Engaging All Mathematical Learners Through Project-Based Design

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Abstract: *The authors describe a project-based activity using TI calculators or Desmos within a high school mathematics classroom. As students learn about functions with technology using a project-based approach, they make connections between school mathematics and topics of personal interest.*

Keywords: *Project-based mathematics, teaching with technology, inquiry*

Introduction

This article describes a project-based activity using TI calculators or Desmos® within a high school mathematics classroom. Opportunities are provided for inclusion and differentiation while teaching about functions. While differentiating content (various functions), process (types of technologies), and product (level of design),the authors provided students with a project that was mathematically meaningful and engaging.

As Lappan et al. (2012) notes, mathematically rich and engaging tasks offer students the opportunity to acquire conceptual mathematical understanding. One example is project-based learning which focuses on self-directed inquiry, process over product, goal setting, and collaboration in groups. These projects promote intrinsic motivation by organizing instructional strategies based on learner interest, effort, and persistence (Borich 2017).

Content Focus: Transformation of Functions with Technology

Mathematical transformations occur throughout the K-12 curriculum. Elementary school students compare and relate objects through movement. Middle school students reflect objects and analyze functions. High school students represent linear and nonlinear functions symbolically, deepen their understanding of relations, and expand their repertoire of familiar functions (NCTM 2000). It is imperative that students have a working understanding of transformations of functions for many real-world applications.

Students are to analyze functions using different representations, first by hand, then using technology, according to Standard `F-IF.7` (National Governors Association, 2010). In this project, a graphing utility is first used to experiment with the properties of functions and their graphs to build equations that model students' designs. Using mathematical tools can help students develop a deeper understanding of real-world phenomena and foster an appreciation of those concepts (NCTM 2000).

Students are to experiment with cases and illustrate an explanation of the effects on the graph using technology according to Standard `F-BF.3` (National Governors Association, 2010). For example, students must understand the effect of the parameters a, h, and k on the function $f(x) = a(x-h)2+k$. In this project, students experiment and explain processes using Desmos® or TI graphing calculator. Students demonstrate an understanding of this concept when they create an equation modeled by a graph and use technology to evaluate whether the conjectured equation is reasonable. Using the technology provides students with a visual representation of the domain restrictions.

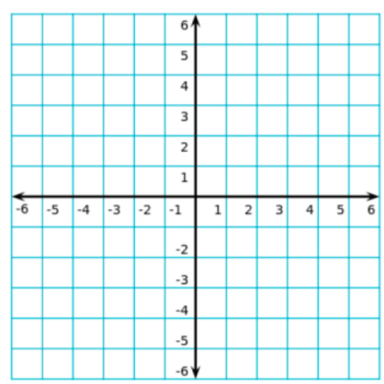
Description of the Project

In this article, two Ohio high school mathematics teachers asked students to complete a graphic design. One teacher was from a rural district and used 10 days to complete the task, while the other teacher was from an urban district and completed the unit in five blocked periods of 90 minutes.

Procedures and Materials

Before beginning their projects, both classes completed a graphic organizer which included a graph and relevant information for all parent functions studied throughout the year (Figure 1).

Figure 1: *Function graphic organizer.*

Type of Function and Equation in Standard Form	**Characteristics**	**Graph**
For each function type, find the max, min, intercepts, and asymptotes if they exist. Then graph the parent function: • Linear • Absolute Value • Quadratic • Cubic • Square Root • Exponential • Logarithmic	Max: Min: Intercepts: Asymptote(s):	

After the introduction to the project, students were asked to create a design using either a hand-held TI-Nspire CAS® graphing calculator or Desmos® graphing calculator (www.desmos.com). The five-part unit leads students through a variety of activities to strengthen their understanding of function and transformation from different perspectives: from identifying the functions used in a provided image, to designing and describing their own images using transformations of parent functions (e.g., linear, quadratic, cubic, exponential, logarithmic, and rational) and conic sections (e.g., hyperbolas, parabolas, ellipses, and circles). We describe each phase of the unit in more detail in the sections that follow. Resources for the activities we discuss are provided in the appendices at the end of this paper.

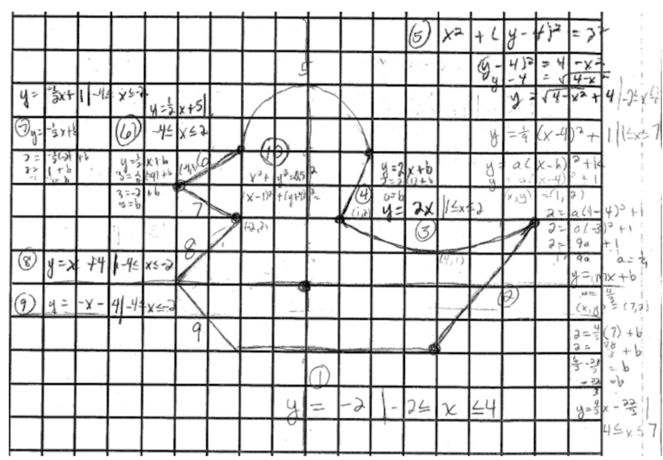
Day 1

Students will review the functions preciously studied this year: Linear, absolute value, quadratic, polynomials and rational functions (Pre-Calc add the trigonometric functions). Domain restrictions will be investigated using the graphing calculator in Desmos.

Students will be given the equations for the duck graphic and a piece of graph paper. They draw the lines that model the equation to form a picture and check it on the graphing calculator. If it draws the duck they are correct.

Figure 2: *Constructing a duck from a list of equations.*

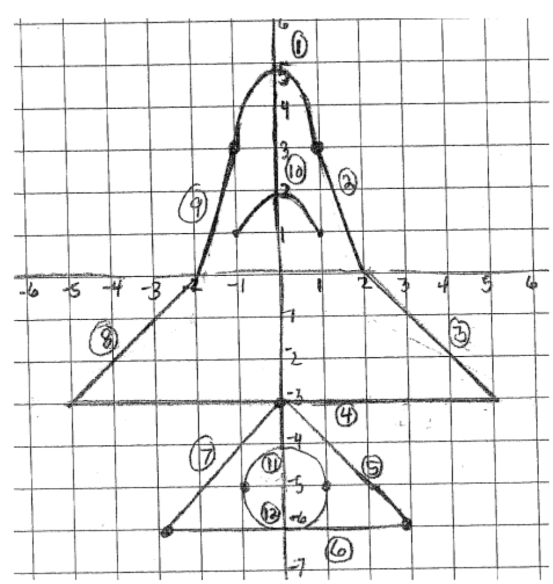
No.	Function	Equation	Domain
1	Linear	$y=-2$	$-2\le x\le 4$
2	Linear	$y=\frac{4}{3}x-\frac{22}{3}$	$4\le x\le 7$
3	Quadratic	$y=\frac{1}{9}(x-4)^2+1$	$1\le x\le 7$
4	Linear	$y=2x$	$1\le x\le 2$
5	Square Root	$y=\sqrt{4-x^2}+4$	$-2\le x\le 2$
6	Linear	$y=\frac{1}{2}x+5$	$-4\le x\le -2$
7	Linear	$y=-\frac{1}{2}x+1$	$-4\le x\le -2$
8	Linear	$y=x+4$	$-4\le x\le -2$
9	Linear	$y=-x-4$	$-4\le x\le -2$
10	Circle top	$y=\sqrt{-x^2-2x-0.75}+4$	——
11	Circle Bottom	$-\sqrt{-x^2-2x-0.75}+4$	——



For additional practice, students will be given the picture of a rocket with numbers by each line. They write the equations next to each number that they believe will draw the picture. When finished, they check them using the graphing calculator to see if the rocket is drawn.

Figure 3: *Constructing equations from a graphic of a rocket.*

No.	Function	Equation	Domain
1	Quadratic	$y=-2x^2+5$	$-1\le x\le 1$
2	Linear	$y=-3x+6$	$1\le x\le 2$
3	Linear	$y=-x+2$	$2\le x\le 5$
4	Linear	$y=-3$	$-5\le x\le 5$
5	Linear	$y=-x-3$	$0\le x\le 3$
6	Linear	$y=-6$	$-3\le x\le 3$
7	Linear	$y=x-3$	$-3\le x\le 0$
8	Linear	$y=x+2$	$-5\le x\le -2$
9	Linear	$y=3x+6$	$-2\le x\le -1$
10	Quadratic	$y=-x^2+2$	$-1\le x\le 1$
11	Circle	$y=\sqrt{1-x^2}-5$	$-1\le x\le 1$
12	Circle	$y=-\sqrt{1-x^2}-5$	$-1\le x\le 1$
13	Quadratic	$y=-x^2+2$	$-1\le x\le 1$

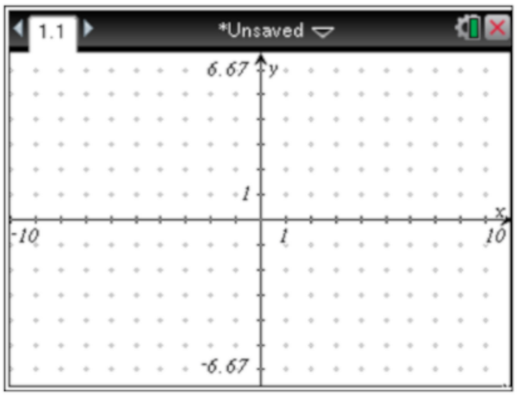


After students work on the rocket and duck tasks, they are encouraged to finish function foldables.

Day 2

Students use the TI-Nspire graphing calculator activity sent wirelessly to their calculators called "Designing with Transformed Functions" (Source). They will use transformed quadratic equations to make a picture of the McDonald's arches and use different colors, and experiment with line width and design. When finished, they will send the picture wirelessly through the graphing calculator. Students will use the graph page and instruction lines to design an emoticon on the last page of the activity and again submit wirelessly through the TI-Navigator system.

Figure 4: *Constructing an emoticon with Nspire.*



Record your equations in the table below along with their domain restrictions. Be sure to identify which part of the image each function creates.

Function Equation	Domain Restriction	What It Creates in the Figure

Additional Day 2 resources are available from Texas Instruments at the following link.

Day 3

Students will use white paper to draw a rough drafts of a design meeting the parameters on the rubric. When approved, they will use clean lines to transfer to graph paper so that intersecting lines meet at integer numbers on the $x - y$ coordinate plane. These rough drafts will be turned in at the completion of the project. The lines of the picture will be numbered 1,2, etc. on the graph paper when approved by the teacher.

Day 4

Students will begin writing the equations for the lines of the design and adding domain restrictions. This is a rough draft that will be turned in at the completion of the project. Students should use every other line to write the equations so if they need to be adjusted, then can just write them below and leave the original equation there.

Day 5

Students will complete the graphic design project and submit wirelessly to the teacher so that a screen shot may be taken and returned to the student to add paint and pictures will be printed. They will be graded according to the rubric. The rough drafts of designs, graphs, and lined instructions will also be submitted along with the final drafts.

Examples of Final Projects

A total of 68 students engaged in this project. Student work samples are shown in Figures 5 through 10. The first three examples were generated in Desmos, while the last three were constructed with TI Nspire. If students wish to explore other designs, the Desmos homepage features winners of the Desmos Global Art Project (https://www.desmos.com/art).

Figure 5: *Snowman constructed in Desmos.*

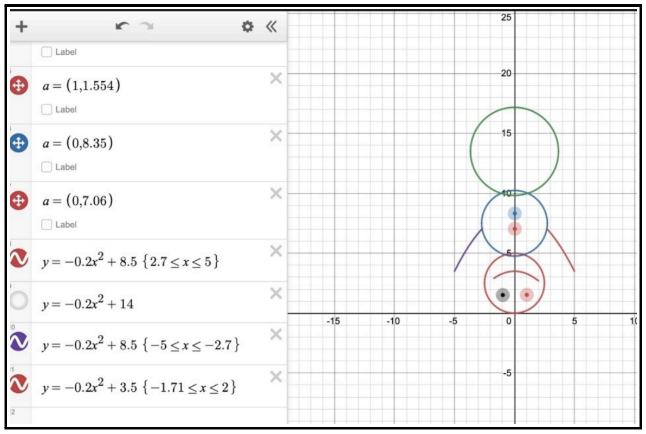


Figure 6: *Rodent creature constructed in Desmos.*

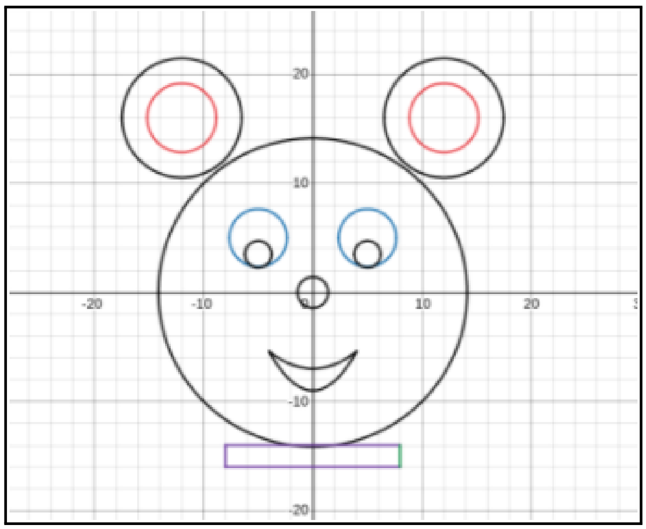


Figure 7: *Roadway constructed in Desmos.*

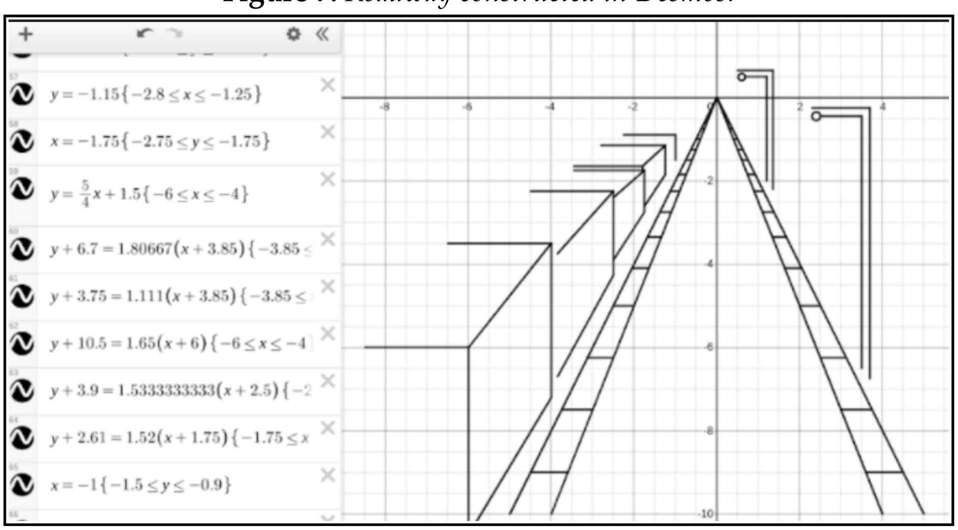


Figure 8: *Musical staff constructed in NSpire CAS.*

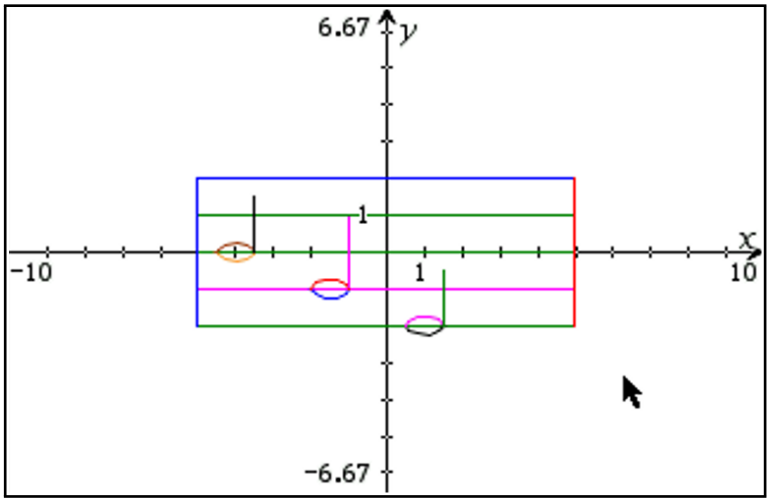


Figure 9: *Fish constructed in NSpire CAS.*

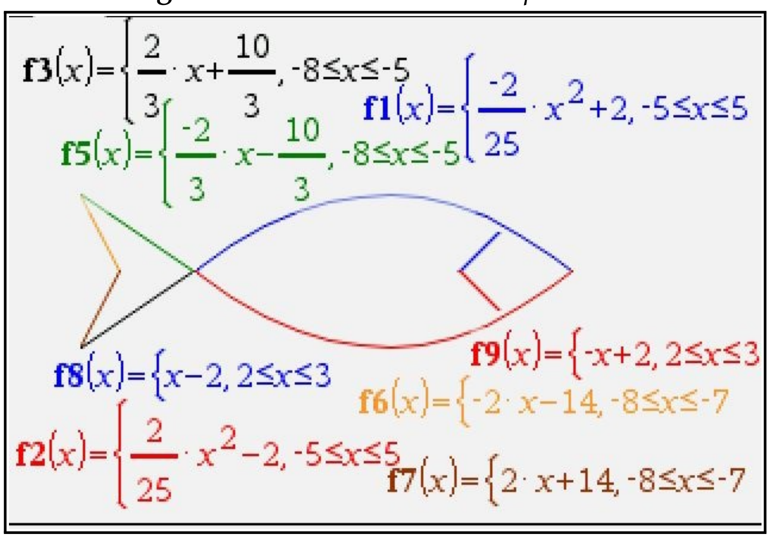
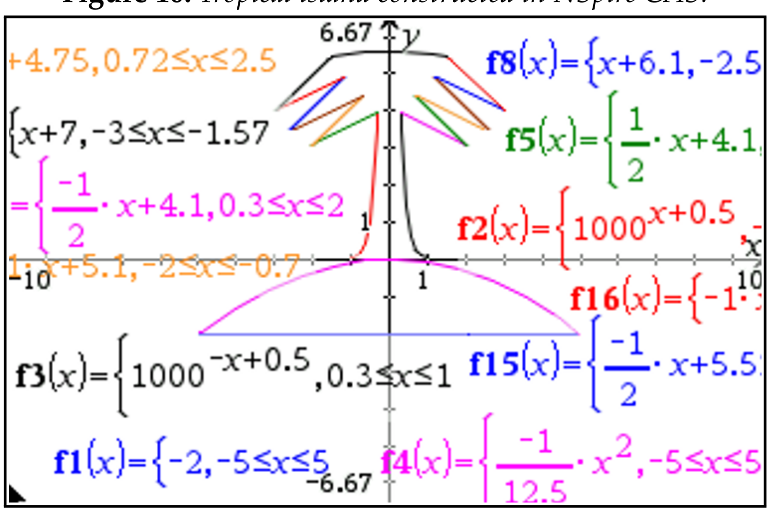


Figure 10: *Tropical island constructed in NSpire CAS.*



To assess student work, both teachers utilized a grading rubric provided in Figure 11.

Figure 11: *Project scoring rubric.*

Criterion	0 (No Attempt)	1 (Limited)	2 (Proficient)	3 (Accomplished)
1. Draws a rough draft of the graphic art.	Does not complete a rough draft.	Draws a rough draft that is incomplete.	Draws an almost complete rough draft of the design.	Draws a complete and connected rough draft of the design.
2. Transfers the graphic art to graph paper.	Does not attempt to transfer the design to graph paper.	Transfers a partial design to graph paper.	Transfers most of the design to graph paper.	Transfers all of the design to graph paper proportionally and accurately.
3. Represents the graphic art using a variety of functions	Names no functions used for the design.	Represents and names at least two types of the functions.	Represents and names at least three types of functions.	Represents and names at four or more types of functions.
4. Lists the domain restrictions for the functions of the graphic design.	Lists no domain restrictions for the functions of the lines of the graphic design.	Lists some domain restrictions for the functions of the lines of the graphic design.	Correctly analyzes most of the functions and lists the correct domain restrictions.	Correctly and explicitly analyzes the functions and lists the correct domain restrictions for all using inequality notation.
5. Graphs the completed design using appropriate settings.	Does not attempt to graph the design.	Graphs part of the design. Some domain restrictions are given and correctly used.	Correctly inputs most of the functions with the domain restrictions to graph most of the art design.	Correctly and explicitly inputs the functions and their domain restrictions to graph the complete art design with no gaps or overlaps.

Results of the project utilizing the rubric and short answer surveys showed:

- The majority of students chose to use 2 or 3 elementary functions with few restrictions.
- Students made the most connections to prior knowledge by transforming functions when creating their pictures.
- Students often utilized a trial and error approach when transforming functions when drawing their pictures and found that it was easier to do so through Desmos.

Student comments regarding the project included:

- "I absolutely loved this, any kind of project that incorporates artwork is right up my alley. It was really fun and creative."

- "I feel like I learned more about how functions work because of this project"
- "The project really made me think about what exactly I had to do. It was really fun and exciting to have my own choice!"
- "This project is an enjoyable summary of all we did this year, and I found it to be quite relaxing and recreational."

Choice and creativity were often cited as sources of student satisfaction and engagement. Students also enjoyed seeing their fellow peers' projects.

Conclusion

This project taps into students' individual interests and allows for differentiated levels of instruction and difficulty. The use of technology in this project enhanced student engagement and allowed students to visualize the effects of function transformations. Upon completion, students were pleased with the quality of their work, thus gaining confidence in the skills they learned while utilizing technology.

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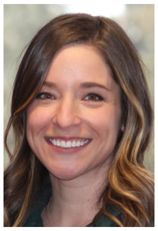
Judy Brown teaches mathematics at National Trail High School in New Paris, Ohio. Judy is a longtime advocate for project-based learning in the STEM disciplines. Ms. Brown is committed to making math meaningful for her students through hands-on exploration and collaboration.

Jennifer Walls teaches mathematics at Firestone CLC in Akron Public Schools. Jenny was recognized as Akron Public Schools Teacher of the Year for 2016. She was also recognized by the Ohio Council of Teachers of Mathematics as the recipient of the state secondary Buck Martin Award in 2020. Jennifer is a member of the Ohio Mathematics Leadership Council (OMELC).

Ann Farrell is a professor emerita in the Department of Mathematics and Statistics at Wright State University. Her primary professional and research interests focus on mathematical lesson study. Ann was awarded the OCTM Christofferson-Fawcett Award in 2016 in recognition for her lifetime contribution to mathematics education in Ohio and beyond.

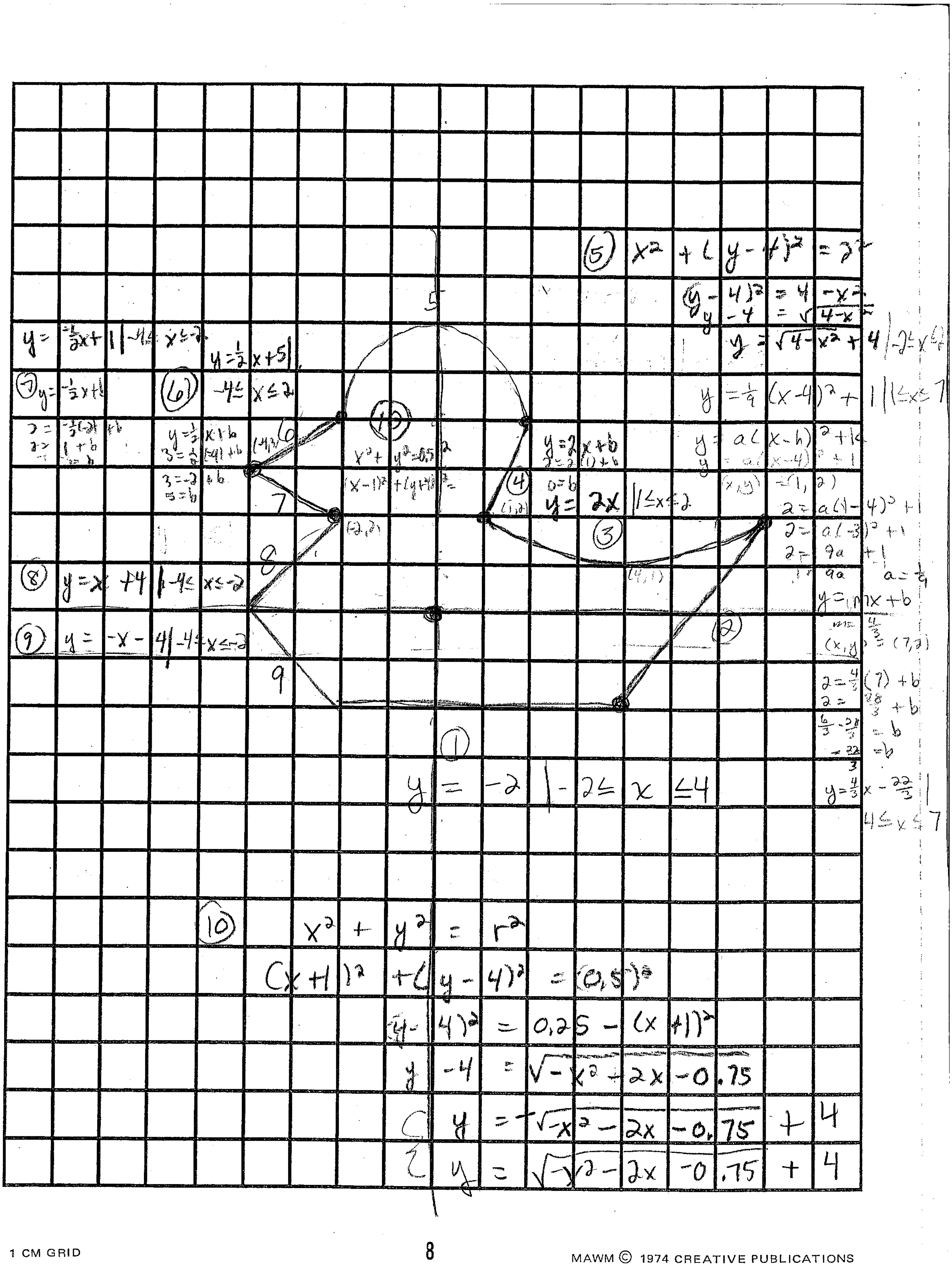


Michelle Meadows is an Associate Professor of Education at Tiffin University in Tiffin, Ohio, where she serves as Chair for their online and on-campus Undergraduate and Graduate Education programs in the School of Arts & Sciences.

Heather Leis is an intervention specialist at National Trail High School specializing in mathematics and science instruction. Heather is committed to providing her students with engaging tasks and projects to help them see mathematics as a creative area of study.

Joanne Caniglia is a Professor in the School of Teaching, Learning, and Curriculum Studies at Kent State University. Her research activities include creating meaningful mathematical tasks to assist secondary and special education students. She is a PI and Co-PI for many National Science Foundation and Board of Regents initiatives.

APPENDIX Day 1: Teaching Materials



⑤ x² + (y − 4)² = 2²
y = √4−x² + 4 | −2 ≤ x ≤ 2
y = 1/9 (x − 4)² + 1 | 1 ≤ x ≤ 7
y = a(x − h)² + k
y = 2x | 1 ≤ x ≤ 2
⑧ y = x + 4 | −4 ≤ x ≤ −2
⑨ y = −x − 4 | −4 ≤ x ≤ −2
⑥ −4 ≤ x ≤ 2
y = 4/3 x − 22/3 | 4 ≤ x ≤ 7
① y = −2 | −2 ≤ x ≤ 4
⑩ x² + y² = r²
(x + 1)² + (y − 4)² = (0.5)²
(y − 4)² = 0.25 − (x + 1)²
y − 4 = √−x² − 2x − 0.75
y = −√−x² − 2x − 0.75 + 4
y = √−x² − 2x − 0.75 + 4
1 CM GRID
8
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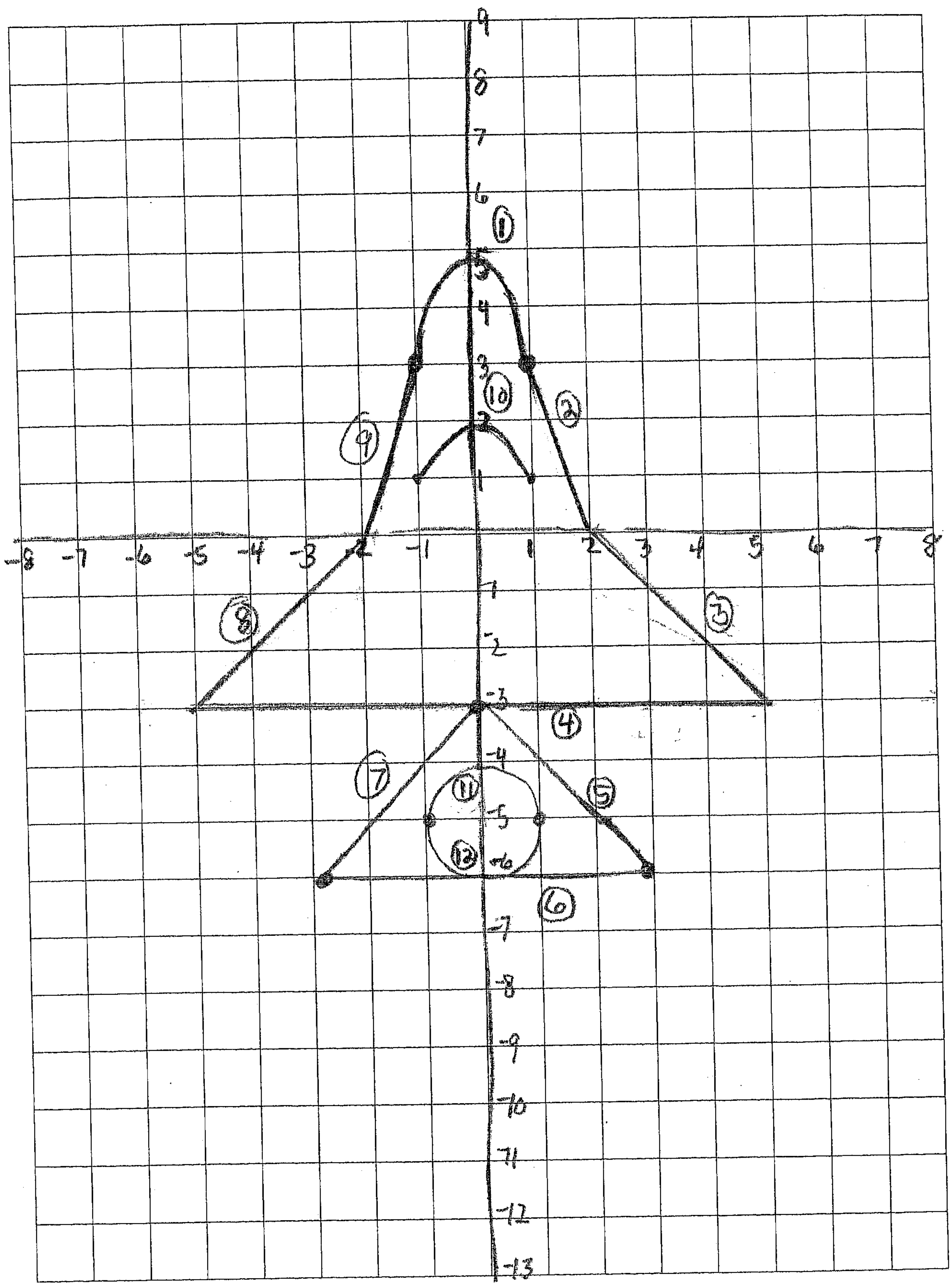
Name________________________ Picture____________________

No.	Function	Equation	Domain
1	Linear	$y = -2$	$-2 \le x \le 4$
2	Linear	$y = \frac{4}{3}x - \frac{22}{3}$	$4 \le x \le 7$
3	Quadratic	$y = \frac{1}{9}(x-4)^2 + 1$	$1 \le x \le 7$
4	Linear	$y = 2x$	$1 \le x \le 2$
5	Square Root	$y = \sqrt{4 - x^2} + 4$	$-2 \le x \le 2$
6	Linear	$y = \frac{1}{2}x + 5$	$-4 \le x \le -2$
7	Linear	$y = -\frac{1}{2}x + 1$	$-4 \le x \le -2$
8	Linear	$y = x + 4$	$-4 \le x \le -2$
9	Linear	$y = -x - 4$	$-4 \le x \le -2$
10	Circle top	$y = \sqrt{-x^2 - 2x - 0.75} + 4$	——
11	Circle Bottom	$-\sqrt{-x^2 - 2x - 0.75} + 4$	——
12		$\sqrt{-x^2 - 2x - 0.75} + 4$	

Name_________________________ Picture_____________________

No.	Function	Equation	Domain

Lesson 1.2.1 Resource Page



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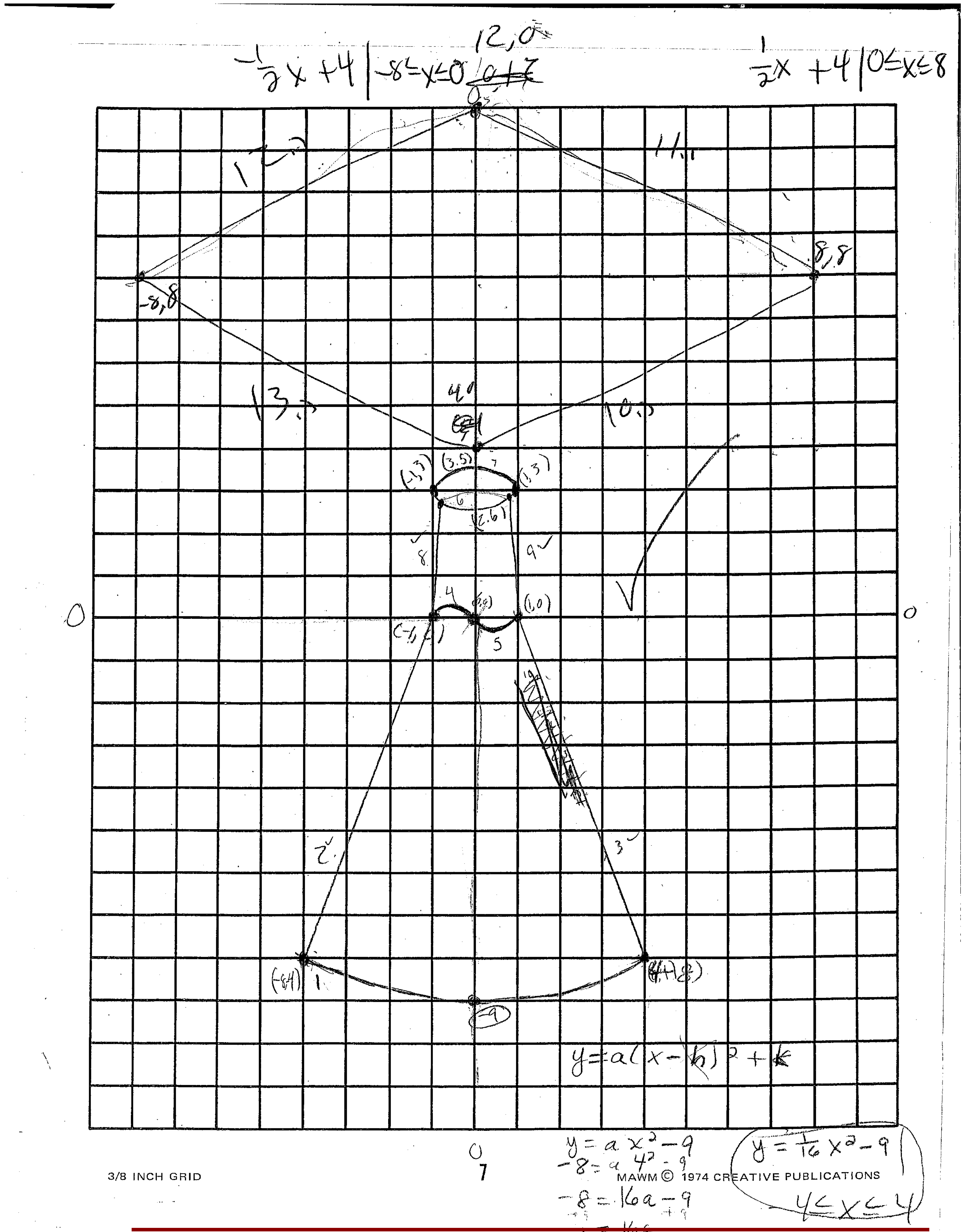
Name Key Picture Rocket

No.	Function	Equation	Domain
1	Quadratic	$y = -2x^2 + 5$	$-1 \le x \le 1$
2	Linear	$y = -3x + 6$	$1 \le x \le 2$
3	Linear	$y = -x + 2$	$2 \le x \le 5$
4	Linear	$y = -3$	$-5 \le x \le 5$
5	Linear	$y = -x - 3$	$0 \le x \le 3$
6	Linear	$y = -6$	$-3 \le x \le 3$
7	Linear	$y = x - 3$	$-3 \le x \le 0$
8	Linear	$y = x + 2$	$-5 \le x \le -2$
9	Linear	$y = 3x + 6$	$-2 \le x \le -1$
10	Quadratic	$y = -x^2 + 2$	$-1 \le x \le 1$
11	Circle	$y = \sqrt{1 - x^2} - 5$	$-1 \le x \le 1$
12	Circle	$y = -\sqrt{1 - x^2} - 5$	$-1 \le x \le 1$
13	Quadratic	$y = -x^2 + 2$	$-1 \le x \le 1$

APPENDIX Day 3: Teaching Materials


$-\frac{1}{2}x + 4 \mid -8 \le x \le 0$
12, 0
$\frac{1}{2}x + 4 \mid 0 \le x \le 8$
-8, 8
8, 8
$y = a(x-h)^2 + k$
$y = ax^2 - 9$
$-8 = a\,4^2 - 9$
$-8 = 16a - 9$
$y = \frac{1}{16}x^2 - 9$
$4 \le x \le 4$
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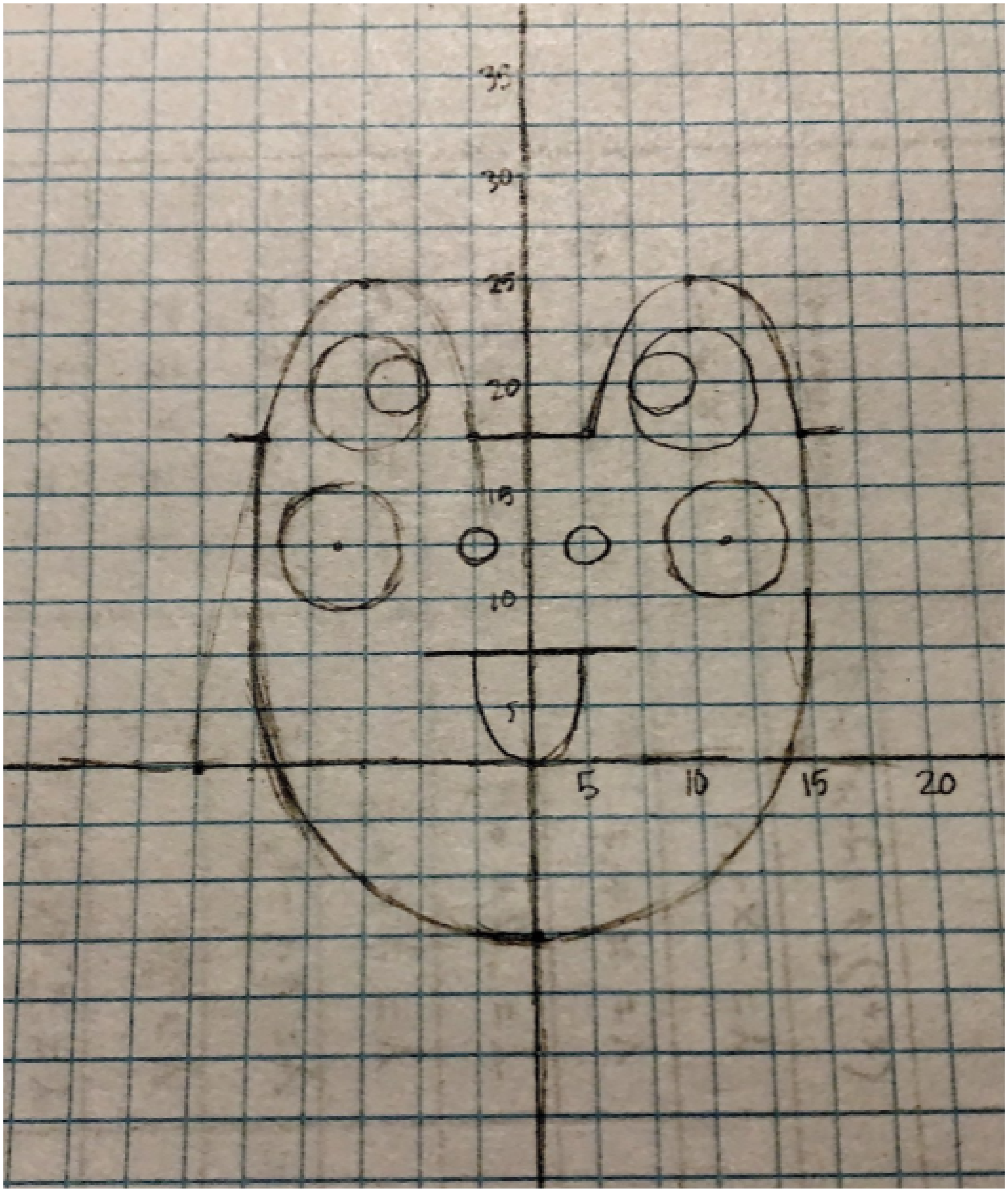
Name [illegible] Picture ______

$\frac{-5}{-3} = 2.)$

$\frac{-5}{3} = 3.)$

No.	Function	Equation	Domain
1.)	Quadratic	$y = \frac{1}{16}x^2 - 9$	$-4 \le x \le 4$
2.)	linear	$\frac{8}{3}x + \frac{8}{3}$	$-4 \le x \le -1$
3.)	linear	$-\frac{8}{3}x + \frac{8}{3}$	$1 \le x \le 4$
4.)	Quadratic	$y = 0.5\sqrt{0.5 - (x+0.5)^2}$	$-1 \le x \le 0$
5.)	Quadra	$y = 0.5\sqrt{0.5 - (x-0.5)^2}$	$0 \le x \le 1$
6.)	Quadratic	$\frac{3.5}{9}x^2 + 2.5$	$-0.8 \le x \le 0.8$
7.)	Quadratic	$\frac{-4.5}{9}x^2 + 3.2$	$-0.8 \le x \le 0.8$
8.)	linear	$13.5x + 13.5$	$-1 \le x \le -0.8$
9.)	linear	$-13.5x + 13.5$	$0.8 \le x \le 1$
10.)	linear	$y = \frac{1}{2}x + 4$	$0 \le x \le 8$
11.)	linear	$y = -\frac{1}{2}x + 12$	$0 \le x \le 8$
12.)	linear	$\frac{1}{2}x + 12$	$-8 \le x \le 0$
13.)	linear	$y = \frac{-1}{2}x + 4$	$-8 \le x \le 0$
14.)			
15.)			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APPENDIX Day 4: Teaching Materials

No.	Function	Equation	Domain

Name______________________________ Picture_________________________