Fruitful Fraction Models with Digital Tools

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Abstract: *With COVID-19 changing the digital landscape of mathematics instruction, teachers turned to digital tools to support remote learning. In this article, the authors explore a digital tool used by students to create fraction bar and circle models while engaged in a task. They discuss considerations for how to teach and model with digital tools in mathematics classrooms.*

Keywords: *Fraction models, virtual manipulatives, digital tools*

Introduction

With COVID-19 changing the digital landscape of education, teachers have had to re-envision their instructional strategies in the context of remote learning. This learning space has both inspired innovation with digital tools and illuminated digital inequities in vulnerable and disadvantaged communities (Kuhfeld et al., 2020; Moldavan et al., 2021). In navigating the digital divide, teachers with knowledge of and access to digital tools have found success in selecting and using such resources for their mathematics classrooms. This article highlights the efforts of one of these teachers, Ms. Richardson, who used an app alongside a task to build students' mathematical knowledge of fractions. We review how Ms. Richardson assessed the app using guiding questions for teaching with digital tools and how the app provided opportunities for students to create fraction bar and circle models. We also share student work samples to highlight features of the app and discuss considerations for using digital tools to model mathematical concepts.

Teaching with Digital Tools

Virtual manipulatives, simulation software, graphing calculators, and other application-based websites can assist students with visualizing mathematical ideas, organizing and analyzing data, and communicating mathematical reasoning and problem-solving (National Council of Teachers of Mathematics [NCTM], 2000, 2014). Digital tools can further support students in developing their mathematical knowledge through modeling and reasoning (Moyer et al., 2002; Reiten, 2018). While there are benefits to using digital tools, teachers must have the skills to select and use such tools. Table 1 provides guiding questions for teaching with digital tools in the mathematics classroom with respect to selection, instructional use, and evaluation.

Table 1: *Guiding Questions for Teaching with Digital Tools.*

Stage of Teaching with Digital Tools	Guiding Questions for Reflection
Selection	• Who has access to the digital tool? • How can you learn how to use the digital tool? • How is the digital tool designed to be accessible?
Instructional use	• How does the digital tool support the learning goals? • How will you implement the digital tool during instruction? • How will students engage with the digital tool?
Evaluation	• How can you monitor and share student work using the digital tool? • What type of feedback does the digital tool provide? • How does the digital tool provide opportunities for self-assessment and reflection tied to the learning goals?

A Digital Tool for Creating Fraction Models

Visual fraction models are important for teaching fraction concepts (NGA Center & CCSSO, 2021). Ms. Richardson can attest to this statement given her experience as a sixth-grade mathematics teacher of students with disabilities. With her students' needs primarily focused on language and auditory processing difficulties, Ms. Richardson tries to find innovative ways for her students to use hands-on materials and models to communicate their mathematical understanding. Remote learning added a layer of complexity to Ms. Richardson's planning, especially when it came to integrating digital tools that could assist with creating fraction models during synchronous zoom sessions. Ms. Richardson pre-assessed her students for their prior understanding of fractions, including fraction equivalence and operations, referencing standards targeted in grades 3–5. With half of her class of twelve indicating misconceptions regarding the relationship between improper fractions and mixed numbers, she designed the Fruity Fraction Salad Task (see Figure 1). To accompany the task, she searched for a digital tool that could be used by her students to create fraction models in a synchronous context with both independent work and whole-class discussion.

In her search for a digital tool, Ms. Richardson considered a free app known as *Fractions* (Math Learning Center, 2020). Teachers and students can use the app to model various fraction concepts (e.g., comparison, equivalency, operations, order), which are essential for students learning fractions beginning in the elementary grades (NGA Center & CCSSO, 2010). The app allows students to create interactive bar and circle models to represent fractions with varying denominators. Students can shade their models to illustrate a part of a whole and use the label option to show or hide the associated numerator and denominator. Other features of the app include duplicating, rotating, or superimposing models onto each other to help students visualize and compare fractions in different ways. The app also has a pen and keypad to write expressions and equations, which can be useful when comparing fractions and performing operations.

Figure 1: *The Fruity Fraction Salad Task used by Ms. Richardson.*

Fruity Fraction Salad Task

Standards:

CCSS.MATH.CONTENT.3.NF.A.1: Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size $1/b$.
CCSS.MATH.CONTENT.3.NF.A.3: Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.
CCSS.MATH.CONTENT.4.NF.B.3.C: Add and subtract mixed numbers with like denominators, e.g., by replacing each mixed number with an equivalent fraction, and/or by using properties of operations and the relationship between addition and subtraction.
CCSS.MATH.CONTENT.5.NF.A.1: Add and subtract fractions with unlike denominators (including mixed numbers) by replacing given fractions with equivalent fractions in such a way as to produce an equivalent sum or difference with like denominators.
CCSS.MATH.PRACTICE.MP2: Reason abstractly and quantitatively.

Apples: $\frac{18}{6}$

Bananas: $\frac{14}{4}$

Pineapples: $\frac{13}{2}$

Oranges: $\frac{11}{3}$

Part I: Gathering Ingredients

Tasha is making her mother's fruit salad. Looking at the recipe, Tasha wants to know the following:

Written as a mixed number in simplest form, how much of each fruit does the recipe call for? Draw a fraction bar or circle model to support your answer.

Part II: Missing Mangoes

Tasha forgot to add mangoes to the fruit salad. She wants to add 67 eighths. Draw a model and create an algorithm that Tasha can use to convert 67 eighths into a mixed number in simplest form.

Part III: A Berry Good Idea

Tasha wants to include strawberries in the fruit salad. If she cuts $8\frac{1}{3}$ strawberries into equal-sized thirds, how many pieces are in the fruit salad? Draw a model and create an algorithm that Tasha can use to convert $8\frac{1}{3}$ into an improper fraction.

Part IV: More Berries

Tasha's son loves strawberries and wants to add more into the fruit salad. If Tasha doubles the number of strawberries, how many equal-sized thirds will she need? Draw a model. Express the total number written as an improper fraction.

Part V: A Hungry Baker

Tasha's daughter decided to help prepare the fruit salad. When she came into the kitchen, she ate half of a banana. How many equal-sized fourths of a banana are left to use in the recipe? Draw a model. Express the total number written as a mixed number in simplest form.

When considering the app, Ms. Richardson consulted the guiding questions for teaching with digital tools in Table 1. The questions prompted Ms. Richardson to consider who has access to the app, how users would learn how to use it, and how to integrate the app into lessons given its accessibility and design. She saw that it was accessible for free by both teachers and students. She also noted the app could be used on various devices and how it provided self-guided, step-by-step directions with visuals to make the app easy to learn without the need of training from experts. Rather than having a rigid tool-directed design, the digital tool offered a flexible design where users could adapt the resource to their needs. With these selection considerations in mind, Ms. Richardson determined that the app met her criteria for her remote context.

As for the app's instructional use, Ms. Richardson believed that she could have her students use the app to make visual fraction models that could demonstrate their understanding of fractions, especially for those who struggled to express mathematical ideas in written form. Students could create models to problematize their misconceptions and explore the relationship between each other's models to make sense of varied representations and extend their thinking. Thus, the app could promote meaningful discourse focused on self-discovery and student-centered learning.

In planning for ways to evaluate student thinking, Ms. Richardson considered questions pertaining to how the app could assist with monitoring and sharing student work, providing feedback, and promoting opportunities for self-assessment and reflection. While the app did not offer opportunities for multi-user collaboration or automated immediate feedback beyond the fraction labels, she considered alternative plans for teacher-directed feedback involving students sharing their screens or taking screenshots. She also noted her need to provide supplemental prompts beyond what the app provided to encourage self-assessment of the learning goals for tracking learning progress. Reflection on the app allowed Ms. Richardson to consider its offerings in meeting both the task's learning goals and her students' needs as well as informing plans for accommodations where necessary.

Next, Ms. Richardson considered how the digital tool could inform model use and goals (see Table 2). For instance, she could see the app encouraging her students to use the interactive bar and circle models to make connections between the numerator and the denominator found in an improper fraction with that of the whole number and fraction found in a mixed number. By using the app to explore various relationships using models, her students could deepen their understanding of fractions to then create and interpret word problems that use fractions to connect applications. They could also use the app to develop strategic modeling, reasoning, and problem-solving strategies to transfer between concepts rather than memorize a set of procedures to use in a particular setting (NCTM, 2000, 2014).

Serving a Fruity Fraction Salad Task

Ms. Richardson purposefully created tasks for remote learning that incorporated digital tools to keep her students engaged and encouraged to express their mathematical ideas. The tasks focused on providing students with opportunities to explore and represent mathematics concepts, think critically about how to apply their knowledge to real-world problems, and reinforce mathematical identities as problem-solvers and doers of mathematics. She knew that providing space for students to create visual fraction models would promote reasoning to make sense of equivalence and comparison, thereby building mathematical knowledge and conceptual understanding (Chval et al., 2013). Thus, she created the Fruity Fraction Salad Task.

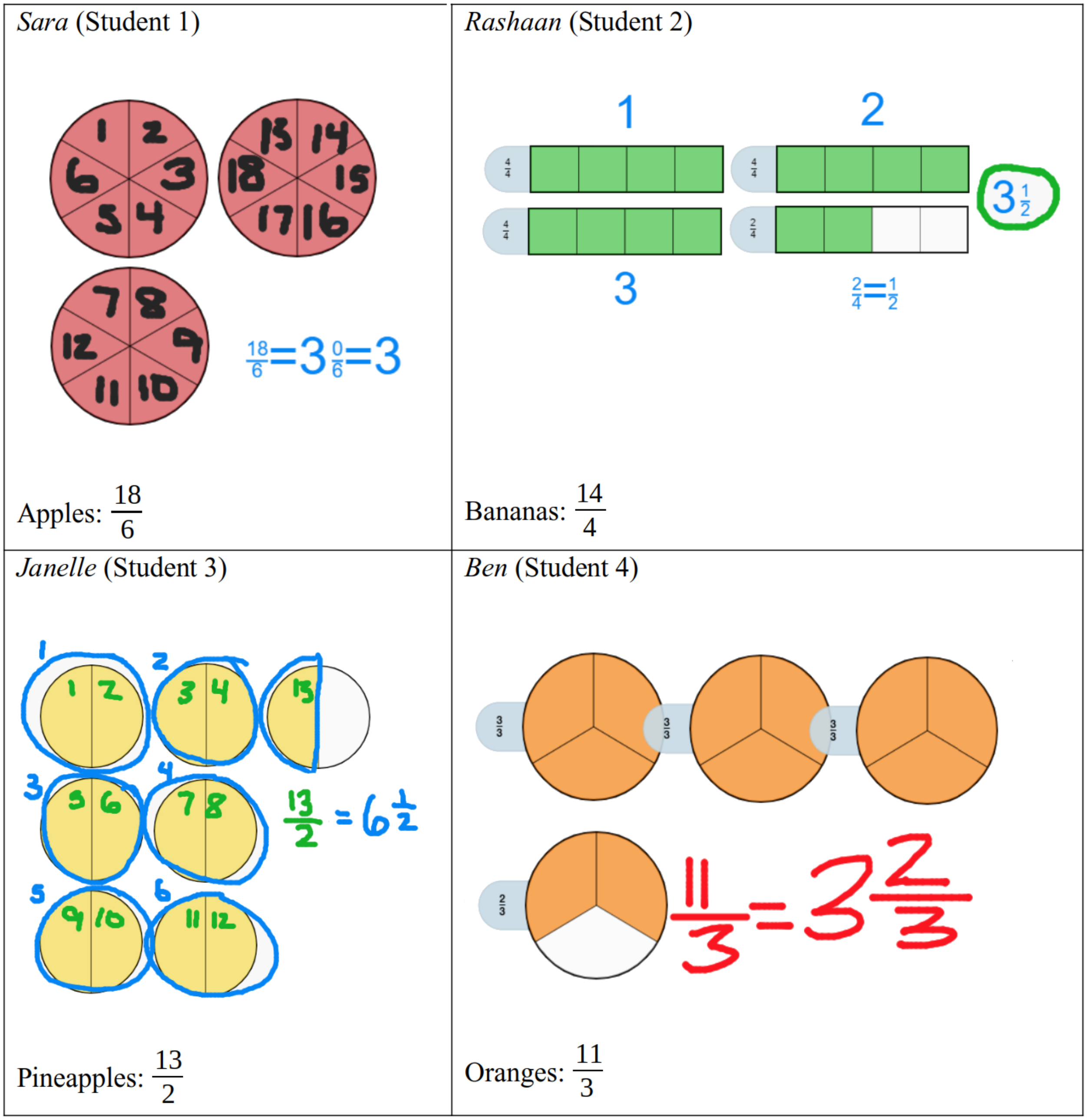
The Fruity Fraction Salad Task is a 45-minute student-centered task that requires students to explain equivalence of fractions between improper fractions and mixed numbers using models and perform operations with mixed numbers with like and unlike denominators (3.NF.A.1, 3.NF.A.3, 4.NF.B.3.C, and 5.NF.A.1). The task also encourages students to reason abstractly and quantitatively (MP2).

Table 2: *Digital Tool Considerations to Inform Model Use and Goals*

Model Goals	Digital Tool Considerations to Inform Model Use
Make explicit connections among mathematical representations to deepen understanding of concepts	*How can the digital tool be used to…* - make models to explore and connect concepts? - assist students in creating multiple representations to justify and explain their reasoning? - probe student thinking about the relationship between models to make sense of mathematical ideas?
Analyze errors and misconceptions with models to promote productive struggle	*How does the digital tool…* - help students use models to locate errors and misconceptions? - ask questions and scaffold students' thinking with appropriate hints and suggestions? - provide feedback in response to correct solutions and/or perseverance in solving problems?
Facilitate meaningful discourse to encourage purposeful sharing with models and extend student thinking	*How can the features of the digital tool be used to…* - create varied models that allow for student explanation and justification? - inform ways to select and sequence multiple representations and solution strategies? - help students compare similar and different representations to make mathematical connections and advance understanding?
Promote reasoning and problem-solving to make real-world connections	*How can the digital tool be used to…* - create simpler problems and/or visual representations to model a situation? - elicit multiple approaches and representations to promote varied problem-solving strategies? - contextualize mathematical ideas to real-world situations?

The task begins with an oddly written recipe that a fictional character named Tasha received from her mother. In Part I, students write how much of each fruit the recipe calls for as a mixed number in simplest form. Referencing the improper fraction given, students are also asked to create visual models using the app *Fractions* to make an equivalent mixed number (see Figure 2). The screenshots show how students used the app to engage with the task.

Figure 2: *Sample student work in Part I.*



As students worked on Part I, Ms. Richardson considered how fraction bar and circle models made on the app could assist students in analyzing their misconceptions and understanding how fractions greater than one are improper with parts decomposed into wholes and fractions. She also considered ways to help students compare similar and different representations to justify reasoning and extend student thinking. She knew providing opportunities for students to engage in productive discussions during the task would help students communicate their mathematical ideas and evaluate the ideas of their peers (Smith & Stein, 2011). The dialogue below is a sample of Ms. Richardson using the digital tool with her students to introduce Part I of the task:

MS. RICHARDSON : What do you notice about the amount of fruit given in the recipe?
SARA : The fruit amounts have large numbers on top.
RASHAAN : And recipes don't usually look like this.

MS. RICHARDSON : I agree! What type of fractions are in the recipe?

JANELLE : Improper.

MS. RICHARDSON : Okay. How do we know these are improper fractions? Try to use *numerator* and *denominator* in your justification.

JANELLE : Because the number on top, like the numerator, is greater than the number on the bottom, like the denominator.

MS. RICHARDSON : Yes, this is true. What would a one in the numerator and a four in the denominator look like?

BEN : A whole cut into four pieces. Like a pizza cut four ways with one piece shaded. Like a fourth shaded.

MS. RICHARDSON : Can you draw a model of a fourth using the app and share your screen? While Ben does this, let's have everyone try to draw a model of what he said on your own device. After, I want you to think how your model might look different or the same.

ASHLEY : Mine looks the same. It has the same number of parts with one part shaded. I can add more parts and keep the same look. Look at this [creates a circle model with eight parts and shades two parts].

MS. RICHARDSON : Can anyone explain to me why Ashley's circle models look different but represent the same amount shaded?

DANIELA : One-fourth and two-eights equal the same value.

MS. RICHARDSON : Great connection! They are equivalent fractions. What if I asked you to shade five-fourths? Would you have to add anything to each model?

Using the workspace provided in the app, students created models of each fruit amount depicted in the task. Then, they discussed how the four improper fractions related to their models' associated wholes and fractions. The following dialogue occurred that referenced the models of Sara's apples and Ben's oranges:

MS. RICHARDSON : Look at the model for apples and oranges. How are the models similar?

RASHAAN : Sara and Ben both used circles.

AAKASH : Both models have 3 whole circles shaded.

MS. RICHARDSON : Great observations. How are the models different?

MING : The apples have no pieces left over, but the oranges have 2 one-thirds.

CRYSTAL : The apples is not a mixed number. It's just 3.

MS. RICHARDSON : Interesting. Without a model, how would you know that 18 sixths would have no pieces left over?

JOSH : 18 divided by 6 is 3. That's what Sara's model says.

MS. RICHARDSON : So, let's look at the model for oranges. How does the model show us 11 divided by 3?

ASHLEY : There are 3 wholes completely shaded. And there are 2 parts shaded on the fourth circle. These parts are left over.

MS. RICHARDSON : Okay, so you're saying there is a remainder of 2. Now using this idea, let's look at Part II.

In Part II, the task asks students to add mangoes to the fruit salad. The improper fraction differs from those previously referenced in that it has a rather large numerator that may take time to represent visually. Thus, the question challenges students to describe an algorithm to convert the improper fraction into a mixed number. For students to be prepared for this part of the task, Ms. Richardson strategically used the models made by Sara and Ben with the app to show how visual representations can promote reasoning and problem-solving strategies. The following dialogue occurred referencing Figure 3 while working on Part II:

MS. RICHARDSON : How would you describe a mathematical process to convert 67 eighths into a mixed number?

ASHLEY : The problem asked for 67 eighths, but 67 is a big number. I know 21 divided by 8 is 2 wholes with 5 one-eighths left over. Because 67 eighths has the same number [denominator], I used the same strategy to get $8\frac{3}{8}$.

MS. RICHARDSON : I like this strategy of solving a simpler problem. Did anyone approach this problem differently?

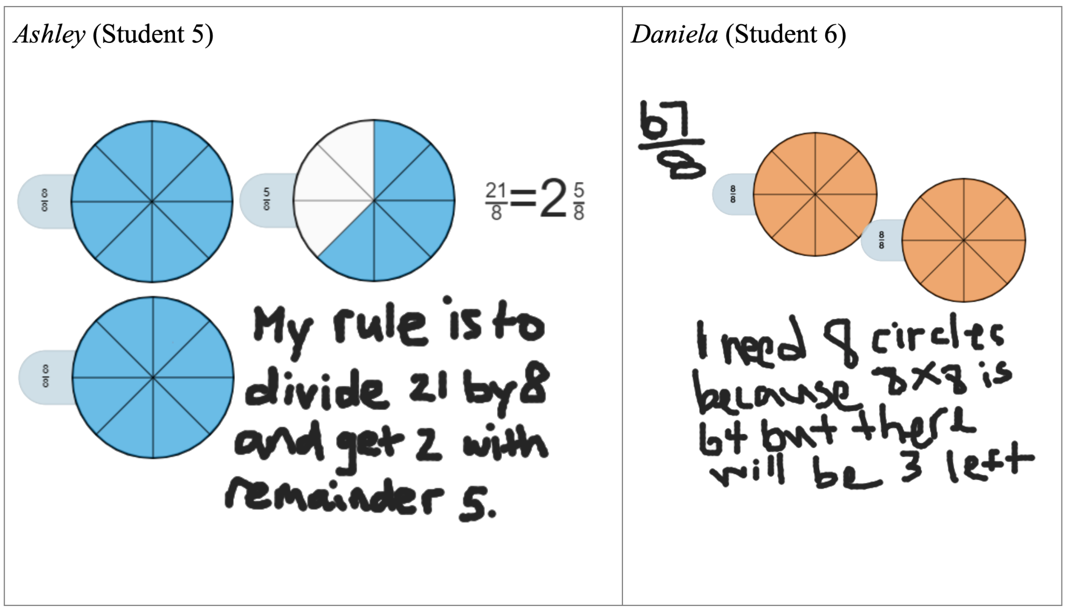
DANIELA : When I started making the whole circles and cutting them up, the pieces increased by 8 each time. I knew 8 times 8 is 64. So there will be 3 pieces left over.

AAKASH : I took out 8 eighths to make a whole and had 59 left. Then I took out 8 more and had 51 left. I kept subtracting 8 until I didn't have enough. I did this 8 times, but then I had 3 pieces left over.

MS. RICHARDSON : Who can explain what Daniela and Aakash mean when they say there are 8 wholes and 3 pieces left over?

DEMI : We have 8 mangoes and $\frac{3}{8}$ of another mango. So, 67 eighths is the same as $8\frac{3}{8}$.

Figure 3: *Sample student work in Part II.*



In Part III, the task adds strawberries to the fruit salad. Students must convert the mixed number of strawberries into an improper fraction. To help facilitate this part of the task, the following dialogue referencing Figure 4 occurred:

MS. RICHARDSON : How did you convert a mixed number into an improper fraction?

AAKASH : I started drawing a model and counting the parts but didn't feel like counting all, so I just counted by 3's.

MS. RICHARDSON : How did you know to count by 3's?

AAKASH : I know 3 thirds make a whole. So, I had 8 wholes. Then, I added the one extra to get 25 thirds.

MS. RICHARDSON : I like this idea. Did someone use different reasoning to explain their algorithm?

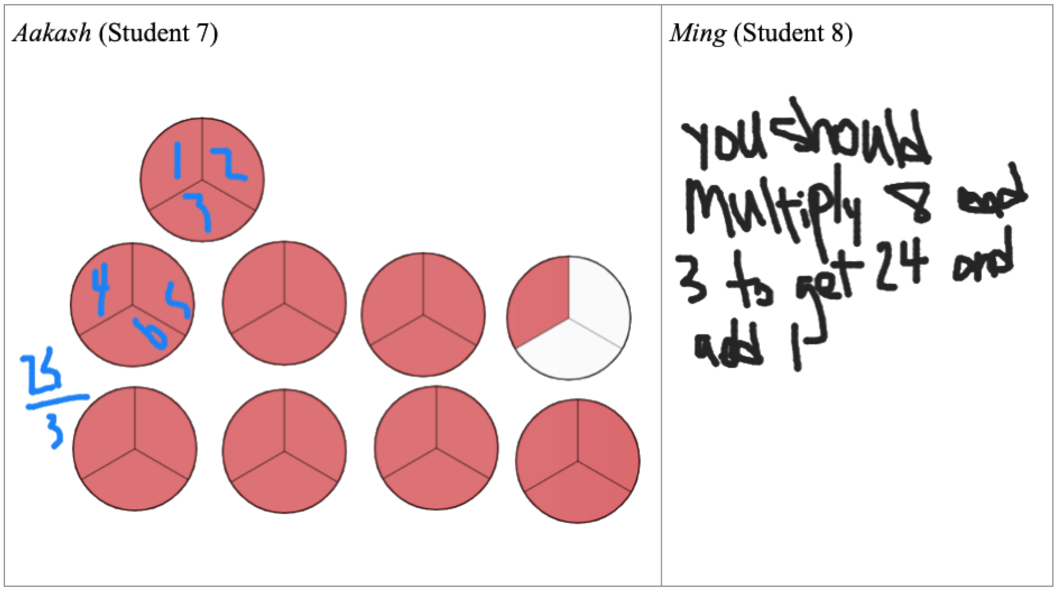
MING : I multiplied 8 and 3 and then added 1.

MS. RICHARDSON : How can we restate this idea using the mathematical vocabulary *numerator* and *denominator*?

RAVEN : Multiply the whole number by the denominator and add the numerator.

As the task continued, Ms. Richardson asked her students to compare their work with others by sharing their screens. This open discussion encouraged students to justify their reasoning using a model and learn how others may have represented the problem using different models.

Figure 4: *Sample student work in Part III.*



In Parts IV and V, students used the app to draw models as they completed an addition problem with mixed numbers with like denominators (see Figure 5) and a subtraction problem with improper fractions with unlike denominators (see Figure 6). Demi's work in Figure 6 shows how the use of a model helped her visualize the subtraction between two fractions with unlike denominators. As demonstrated throughout this task, Ms. Richardson used the app to encourage students to use models to make mathematical connections among varied representations and justify their reasoning.

Figure 5: *Sample student work in Part IV.*

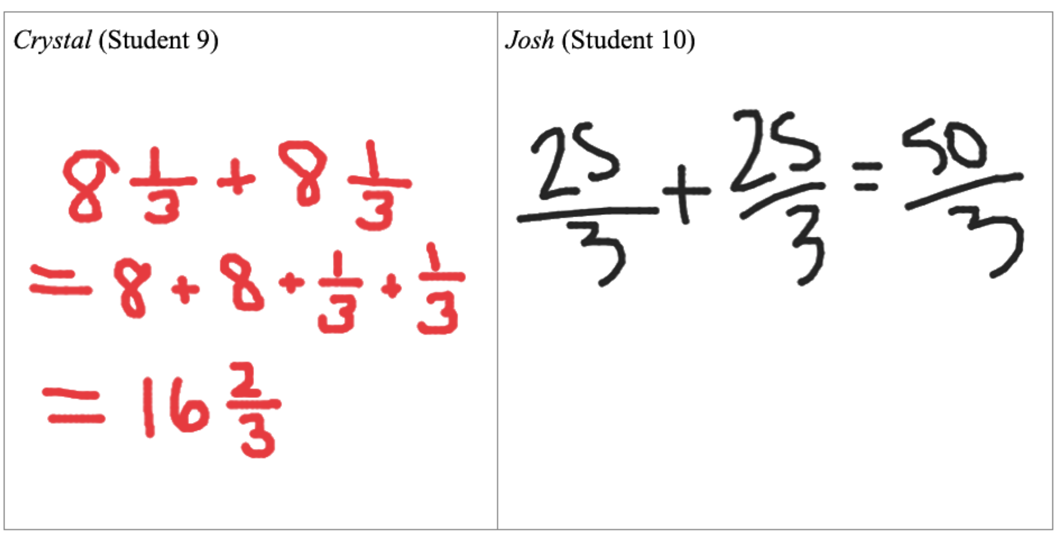
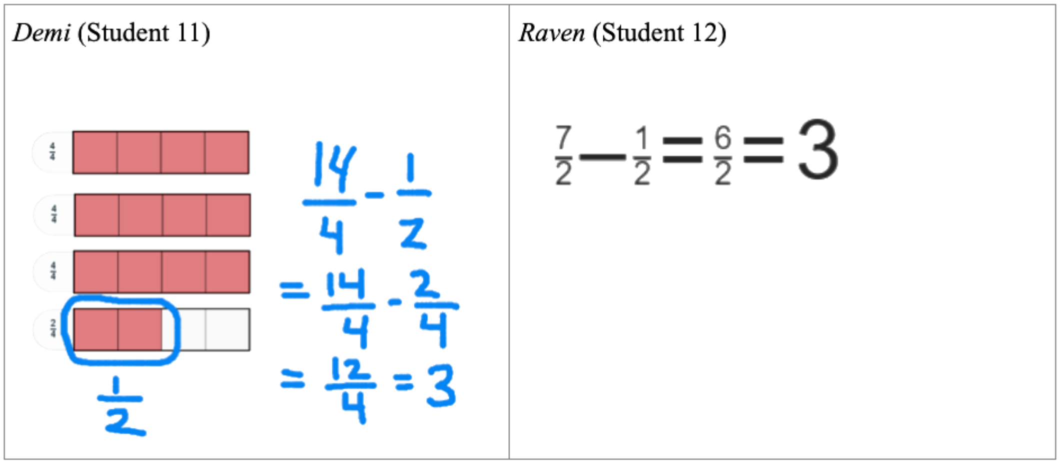


Figure 6: *Sample student work in Part V.*



Adding Digital Tools to Your Plate

Teaching and learning fractions can be challenging, especially when trying to create and share visual fraction models in remote settings. Ms. Richardson created an engaging Fruity Fraction Salad Task to assist her students in creating fraction models with the use of the app *Fractions*. When reflecting on her experience engaging with the task, Ms. Richardson shared how her students benefited from using the app to model and develop their mathematical knowledge of fractions. She also noted how her students remained engaged in the task because they had choice in making models that allowed for creativity and ownership. While there are benefits to using this app, Ms. Richardson did express concern about the app not allowing her to see the students' workspace in real-time. Thus, she asked students to submit screenshots or share their screens to document their work. Nevertheless, Ms. Richardson's use of the app alongside the task allowed her students to explore fraction concepts necessary to access grade-level material.

Digital tools can offer students multiple entry points to engage with various representations of mathematical concepts, provide access to continued learning opportunities outside of the classroom, and enrich conceptual understanding so students can make meaningful connections to solve real-world problems (NCTM, 2014). While these benefits justify the need for teachers to advocate for technology integration to enhance mathematics instruction, many teachers and students face the pervasive challenge of gaining access and knowledge to teach and learn with digital tools. Mathematics education must prioritize digital equity, where both access and knowledge to use digital tools are readily available for teachers and students. Thus, the field must rethink what resources can bridge transitions from in-person to remote instruction to ensure students stay engaged and the opportunity gap to learn with digital tools is not widened. When resources are available, teachers must then consider how they can teach with digital tools and use them to model in the mathematics classroom. Studying teacher-initiated efforts, like that of Ms. Richardson, can illuminate the various ways to integrate digital tools to enhance instruction and empower students with skills to develop their mathematics learning.

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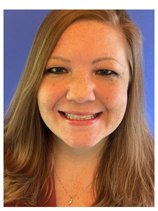

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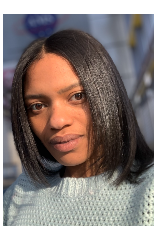

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