item five indicated that both PSTs and ISTs showed unanimous interest in attending a second session about the SMPs. They fully recognized that there was much to learn about the SMPs; using the lens of teachers' promotion of SMPs offered a needed and novel perspective for supporting their future classroom practice.

Overall, PSTs and ISTs left the PD with a better perspective of the role in which teachers play when promoting the SMPs in classroom instruction. The shift in focus by participants was initiated through the use of a teacher-focused protocol during the PD experience. Finally, future PD experiences were both warranted and desired by PSTs and ISTs.

5 Implications for Protocol-Centered Teacher-Focused Professional Development

As discussed previously, it seems that PSTs and ISTs focus on students' perspectives of engaging in the SMPs through the process of analyzing math tasks (Bleiler et al., 2015; Bostic & Matney, 2014; Graybeal, 2013; Johns, 2016; Kolb, 2015; Peretin, 2014; Stohlmann et al., 2015). However, if PSTs and ISTs are not encouraged to shift their perspective from student engagement in the SMPs to ways that they might promote them during instruction, then PSTs and ISTs may not be able to effectively foster engagement in the SMPs. The ideas found in the pre-PD survey data provide evidence about participants' lack of familiarity with and potentially ineffective promotion of the SMPs.

Prior to the PD, PSTs and ISTs failed to offer observable math teaching behaviors that would encourage students to engage in the SMPs for grades 4-12 math. We agree with Olson et al.'s (2014) proposal for a more critical analysis of teacher behaviors that encourage a more thorough and meaningful engagement in the SMPs. The *Revised SMPs Look-for Protocol* includes a variety of math teaching behaviors that might lead to deep engagement in the SMPs. For example, an indicator for SMP3 is: "Provide and orchestrate opportunities for students to listen to the solution strategies of others, discuss alternative strategies or solutions, and defend their ideas" (See Appendix A). This indicator is a math teaching behavior in which the teacher's behavior encourages students to be engaged in listening to, discussing, and defending ideas. Such an indicator results in a deeper level of student engagement in the SMP in contrast to the indicators expressed in the pre-PD survey. Therefore, a protocol-centered PD experience, as presented here, that allows for a critical analysis of a protocol focusing on the teacher perspective of the SMPs appears to foster a better perspective of productive ways to engage students in the SMPs through specific math teaching behaviors.

Our results contribute to the knowledge base for PD, specifically scholarship focusing on de-

veloping teachers' perspective about math teaching behaviors. Examining a protocol that focused on teachers' behaviors to promote the SMPs was a unique aspect of this PD experience. It challenged participants to transition from surface-level behaviors related to the SMPs, like those described by Olson et al. (2014), to developing more appropriate ideas about ways to promote the SMPs during classroom instruction. We suggest that math teacher educators, including curriculum coaches, professional development specialists, and teacher leaders, take time to facilitate discussions with PSTs and ISTs about observable math teaching behaviors related to fostering the SMPs through a protocol-centered PD based on our data. We emphasize that results from this study suggest that a brief PD experience about the SMPs may be successful at encouraging a shift in teachers' perspectives of the SMPs and support their interest to participate in more sustained PD opportunities. Such PD opportunities may have a goal of further enriching teachers' knowledge and practice using the SMPs. A protocol, such as the *Revised SMPs Look-for Protocol*, as part of a PD focusing on the SMPs, may be a resource for math teacher educators to spark or continue conversations about the teacher's role in promoting the SMPs.

6 Final Thoughts

Teachers are critical to students' development of and engagement in the SMPs. We aimed to explore a protocol-centered PD for grades 4-12 ISTs and PSTs. Specifically, our frame was exploring their perspective about the teacher's role in promoting the SMPs during classroom instruction and their interest to learn more about the SMPs. Our results indicated that a brief protocol-centered PD focused on math teaching behaviors related to the SMPs supported PSTs and ISTs' perspective and desire to learn more about the SMPs. These results supplement, not replace, PD that supports teachers' exploration of mathematical practices through examination of students' work. We believe a brief PD experience, as described in this report, can be instrumental in beginning the conversation about teacher-focused SMP actions and encourage a teacher's desire to learn more. We recommend further PD on the meaning of the SMPs for classroom teachers and the teacher's role in promoting the SMPs with a goal of impacting teachers' classroom practices.

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Mathematical Practices	Teachers
SMP1: Make sense of problems and persevere in solving them.	**A** Involve students in rich problem-based tasks that encourage them to persevere in order to reach a solution. **B** Provide opportunities for students to solve problems that have multiple solutions and/or strategies. **C** Encourage students to represent their thinking while problem solving. NOTE: Task must be grade-level and developmentally-appropriate. That is, a solution is not readily apparent, the solution pathway is not obvious, and more than one pathway is possible.
SMP2: Reason abstractly and quantitatively.	**A** Facilitate opportunities for students to discuss representations or use representations to make sense of quantities and their relationships. **B** Encourage the flexible use of properties of operations, tools, and solution strategies when solving problems. **C1** Provide opportunities for students to decontextualize (abstract a situation) the mathematics within a mathematics task. **C2** Provide opportunities for students to contextualize (identify referents for symbols involved) the mathematics within a mathematics task. NOTE: Must have C1 and C2 to receive credit for indicator.
SMP3: Construct viable arguments and critique the reasoning of others.	**A** Provide and orchestrate opportunities for students to listen to the solution strategies of others, discuss alternative strategies or solution(s), and defend their ideas. **B** Ask higher-order questions which encourage students to defend their ideas, consider student(s) response(s) before making code. **C** Provide prompts/tasks that encourage students to think critically about the mathematics they are learning, must be related to argumentation or proving events. **D** Engage students in proving events that encourage students to develop and refine mathematical arguments (including conjectures) or proofs.
SMP4: Model with mathematics.	**A** Use mathematical models appropriate for the focus of the lesson. **B** Encourage student use of developmentally and content-appropriate mathematical models (e.g., variables, equations, coordinate grids). **C** Remind students that a mathematical model used to represent a problem's solution is "a work in progress" and may be revised as needed. **D** Employ problems arising from everyday life, the local community, society, and workplace such that the solution is a model to reuse. NOTE: Must have D to be considered a task embedded within instruction promoting modeling with mathematics.

Mathematical Practices	Teachers
SMP5: Use appropriate tools strategically.	**A** Use appropriate physical and/or digital tools to represent, explore and deepen student understanding. **B** Help students make sound decisions concerning the use of specific tools appropriate for the grade level and content focus of the lesson. **C** Provide access to materials, models, tools, and/or technology-based resources that assist students in making conjectures necessary for solving problems. Students must use the resources. NOTE: Representations do NOT count as tools.
SMP6: Attend to precision.	**A** Emphasize the importance of precise communication by encouraging students to focus on clarity of the definitions, notation, and/or vocabulary used to convey their reasoning. **B** Encourage accuracy and efficiency in computation and problem-based solutions, expressing numerical answers, data and/or measurements with a degree of precision appropriate for the context of the problem. **C** Foster explanations and justifications using clearly articulated oral and/or written communication and grade-level appropriate conventions. Explanation or justification must go beyond IRE.
SMP7: Look for an make use of structure.	**A** Engage students in discussions emphasizing relationships between particular topics within a content domain or across content domains. **B** Recognize that the quantitative relationships modeled by operations and their properties remain important regardless of the operational focus of a lesson. **C** Provide activities in which students demonstrate their flexibility in representing mathematics in a number of ways (e.g., $76 = (7 \times 10) + 6$; discussing types of quadrilaterals, etc.) **D** Encouraging examinations of a 'signal' and 'noise' in statistics-related tasks.
SMP8: Look for regularity in repeated reasoning.	**A** Engage students in discussion related to repeated reasoning that may occur while executing a problem-solving strategy in a problem's solution. **B** Draw attention to the prerequisite steps necessary to consider when solving a problem. **C** Urge students to continually evaluate the reasonableness of their results during problem solving.