More of the Story: Ruminations on "How Mathematics Education in Ohio Impacted the Nation"

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Abstract

This is one of three responses to two articles by James E. Schultz on how mathematics education in Ohio—and especially at Ohio State University—has influenced mathematics teaching and learning across the United States and beyond. Every mathematics teacher in Ohio should know this story. Schultz's two contributions are oral histories in written form and possess a biographical and even autobiographical flavor. My response will follow suite. After a brief introduction, I reflect on—and add to—Schultz's "Laying the Groundwork" article (2024a) and his "Incorporating Technology" piece (2024b). I then consider how the issues and legacy presented in these articles connect to mathematics teaching and learning today and how Ohio continues to influence mathematics education beyond its borders. I close with an epilogue on how I first met several key characters in this mathematical narrative.

Keywords: Curriculum reform; Mathematical pathways; Teacher professional development; Graphing calculators; Multiple linked representations; Research in mathematics education

My Connections to Mathematics Education in Ohio

Though I started school in San Antonio, Texas, it was three teachers in Kettering, Ohio, that sparked my deep interest in mathematics and inspired me to help others learn its scope and beauty: Sr. Gemma Glutz in sixth grade and my high school teachers Mr. Frank C. Maus in Geometry and Mr. Alvin K. Funderburg in Algebra III/Trig and Calculus. After graduating from high school in Kettering, I spent the next 13 years studying mathematics and mathematics education at the University of Texas at Austin and teaching mathematics in Austin and Houston.

Then, through a series of fortunate events—detailed in the epilogue—upon completing my Ph.D. in 1986, I was hired as an assistant professor of mathematics education in the Department of Educational Theory and Practice at the Ohio State University (OSU)—with a courtesy appointment in the Department of Mathematics. For the next 4 years, I worked closely with Alan R. Osborne, Franklin D. Demana, and Bert K. Waits on the OSU Calculator and Computer PreCalculus (C^2PC) Project. Although Al Osborne played a supporting role in Jim's narrative, Al was my main mentor and my connection to Frank and Bert. In the C^2PC project, Frank and Bert wrote the textbook materials; Al and I developed support materials for teachers. Frank, Bert, and I pilot-tested the emerging loose-leaf student materials, working side-by-side with partner teachers at high schools in the Columbus area from January 1987 through June 1988. The students used graphing technology on a daily basis to explore families of functions and to solve mathematical and applied problems.

The resulting textbook is now in its 11th edition (Demana et al., 2024) and the associated professional development has evolved into the Teachers Teaching with Technology (T^3) program, which is supported by Texas Instruments, Inc. Bert's Master Grapher (Waits & Demana, 1988c) software served as the starting point for the Texas Instruments TI-81, TI-82, TI-83, TI-84 series of handheld graphing calculators, used by students around the world. And, as noted by Schultz (2024b), Frank and Bert established multiple international conferences on technology that continue to this day.

After leaving Ohio State in 1990, I continued to collaborate with these mentors in various ways. Currently, I am a national instructor in the T^3 program, and I have been the lead author of *Precalculus: Graphical, Numerical, Algebraic* for the past three editions. In 2007, I returned to Ohio, following Jim Schultz as Robert L. Morton Professor of Mathematics Education at Ohio University in Athens. It is from these perspectives that I reflect on Jim's articles.

Reflections on "Laying the Groundwork"

Now that I have provided some of my connections to Jim Schultz's historical accounts, let me turn to his first installment—"Laying the Groundwork for Reform" (2024a). How mathematics education in Ohio has influenced the rest of the nation is a story that every mathematics teacher in Ohio should know.

First off, Jim paints a beautiful picture of Arnold E. Ross, exploring many details that were new to me. Arnold was an amazing person who broke down barriers and built bridges between persons and groups. In a personal stroke of good luck, I met Arnold at dinner on the first day of my on-campus interview at Ohio State in May 1986. Al Osborne and some other faculty took me to the OSU Faculty Club; Arnold was dining by himself, and Al invited him to join us. Arnold sat next to me and told stories throughout the evening. During dinner, Arnold taught me that all mathematicians, mathematics educators, and mathematics teachers are colleagues within a single mathematics community. Arnold must have used the term *colleague* a hundred times that evening. Throughout his career, Arnold built community through his words and his actions.

Perhaps the most important point of Jim's recollections of Arnold is that he created a unified, welcoming environment in the Department of Mathematics at Ohio State University. Without Arnold Ross at the helm, mathematics educators Jim Schultz and Bert Waits would never have been hired. Nor in 1965 would the married couple of mathematicians James R. C. and Joan R. Leitzel have been hired. In her October 2024 acceptance speech for the Christofferson-Fawcett Award (Figure 1), Joan related how Arnold "pulled strings" to make it possible for them to come together to the Ohio State Mathematics Department.

Figure 1: Joan Leitzel receives the 2024 Christofferson-Fawcett Award from OCTM president Mike Lipnos, as 2023 awardee Mike Mikusa proudly looks on.

Both Jim and Joan Leitzel went on to become influential figures in mathematics and education in the United States: Jim, who at Ohio State was known for shepherding the Master of Arts in Mathematics program for schoolteachers, was instrumental in establishing the Mathematical Association of America's Project NExT (**N**ew **Ex**periences in **T**eaching), which to this day helps new Ph.D.s in mathematics make the transition into careers as professors. Joan, who was actively involved in mathematics curriculum development at Ohio State, went on to hold major leadership positions with the National Science Foundation (division director) and the Mathematical Sciences Education Board (chair, 2000–2005)—and served as the President of the University of New Hampshire (1996–2002).

As Jim Schultz noted, Arnold Ross would frequently reach out to others in the campus community and beyond. Yes, Arnold knew Woody Hayes and consequently had access to the governor. But just as important, Arnold knew everyone at the Faculty Club, and everyone knew him. This made it natural for mathematicians, such as Frank Demana and Jim and Joan Leitzel to work with education colleagues across campus and to collaborate with colleagues from the state of Ohio's Department of Education.

Three related projects in the 1980s, which Jim Schultz did not mention or emphasize, set the stage for the Demana-Waits collaboration and foreshadowed today's mathematical pathways:

- **Early Mathematics Placement Testing (EMPT) program.** In this project, high school juniors across Ohio were tested to see which mathematics course they would place into if they were to attend Ohio State University. This revealed that many students who had successfully completed Algebra 1, Geometry, and Algebra 2 would nonetheless place into remedial mathematics. Laughbaum (2024) provides further details about the Ohio EMPT program, which was replicated in other states.
- **Transition to College Mathematics.** Once EMPT made it clear that many high school students were not learning algebra in a way that would stick with them, Frank Demana and Joan Leitzel in the OSU Department of Mathematics collaborated with Joe Crosswhite and Alan Osborne in the College of Education to create a new path through algebra. This innovative approach focused on solving problems grounded in real-world contexts—first numerically using tables with calculator support, then using the associated hand-drawn graphs, and finally using algebraic equations. This new way of teaching algebra became a textbook for high school seniors and college freshmen—*Transition to College Mathematics* (Demana et al., 1984). Demana and Joan Leitzel (1988) explained this approach to the rest of the nation in their NCTM *Ideas of Algebra* yearbook chapter entitled Establishing Fundamental Concepts Through Numerical Problem Solving.
- **Approaching Algebra Numerically (AAN).** After Joe Crosswhite left for Reston, Virginia, to become the full-time NCTM President (Schultz, 2024a), Frank Demana, Joan Leitzel, and Al Osborne collaborated with Suzanne K. Damarin, Jim Leitzel, and then Ph.D. student Charles B. (Chuck) Vonder Embse to create supplemental curriculum materials for seventh and eighth graders who were *Getting Ready for Algebra* (Demana et al., 1988a, 1988b). These two books used the same three-step approach to algebra as *Transition to College Mathematics*. In addition, the project team created the AAN Calculator—"a computer-based simulation of a hand-held calculator designed with novice learners in mind" (Vonder Embse et al., 1988, p. 12).

The combination of education research and instructional engineering used in these projects is an early example of what is now called *design science* (Cobb, 2007). These projects are also lovely examples of interdisciplinary collaboration, the use of *multiple linked representations*, and the strategic use of technology. This was cutting edge stuff!

Due to space limitations, Jim Schultz did not to tell complete stories of Harold Fawcett or Marilyn Suydam. Nor did he even mention Suzanne Damarin or Jim Leitzel. Here I will add just a tad. Donna Berlin and Art White were two other professors in the College of Education who were nationally known for their work with the School Science and Mathematics Association. Donna started at one of OSU's regional campuses, and Art was a science educator, but he taught research methods to all of OSU's mathematics education Ph.D. students. And then there was Peggy Kasten, who Jim Schultz mentions as a coauthor and correspondent. When Peggy and I joined the education faculty at OSU in 1986, we became Professors 13 and 14 doing work in mathematics education! Thus, there remains much to tell about Ohio State University in its mathematics education heyday. An entire book would be needed to do justice to this story. Moreover, Ohio has had many influential figures who never toiled at Ohio State University; these include Halbert C. Christofferson of Miami University, Kenneth Cummins of Kent State University, Linda M. Gojak of John Carroll University, Lenard Pikaart of Ohio University, and William R. Speer of Bowling Green State University, to name but a few. To explore further the story of Ohio's influence on U.S. mathematics education, visit the Awards and History and Legacy pages of OCTM's website.

Reflections on "Incorporating Technology to Carry Out the Vision"

Now I turn to Jim Schultz's second article—"Incorporating Technology to Carry Out the Vision" (2024b). Through good fortune, I was able to witness much of this part of the story firsthand—and even participate in it to a degree.

In 1986 as I arrived at Ohio State, Frank Demana and Bert Waits had just joined forces in the Calculator and Computer PreCalculus (C^2PC) project. C^2PC (pronounced "C-squared P C") was a natural extension of the Transition to College Mathematics project. The motivating question for C^2PC was, *What should high school juniors or senior study if they have successfully learned Algebra 1, Geometry, and Algebra 2—as demonstrated by their performance on the EMPT test?* The answer in 1986 was to study functions and their graphs in preparation for calculus. (The increasing importance of statistics was emerging and would be incorporated into precalculus a decade or so later.) In working on this design challenge, Frank and Bert knew from EMPT results that traditional precalculus mathematics did not take hold for many students. Plus, Transition to College Mathematics had convinced Frank of the importance of (a) real-world problems to motivate students and help them see how mathematics connected to the world around them, (b) the strategic use of technology, (c) using numerical and graphical representations to support the understanding of algebraic problem solving. Meanwhile, Bert had developed Master Grapher software that could graph functions (as well as polar and parametric equations and even real-valued functions of two variables as surfaces). Frank and Bert were ready to design and engineer a curriculum that built on Frank's prior collaborative work and made full use of Bert's Master Grapher software, which ran on the then popular Apple IIe microcomputer.

Frank and Bert started writing the student instructional materials in 1986. Using the typical table of contents of the time, Frank created the section narratives, examples, and solutions. Bert wrote the exercise sets. The use of Master Grapher was incorporated throughout the narrative, and the concepts of viewing window, standard approximation, complete graph, and end behavior emerged. The book would become *Precalculus Mathematics: A Graphing Approach* (Demana et al., 1990).

Starting in January 1987, to pilot-test the materials Frank taught at Centennial High School in Columbus City Schools with Pam Dase; Bert taught at Upper Arlington High School with Bruce Blackston; and I taught at Worthington Alternative High School with Char Wilkerson. I received a Research Challenge Program Investigators' Fund Award from Ohio State to buy

$1,000 worth of Casio fx-7000G graphing calculators for students to use at Worthington Alternative. For the 1987–1988 school year, with the help of a $17,900 University Seed Grant from the OSU Office of Research and Graduate Studies, I upgraded to the Casio fx-8000G and moved to Walnut Ridge High School in Columbus to teach with Fred Koenig while Frank and Bert continued with Pam and Bruce. In addition, Ron Meyer taught solo using the loose-leaf draft of *Precalculus Mathematics: A Graphing Approach* at Franklin Heights High School in South-Western City Schools during 1987–1988.

This partnering with practicing teachers and engaging a variety of students across diverse schools helped to make these cutting-edge materials teachable across the country. In addition, we discovered that the hand-held devices allowed students to graph in a regular classroom setting without the teacher reserving a computer lab and thus allowed students daily access to graphing technology. Students could borrow graphing calculators like textbooks and use them both in class and at home.

Frank wrote grant proposals to the Ohio Board of Regents (now the Ohio Department of Higher Education) and Standard Oil of Ohio (now, British Petroleum) to support the pilot testing. Frank and Alan wrote separate proposals to the U.S. National Science Foundation to support related teacher professional development during 1987–1990. Meanwhile, as noted by Schultz, Bert was busy writing the 1989 NCTM *Curriculum and Evaluation Standards for School Mathematics*, which triggered a major curriculum reform movement at the school level that coincided with the U.S. calculus reform movement at the university level. Plus, the C^2PC team produced an amazing number of journal articles and book chapters related to our work, beyond those cited by Jim Schultz (Demana et al., 1990, 1993; Demana & Waits, 1987, 1988, 1989, 1990a, 1990b; Foley, 1987a, 1987b, 1990a, 1990b, 1992; Waits & Demana, 1987, 1988a, 1988b, 1988d, 1989, 1993).

Summer institutes for high school teachers—and workshops for college and university faculty—began in 1988. Field testing of the softbound, two-volume preliminary edition of *Precalculus Mathematics: A Graphing Approach* followed during the 1988–1989 school year. The two-day C^2PC follow-up workshop in December 1988 at Ohio State grew into what is now the annual T^3 International Conference.

This flurry of project activity in the late 1980s and the 1990s created a rich environment for the 20–30 students in the mathematics education Ph.D. program at any given time—many of whom had received MA degrees from the Mathematics Department and many of whom were graduate teaching assistants in Mathematics. Al Osborne, mathematics education professor Richard J. Shumway, and mathematics professor and calculus reformer William Davis directed at least 10 Ph.D. dissertations related to enhancing mathematics teaching and learning via technology: Browning (1989), Crocker (1991), Dunham (1991), Farrell (1990), Martínez-Cruz (1993), Nichols (1992), Palmiter (1986), Tuska (1993), Vonder Embse (1987), and Wayand (1998). Many of these dissertations led to related publications (e.g., Dunham & Osborne, 1991; Farrell, 1996, Vonder Embse, 1990).

Reflections on How Mathematics Education in Ohio Has Influenced Mathematics Education Across the Nation and Beyond

Harold P. Fawcett (1894–1976), Arnold E. Ross (1906–2002), and F. Joe Crosswhite (1929–2020) were giants in mathematics education. Their commitment to mathematics learning, their love of people, and their hard work led to long lists of accomplishments during their lives. Their true legacies, however, lie in the foundation they laid for the future flourishing of others who followed in their footsteps at the Ohio State University.

Research in mathematics education

Fawcett's (1938) *The Nature of Proof*, which NCTM reprinted in 1995, was a seminal work in research in mathematics education. Thus, in 1980, Richard J. Shumway was following in Fawcett's footsteps when he edited *Research in Mathematics Education* for NCTM. Shumway (1980) served as a key reference for scholars in mathematics education until it was superseded by Grouws (1992), which was later supplanted by Lester (2007) and then Cai (2017). Similarly, as noted by Schultz (2024a), Marilyn N. Suydam was an international leader in the dissemination of research on calculators, but more importantly, from 1971 through 1994, she created an annual international summary of all research in mathematics education first in the *Arithmetic Teacher* and—once it was established—in the *Journal for Mathematics in Mathematics Education*. In the final few years, Ph.D. student Deborah A. Crocker and then fellow faculty member Patricia Brosnan served as Marilyn's coauthors. Due to the ever increasing number of research articles in mathematics education around the word, continuing this amazing tradition ultimately became impossible. To say that Ohio State was a national leader in supporting research in mathematics education during this time frame is an understatement.

Interdisciplinary collaboration

Meanwhile, as Schultz (2024a) plainly asserts, Arnold Ross and Joe Crosswhite created an amazingly productive environment of collaboration and mutual support between the OSU Department of Mathematics and College of Education—a confluence rarely witnessed anywhere, and perhaps unequaled. In the 1980s, faculty in Mathematics teamed with faculty in Education to develop three innovative curricula that leveraged emerging technologies and multiple linked representations of functions to give students new insights and to solve realistic problems: Transition to College Mathematics, AAN, and C^2PC.

Curriculum reform

In the wake of the 1989 NCTM Standards, the U.S. National Science Foundation (NSF) put out a call for creating curricula aligned with the new standards. Frank, Bert, Joan Leitzel, Alan Osborne, and others at Ohio State—and as Jim Schultz notes, Zalman Usiskin and his University of Chicago School Mathematics Project (UCSMP) were ahead of this curricular tsunami and were already developing student learning materials that had influenced the creation of the NCTM Standards. Ohio State's significant role in curriculum reform was generally overlooked in books written about the design and development of innovative school mathematics materials during this era (Hirsch, 2007; National Research Council, 2004; Reys et al, 2010; Senk & Thompson, 2003) because the OSU curricula were developed without NSF funding.

Calculus reform. In the 1990s, Frank and Bert teamed with successful calculus authors Ross L. Finney and George B. Thomas, Jr., to create a calculus book in the spirit of C^2PC and calculus reform. Over time, this calculus curriculum development project has evolved into Demana et al. (2020) *Calculus: Graphical, Numerical, Algebraic* (Advanced Placement® sixth edition). Textbooks by Demana and Waits have been translated into Spanish and Portuguese and have international English editions. Since the 1980s, tens of millions of students have learned precalculus, calculus, or both from books written by Frank Demana and Bert Waits—thus having a direct effect on the mathematical education of generations of students across the country and around the world. In addition, OSU mathematician Bill Davis was actively involved in using computer algebra for calculus reform.

National curriculum standards. Ohioans not yet mentioned have guided the development of documents that have set standards for mathematics education at the national level. Brad

Findell, Associate director of mathematics programs for teachers in the OSU Department of Mathematics, was one of three editors of the influential report *Adding It Up: Helping Children Learn Mathematics* (National Research Council, 2001), which defined *mathematical proficiency* as the interaction among conceptual understanding, procedural fluency, strategic competence, adapting reasoning, and productive deposition—a definition that continues to guide U.S. mathematics curriculum at all levels, not just for children. Ohioans Richelle (Rikki) Blair of Lakeland Community College and Nancy J. Sattler of Terra State Community College, both of whom served as presidents of the American Mathematical Association of Two-Year Colleges (AMATYC), edited *Beyond Crossroads: Implementing Mathematics Standards in the First Two Years of College* (AMATYC, 2006) and *IMPACT: Improving Mathematical PROWESS and College Teaching* (AMATYC, 2018), respectively. These two signature documents of AMATYC are guiding lights for teaching lower-division undergraduate mathematics in the United States and Canada.

Mathematical pathways

Jim claims Ohio's leadership in "Providing appropriate pathways for all students" (Schultz, 2024a, p. 30). In their position statement of pathways, AMATYC (2021) identified three principal mathematical pathways: statistics, quantitative reasoning, and algebra to calculus.

Statistics. Jerry Moreno of John Carroll University was an author of *Guidelines for Assessment and Instruction in Statistics Education (GAISE) Report: A pre-K–12 Curriculum Framework*, published by the American Statistical Association (ASA, 2007). Ohioans Michelle Everson (OSU) and Jeffrey Witmer (Oberlin College) have been authors of the College GAISE Reports (ASA, 2005, 2016). The reports have guided curricular reform in statistics. And as noted by Schultz (2024), the UCSMP *Functions, Statistics, and Trigonometry* (Rubenstein et al., 1992) was a pioneer of integrating statistics with traditional high school mathematics topics.

Quantitative reasoning. Ohio has been a national leader in the quantitative reasoning pathway. The roots of this pathway can be seen in the Transition to College Mathematics project. Based on 3 years of pilot testing, Foley and colleagues (2014) wrote the first high school textbook for a course in Quantitative Reasoning. Ohio has now rolled out a course for high school juniors and seniors called Mathematical Modeling and Reasoning (Miller & Walls, 2021) and gateway postsecondary course in Quantitative Reasoning (Foley & Wachira, 2021). Once again, Ohio has been ahead of the curve and was seen as being too far along to be invited to participate in the nationwide Launch Years project of the Dana Center of the University of Texas at Austin, which Burrill (2024) cites in her response to Schultz's articles. Indeed, in November 2023, the Dana Center invited a group from Ohio to present a webinar on *Ohio's Secondary to Postsecondary Quantitative Reasoning Pathway* to representatives from other states participating in the Launch Years Initiative (Foley et al., 2023). Moreover, with NSF support, Foley and Wayand (2022–2026) have created QuantNet Ohio, a statewide professional development network for Quantitative Reasoning instructors, which will be replicated in other state starting in 2025.

AP®mathematics. Ohio has been a national leader first in technology-enhanced Advanced Placement Calculus and more recently in AP Precalculus. The Demana-Waits *Calculus: Graphical, Numerical, Algebraic*, now in its sixth edition, set the textbook standard for AP Calculus AB and AP Calculus BC since graphing calculators were required for testing beginning in 1995. The 11th edition of Demana-Waits *Precalculus: Graphical, Numerical, Algebraic* (2024) is the only textbook developed to align with the College Board AP Precalculus, which was first offered during the 2023–2024 school year. This demonstrates the power of Frank and Bert's legacy in AP®mathematics. The evidence listed above supports Jim's first claim.

Professional development

Schultz (2024a, p. 30) also contends that Ohio has been a leader in "Developing courses to prepare teachers to teach [innovative] curriculum." As we have seen, the professional development component of C^2PC led to T^3—one of today's finest international professional development enterprises for teachers of mathematics. On a personal level, C^2PC led to me to create a lifetime of professional development (PD) opportunities for countless colleagues:

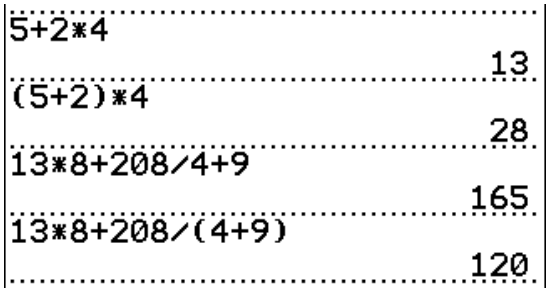
- CalcNet PD network for high school calculus teachers, first for central Ohio while at OSU and then nationally at Sam Houston State University
- South Central Calculus Coalition for college and university faculty while at SHSU
- Mathematics Education Leadership Training (MELT) for teachers while at Appalachian State University
- Quantifying Uncertainty and Analyzing Numerical Trends (QUANT) and Modeling and Spatial Reasoning (Modspar) for high school mathematics teachers and currently the QuantNet PD network for college and university Quantitative Reasoning instructors

Many participants in C^2PC, T^3, and these other PD programs have themselves become PD instructors, thus creating a ripple effect positively impacting tens of thousands of teachers!

Technology

I will close with Schultz's (2024a, p. 30) claim of Ohio's leadership in "Incorporating technology, especially graphing calculators" into K–12 curriculum. Both the AAN Calculator and Master Grapher have been built into the Texas Instruments TI-81, TI-82, TI-83, TI-84 series of graphing calculators and into the curricula and professional development detailed above. Burrill (2024) addresses this aspect of the legacy quite well, so I will only make one additional point that relates to the mostly forgotten AAN Calculator. Figure 2 illustrates two of Vonder Embse et al.'s (1988) examples of the role and importance of grouping symbols. These arithmetic examples remind me of Arnold Ross's motto to "think deeply of simple things" as well as *Adding It Up*'s notion that *mathematical proficiency* is founded on the twin pillars of conceptual understanding and procedural fluency. 'Aha! moments' need not be deep insights about the behavior of functions; they can occur any day at any level when we engage students in doing and exploring mathematics.

Figure 2: Students can discover the role of grouping symbols in the order of operations when computing arithmetic expressions using a multi-line calculator (cf. Vonder Embse et al., 1988).



Epilogue: Being at the Right Place at the Right Time

As a Ph.D. student at the University of Texas in the 1980s, my advisor L. Ray Carry taught me about the Big Three in mathematics education in the United States at that time: the University of Georgia, the University of Wisconsin, and the Ohio State University. I had read book chapters and articles by F. Joe Crosswhite, Alan R. Osborne, James E. Schultz, Richard J. Shumway, Marilyn N. Suydam, and Arthur L. White—all from Ohio State University.

In April 1986, at the annual meeting of the National Council of Teachers of Mathematics in Washington, DC, Ray Carry happened across Al Osborne, who mentioned that Ohio State had just received funding to hire an assistant professor in mathematics education. My wife (Jo) and I were at NCTM with our 4-month-old son. I was thrilled when Jo told me that Dr. Carry had called our room and had some good news to share. This chance meeting between Ray and Al led to my being invited to lunch the next day with Frank Demana, Al Osborne, Jim Schultz, and Bert Waits. They wanted to talk to me about the OSU assistant professorship. Little did I know where this encounter might lead!

During lunch, I learned that Bert had written a microcomputer program called Master Grapher, and Frank and Bert wanted to develop a curriculum for Precalculus that integrated this computer graphing utility in the hands of students daily in class. Frank and Bert were working with Al Osborne to get grant funding and find high school classrooms to pilot this curriculum. In fact, right after lunch, Bert and Frank were going to meet with representatives of Addison-Wesley Publishing Company to pitch their idea. I could tag along if I didn't have other plans. I choose to join them. Two months later, I was hired at Ohio State. Frank, Bert, and Al were established professors at the height of their careers. They took me under their wings and embraced me as a colleague and friend. I was on the ground floor of the C^2PC project and its associated textbook and professional development. These mentors at Ohio State—especially Alan Osborne—introduced me to scores of colleagues across the United States. Frank, Bert, and Al set me on a career arc that has focused writing grant proposals, developing curriculum, serving professional organizations, and conducting professional development for mathematics teachers and college faculty. These mathematical mentors created opportunities for me to present at conferences across the nation and around the globe. For the many doors they opened for me, I will be forever grateful!

References

American Mathematical Association of Two-Year Colleges. (2006). *Beyond crossroads: Implementing mathematics standards in the first two years of college*. Blair, R. (Ed.). https://amatyc.org/general/custom.asp?page=beyondcrossroads

American Mathematical Association of Two-Year Colleges. (2018). *IMPACT: Improving mathematical PROWESS and college teaching*. Orrange, M. B., & Sattler, N. J. (Eds.). https://amatyc.org/page/amatycimpact

American Mathematical Association of Two-Year Colleges. (2021, November 6). Mathematics pathways [Position statement]. https://amatyc.org/page/PositionMathematicsPathways

American Statistical Association. (2005). *Guidelines for assessment and instruction in statistics education (GAISE): College report*. Garfield, J. B. (Chair), Aliaga, M., Cobb, G., Cuff, C., Gould, R., Lock, R., Moore, T., Rossman, A., Stephenson, B., Utts, J., Velleman, P., & Witmer, J.

American Statistical Association. (2007). *Guidelines for assessment and instruction in statistics education (GAISE) report: A pre-K–12 curriculum framework*. Franklin, C., Kader, G., Mewborn, D., Moreno, J., Peck, R., Perry, M., & Scheaffer, R.

American Statistical Association. (2016). *Guidelines for assessment and instruction in statistics education (GAISE) college report 2016*. Everson, M. (Cochair), Mocko, M. (Cochair), Carver, R., Gabrosek, J., Horton, N., Lock, R., Rossman, A., Rowell, G. H., Velleman, P., Witmer, J., & Wood, B. http://www.amstat.org/education/gaise/GaiseCollege_Full.pdf

Browning, C. A. (1989). Characterizing levels of understanding of functions and their graphs (Doctoral dissertation, The Ohio State University, 1988). *Dissertation Abstracts International, 49*, 2957A.

Burrill, G. (2024). Reflections on how mathematics education in Ohio impacted the nation. *Ohio Journal of School Mathematics, 98*, 54–60. https://ohiomathjournal.org/index.php/OJSM/article/view/10279/8291

Cai, J. (Ed.). (2017). *Compendium for research in mathematics education*. National Council of Teachers of Mathematics.

Cobb, P. (2007). Putting philosophy to work: Coping with multiple theoretical perspectives. In F. K. Lester, Jr. (Ed.), *Second handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics* (Vol. 1, pp. 3–38). Information Age.

Crocker, D. A. (1991). A qualitative study of interactions, concept development, and problem solving in a calculus class immersed in the computer algebra system Mathematica™. (Doctoral dissertation, The Ohio State University). *Dissertation Abstracts International, 52*(8). Dissertation 9201643. ProQuest 303941403.

Demana, F., Dick, T., Harvey, J., Kenelly, J., Musser, G., & Waits, B. (1990). Graphing calculators: Comparisons and recommendations. *The Computing Teacher, 17*(7), 24–31.

Demana, F., Harvey, J., Foley, G. D., & Dion, G. (1993). Using graphing calculators to enhance the teaching and learning of university mathematics: The United States experience. In P. Jaworski (Ed.), *Proceedings of the Conference on Technology in Mathematics Teaching* (p. 572). Birmingham, England: University of Birmingham.

Demana, F. D., & Leitzel, J. R. (1988). Establishing fundamental concepts through numerical problem solving. In A. F. Coxford & A. P. Shulte (Eds.), *The ideas of algebra, K–12: 1988 yearbook* (pp. 61–68). National Council of Teachers of Mathematics.

Demana, F., Leitzel, J. R. (Primary Authors), Crosswhite, F. J., & Osborne, A. (1984). *Transition to college mathematics*. Addison-Wesley.

Demana, F., Leitzel, J. R. (Primary Authors), Crosswhite, F. J., & Osborne, A. (1989). *Essential algebra: A calculator approach*. Addison-Wesley.

Demana, F., Leitzel, J. R., & Osborne, A. (1988a). *Level 1: Getting ready for algebra*. D. C. Heath.

Demana, F., Leitzel, J. R., & Osborne, A. (1988b). *Level 2: Getting ready for algebra*. D. C. Heath.

Demana, F. D., & Waits, B. K. (1987). Enhancing problem-solving skills in mathematics through microcomputers. *Collegiate Microcomputer, 5*, 72–75.

Demana, F. D., & Waits, B. K. (1988). The Ohio State University Calculator and Computer Precalculus Project: The mathematics of tomorrow today! *The AMATYC Review, 10*(1), 46–55.

Dunham, P. H. (1991). Mathematical confidence and performance in technology-enhanced precalculus: Gender-related differences (Doctoral dissertation, The Ohio State University, 1990). *Dissertation Abstracts International, 51*, 3353A.

Dunham, P. H., & Osborne, A. (1991). Learning how to see: Students' graphing difficulties. *Focus on Learning Problems in Mathematics, 13*(4), 35–49.

Farrell, A. M. (1990). Teaching and learning behaviors in technology-oriented precalculus classrooms (Doctoral dissertation, The Ohio State University, 1989). *Dissertation Abstracts International, 51*, 100A.

Farrell, A. M. (1996). Roles and behaviors in technology-integrated precalculus classrooms. *Journal of Mathematical Behavior, 15*, 35–53.

Fawcett, H. P. (1938). *The nature of proof: 13th yearbook of the National Council of Teachers of Mathematics*. Bureau of Publications, Teachers College, Columbia University. (Reprinted 1995, National Council of Teachers of Mathematics).

Foley, G. D. (1987a). Future shock: Hand-held computers. *AMATYC Review, 9*(1), 53–57.

Foley, G. D. (1987b). Zoom revisited. *Mathematics Teacher, 80*(8), 606. https://doi.org/10.5951/MT.80.8.0605

Foley, G. D. (1990a). Accessing advanced concepts with technology. *AMATYC Review, 11*(2), 58–64.

Foley, G. D. (1990b). Using hand-held graphing computers in college mathematics. In F. D. Demana, B. K. Waits, & J. Harvey (Eds.), *Proceedings of the Conference on Technology in Collegiate Mathematics* (pp. 28–39). Addison-Wesley.

Foley, G. D. (1992). The power of parametric representations. In J. T. Fey & C. R. Hirsch (Eds.), *Calculators in mathematics education: 1992 yearbook of the National Council of Teachers of Mathematics* (pp. 138–144). Reston, VA: National Council of Teachers of Mathematics.

Foley, G. D., Butts, T. R., Phelps, S. W., & Showalter, D. A. (2014). *Advanced quantitative reasoning: Mathematics for the world around us* (Common Core edition). AQR Press.

Foley, G. D., Cannelongo, A., Gorsuch, R., Miller, S. W., & Wayand, L. (2023, November 30). *Ohio's secondary to postsecondary quantitative reasoning pathway: Curricula, articulation, and professional development* [Webinar]. 10th Cross-State Learning Session, Launch Years Initiative, Charles A. Dana Center, University of Texas at Austin, United States.

Foley, G. D., & Wachira, P. W. (2021). From gatekeeper to gateway: The role of quantitative reasoning. *Ohio Journal of School Mathematics, 87*, 29–36. https://ohiomathjournal.org/index.php/OJSM/article/view/8339/6125

Foley, G. D. (principal investigator), & Wayand, L. (coprincipal investigator). (2022, June–2026, May). *QuantNet: Developing statewide professional development networks for effective teaching of undergraduate Quantitative Reasoning courses*. National Science Foundation, Improving Undergraduate STEM Education, Award No. 2216197.

Grouws, D. A. (Ed.). (1992). *Handbook of research on mathematics teaching and learning* [A project of the National Council of Teachers of Mathematics]. Macmillan.

Hirsch, C. R. (Ed.). (2007). *Perspectives on the design and development of school mathematics curricula*. National Council of Teachers of Mathematics.

Laughbaum, E. D. (2024). Time with Demana/Waits. *Ohio Journal of School Mathematics, 98*, 50–53. https://ohiomathjournal.org/index.php/OJSM/article/view/10222/8289

Lester, F. K., Jr. (Ed.). (2007). *Second handbook of research on mathematics teaching and learning: A project of the National Council of Teachers of Mathematics* (2 vols.). Information Age.

Martínez-Cruz, A. M. (1993). Knowledge and development of functions in technology-enhanced high school precalculus class: A case study (Doctoral dissertation, The Ohio State University, 1993). *Dissertation Abstracts International, 54*, 452A.

Miller, S., & Walls, J. L. (2021). Advocacy corner: The case of Ohio's Mathematical Modeling and Reasoning course. *Ohio Journal of School Mathematics, 88*, 1–7. https://ohiomathjournal.org/index.php/OJSM/article/view/8298/6205

National Council of Teachers of Mathematics. (1989). *Curriculum and evaluation standards for school mathematics*.

National Research Council. (2001). *Adding it up: Helping children learn mathematics*. Kilpatrick, J., Swafford, J., & Findell, B. (Eds.). Mathematics Learning Study Committee, Center for Education, Division of Behavioral and Social Sciences and Education. National Academies Press. https://doi.org/10.17226/9822

National Research Council. (2004). *On evaluating curricular effectiveness: Judging the quality of K–12 mathematics evaluations*. Confrey, J., & Stohl, V. (Eds.). Committee for a Review of the Evaluation Data on the Effectiveness of NSF-Supported and Commercially Generated Mathematics Curriculum Materials. Mathematical Sciences Education Board, Center for Education, Division of Behavioral and Social Sciences and Education. National Academies Press. https://doi.org/10.17226/11025

Nichols, J. A. (1992). The use of graphing technology to promote transfer of learning: The interpretation of graphs in physics (Doctoral dissertation, The Ohio State University, 1992). *Dissertation Abstracts International, 53*, 748A.

Palmiter, J. R. (1986). The impact of a computer algebra system on college calculus. *Dissertation Abstracts International, 47*, 1640A.

Reys, B. J., Reys, R. E., & Rubenstein, R. N. (Eds.). (2010). *Mathematics curriculum: Issues, trends, and future directions: 72nd yearbook*. National Council of Teachers of Mathematics. [includes an associated compact disc of important works related to mathematics curriculum].

Rubenstein, R. N., & Schultz, J. E., et al. (1992). *Functions, statistics, and trigonometry* [University of Chicago School Mathematics Project]. Scott-Foresman.

Schultz, J. E. (2024a). How mathematics education in Ohio impacted the nation: Laying the groundwork for reform. *Ohio Journal of School Mathematics, 97*, 30–42. https://ohiomathjournal.org/index.php/OJSM/article/view/9990

Schultz, J. E. (2024b). How mathematics education in Ohio impacted the nation: Incorporating technology to carry out the vision. *Ohio Journal of School Mathematics, 98*, 30–40. https://ohiomathjournal.org/index.php/OJSM/article/view/10182

Senk, S. L., & Thompson, D. R. (Eds.). (2003). *Standards-based school mathematics curricula: What are they? What do students learn?* Erlbaum.

Shumway, R. J. (Ed.). (1980). *Research in mathematics education*. National Council of Teachers of Mathematics.

Tuska, A. (1993). Students' errors in graphing calculator-based precalculus classes (Doctoral dissertation, The Ohio State University, 1992). *Dissertation Abstracts International, 53*, 2725A.

Vonder Embse, C. B. (1987). An eye fixation study of time factors comparing experts and novices when reading and interpreting mathematical graphs (Doctoral dissertation, The Ohio State University, 1987). *Dissertation Abstracts International, 48*, 1141A–1142A. (ERIC Document Reproduction Service No. ED 283 672).

Vonder Embse, C. B. (1990). Research on reading and interpreting computer-generated graphs using eye-tracking technology. In F. Demana, B. K. Waits,& J. G. Harvey (Eds.), *Proceedings of the Conference on Technology in Collegiate Mathematics* (pp. 278–281). Addison-Wesley.

Vonder Embse, C. B., Whiteman, F. C., Damarin, S. K., & Leitzel, J. R. C. (1988). The AAN calculator: A tool for enhancing instruction. *Arithmetic Teacher, 35*(7), 12–17. https://doi.org/10.5951/AT.35.7.0012

Wayand, L. S. (1998). *Identifying communication obstacles that arise when translating the modern mathematics classroom to distance*. (Doctoral dissertation, The Ohio State University). Dissertation No. 9834095. ProQuest No. 304441345.

Waits, B. K., & Demana, F. D. (1987). Solving problems graphically using microcomputers. *UMAP Journal, 8*(1), 1–7.

Waits, B. K., & Demana, F. D. (1988a). Is three years enough? *Mathematics Teacher, 81*, 11–14.

Waits, B. K., & Demana, F. D. (1988b). Manipulative algebra—the culprit or the scapegoat? *Mathematics Teacher, 81*, 332–334.

Waits, B. K., & Demana, F. D. (1988c). *Master grapher* [Computer software]. Addison-Wesley.

Waits, B. K., & Demana, F. D. (1988d). Microcomputer graphing: A microscope for the mathematics student. *School Science and Mathematics, 88*, 218–224.

Waits, B. K., & Demana, F. D. (1989). Computers and the rational-root theorem—another view. *Mathematics Teacher, 82*, 124–125.

Waits, B. K., & Demana, F. D. (1993). Using graphing calculators to enhance the teaching and learning of university mathematics: The United States experience. In P. Jaworski (Ed.), *Proceedings of the Conference on Technology in Mathematics Teaching* (pp. 73–80). Birmingham, England: University of Birmingham.

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