Factoring in Minecraft

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Abstract: In this paper, the author describes a method for using Minecraft as a manipulative for factoring composite numbers and quadratic equations. These concepts are extended to an investigation of perfect square trinomials, completing the square, and Pythagorean triplets.

Keywords: Minecraft, technology, visualization, factoring

What is Minecraft?

Minecraft is an extremely popular video game with over 112 million people playing it every month (Gilbert, 2019). It is an immersive 3D environment where all things are built with blocks. Even the characters are designed using rectangular prisms. When playing in a randomly generated world, landscapes are terraced. There are rectangular tree trunks and stepped hills. There are water, sand, and grass. Animals wander and fly through the terrain. There is day and night, and it occasionally rains. These nods to real world landscapes and everyday experience lead to a feeling of relatedness, yet there is no attempt at realism.

Everything exists in a cubic three-dimensional grid in Minecraft. This creates a blocky appearance that is immediately recognized by players. Although it is a video game and not a physical toy, Minecraft is similar to Legos, TinkerToys, and Erector Sets, in that it is used to build imaginative constructions. The simplistic nature of these toys facilitates creativity. See Figure 1 for a surface view of a randomly generated world in Minecraft.

Figure 1: *Randomly generated Minecraft world.*

When playing Minecraft, people often do not adhere to any structured goals. Play is typically about digging/mining for materials and then using these materials to build things. As players mine, they discover underground caverns and hidden treasures. They collect tools and materials as they explore. Then they use these tools and materials to build creative structures of their own. When playing in survival mode, where tools and materials must be collected before they can be used, there are roaming monsters, and it is possible to be killed and die.

Creative Mode

One can also play in creative mode. In this mode there is no death or killing, and it allows players the use of any materials or tools they want for their own builds. In this mode, players focus on building, and they can build anything they can imagine. This safe open-ended aspect of play in creative mode and the three-dimensional unit grid make Minecraft a useful platform for modeling discrete mathematical concepts. Minecraft is recommended for children over ten, but Williams (2016) found that in creative mode it is appropriate for children between six and ten as there are no monsters and no violence.

Mathematical Modeling

For mathematical modeling, I set Minecraft up in creative mode and generate a superflat world. Thus, play begins on a completely flat grassy plane. Because there are none of the terraced hills and random trees that exist in a default world, it is easier to focus on building. I have created a youtube channel—*Math in Minecraft with Dr B*—with tutorials for setting up and using Minecraft to model mathematical concepts (*Editors' note*: We encourage you to access the YouTube channel at this link: https://tinyurl.com/math-in-minecraft-channel).

Why use Minecraft?

Play

Polya (2004) realized the importance of curiosity, inventive faculties, and challenge for a student's mathematics disposition. He thought by providing opportunities to be creative with mathematics and using tasks that promote success, students can enjoy the triumph of discovery, and this feeling of discovery may lead to a lifetime love of mathematics. Others see this potential. Malone (1981) explains that the fantastic aspects of an interactive environment may have cognitive and emotional benefits. Vorderer (2000) states people may have a desire for an interactive environment. Shafer, Carbonara, and Korpi, (2019) explain how player enjoyment may be influenced by the sense of spatial presence afforded by a three-dimensional (3D) virtual environment. Minecraft provides students the opportunity to engage with mathematics in a setting associated with play. By utilizing an environment associated with play, creativity is encouraged.

Multiple representations

There are great opportunities in using Minecraft as a learning platform for modeling discrete mathematical concepts, as it affords new representations of content. By engaging mathematics in new ways, students reinforce mathematical understandings. "Effective teaching helps students make connections among visual, symbolic, verbal, contextual, and physical mathematical representations" (NCTM, 2014, p. 535). By visualizing math in multiple settings, students get the opportunity to activate new lenses and different perspectives, and this encourages mastery (Tripathi, 2008). Minecraft's 3D immersive environment encourages new ways of thinking about old problems. By promoting new representations for familiar tasks, conceptual understanding is improved.

How do I use Minecraft?

In this article, I demonstrate how discrete numeric and algebraic structures can be built in Minecraft. Most of the methods shown are an extension of counting blocks, multiplication tiles, or algebra tiles. For the most part, I use Minecraft as a new representation for traditional techniques associated with an area model of multiplication. I also extend this model to volume to create a new representation for factoring third degree polynomials. I discuss perfect squares and Pythagorean triplets. The

purpose of this article is to show a few ways that Minecraft—an environment that students associate with play—can be used across different grade levels to model various mathematical structures.

Arithmetic

Decimal system

According to Common Core State Standard `1.NBT.B.2`, first-grade students learn that 10 is a bundle of 10 ones, and 11 through 19 can be thought of as a ten and some ones. See Figure 2 for a Minecraft representation of twelve as a ten bundle and two ones. Students at this age should be able to add a single digit number to a two-digit number, and they should also be able to add a multiple of 10 to a two-digit number. This promotes an understanding of place value. When using a visual representation, the difference is stark. See Figure 3 for $20 + 12$ and $2 + 12$. Any exercise that traditionally uses blocks can be recreated in Minecraft.
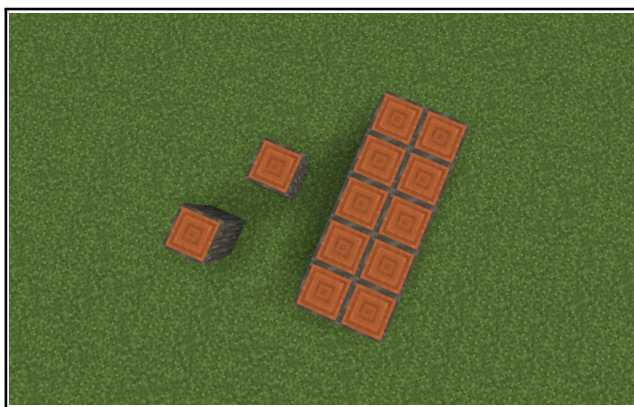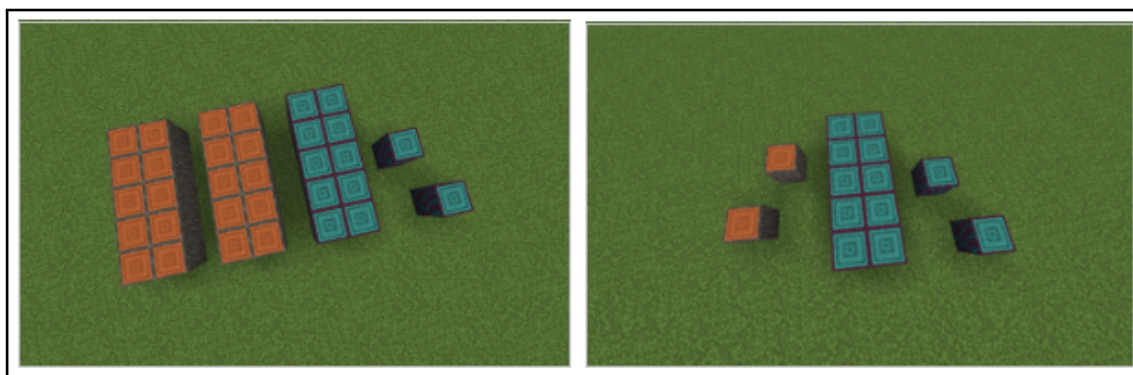

Figure 2: 12 *as a ten bundle and two ones.*

Figure 3: *(Left)* $20 + 12$; *and (Right)* $2 + 12$.

Composite numbers

In third grade, students begin to interpret products of whole numbers as n groups of m (`3.OA.A.1`). Thus, 20 could be represented as two groups of 10 tiles, or five groups of four tiles, or four groups of five tiles. When tiles are arranged into rectangles the factors appear at the margins. See Figure 4 for different visual representations of 20. This visual representation of factors makes the structure evident.
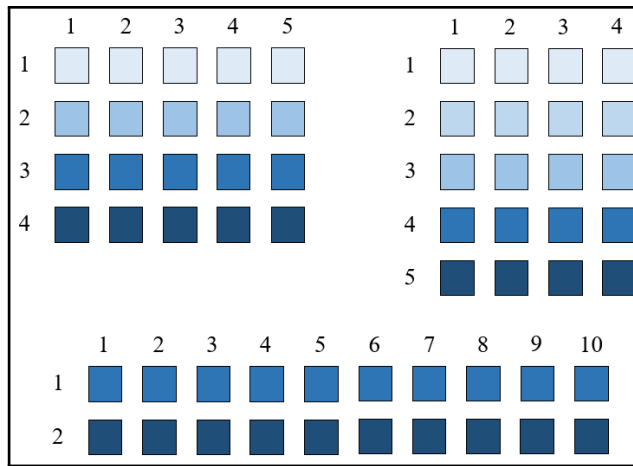
Figure 4: *(Top left)* 20 *factored as* 4 *groups of* 5; *(Top right)* 5 *groups of* 4; *and (Bottom)* 2 *groups of* 10.

When using Minecraft as a platform, students get to play with these representations in a game-like setting, and this may improve the student's relationship towards math. This blending of math and play, and visual and virtual leads to interesting connections. When asking a student if order mattered, "is it four times five or five times four?" She said it didn't matter, it is just your point of view, as illustrated in Figure 5.
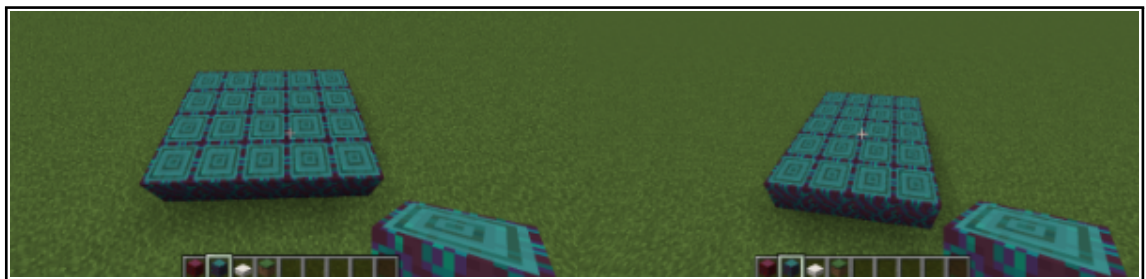

Figure 5: *Commutative property as a change of perspective.*

By portraying all possible groupings as blocks on a grassy plain, a conceptual understanding of multiplication is reinforced, and students begin to visually see the various factorizations. Figure 6 shows three different ways to factor 18. Two groups of nine, three groups of six, and six groups of three.
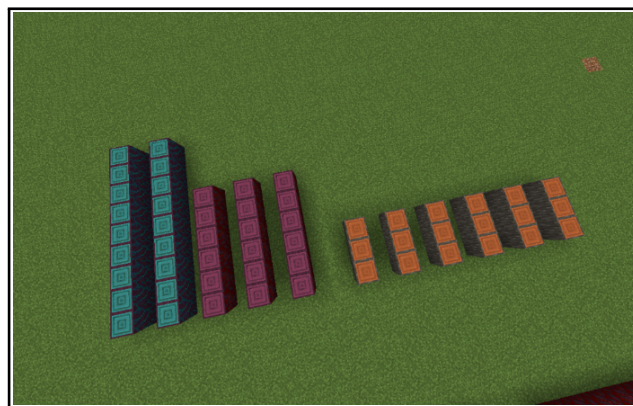

Figure 6: *Factorizations of* 18*.*

To start this lesson, I like to work with blocks or tiles and let students explore the various groupings. Then I allow students to turn in pictures of their factor models. Some students choose to use screen shots from Minecraft for their pictures. This allows students to engage the content on their own terms in an environment they relate to as being fun and relaxing. Both the physical and the computer-generated explorations into the mathematical structure require strategic thinking, and by revealing structure, these exercises move students toward conceptual understanding.

Area and Volume

In the third grade, students begin to understand a unit square is one square unit of area, and a rectangular figure made up of n unit squares has an area of n square units (3.MD.C.5). Around the fifth grade, students extend this concept to volume (5.MD.C.3). A unit cube is one cubic unit of volume and a rectangular box made up of n unit cubes has a volume of n cubic units. Physical representations of these concepts can be built in Minecraft. To represent area, each building block when viewed from above can represent a unit square. Figure 7 shows a 3×4 rectangle representing 12 square units of area.

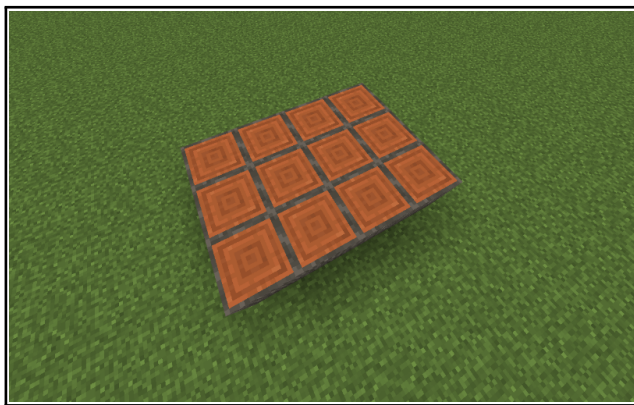**Figure 7:** *A 3×4 rectangle demonstrating 12 square units of area.*

To represent volume, each building block when viewed as a 3D object can represent a unit cube. These building blocks can be arranged into larger rectangular boxes to represent volume. Figure 8 shows a box that is 3 units long, 4 units wide, and 2 units high that has a volume of 24 cubic units.
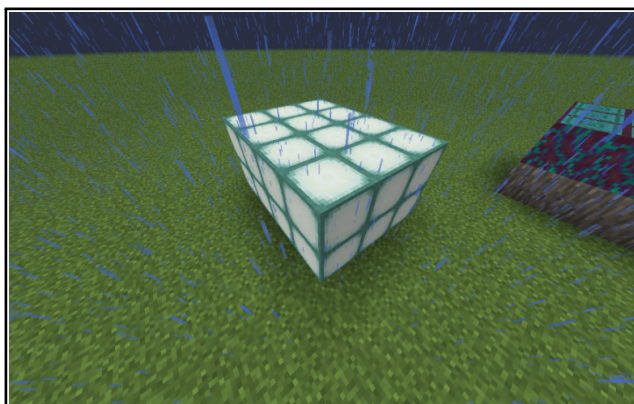

Figure 8: *A $3 \times 4 \times 2$ box representing 24 cubic units of volume in the rain.*

Since Minecraft allows players to view the same object from any viewpoint, whole numbers with at least 3 factors also have a 3D visual representation. Figure 9 shows 18 factored into 3 groups of 3, 2 layers high.
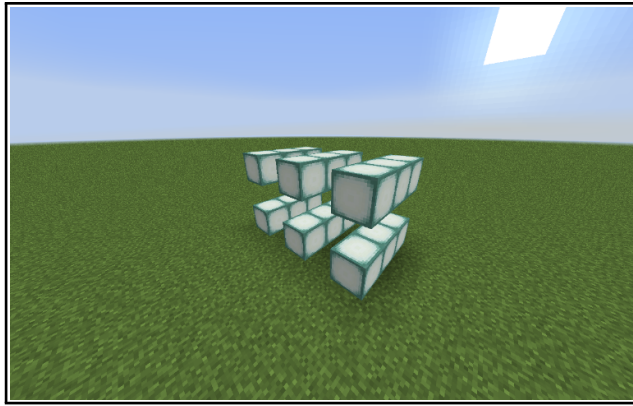
Figure 9: 18 *factored as* 3 *groups of* 3 *blocks* 2 *blocks high.*

Algebra

REPRESENTING ALGEBRAIC EXPRESSIONS

Blocks can also be arranged to represent algebra tiles and algebraic expressions. To simulate algebra tiles in Minecraft, I represent x as a unit wide/height rod of some arbitrary length. I usually use five as my arbitrary length, but any length greater than one will work. Following this same pattern, x^2 is a square with sides of this same arbitrary length. Thus, I represent x^2 in Minecraft as a unit height slab that is x units wide and x units long. Figure 10 shows a representation of 1, x, and x^2, and how these correspond to the unit, rod, and slab that are traditionally used in algebra tiles.
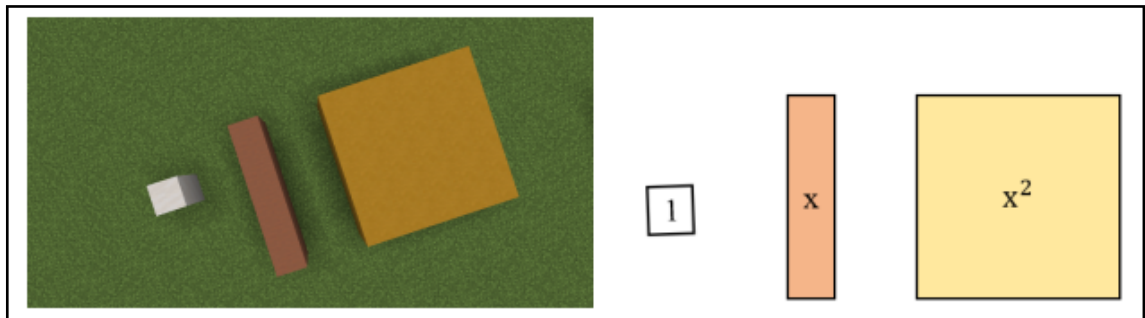

Figure 10: *Minecraft representation of unit, rod, and slab representing* 1, x, *and* x^2, *respectively.*

The algebraic expression can be represented with one slab representing x^2, three rods representing $3x$, and two units representing the 2. See Figure 11 for both the traditional and the Minecraft representation.
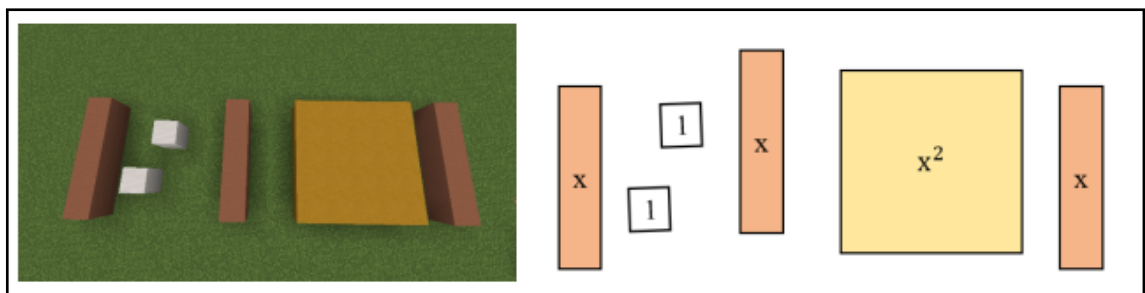

Figure 11: *Minecraft and traditional representation of* $x^2 + 3x + 2$.

Factoring Trinomials

Similar to how composite numbers reveal their factors in an area model of multiplication, algebraic expressions can be arranged into rectangles to reveal their factors. This aligns with the common core state standard HSA.SSE.B.3.A. Figure 12 demonstrates $x^2 + 5x + 6$ factored into $(x+3)(x+2)$.
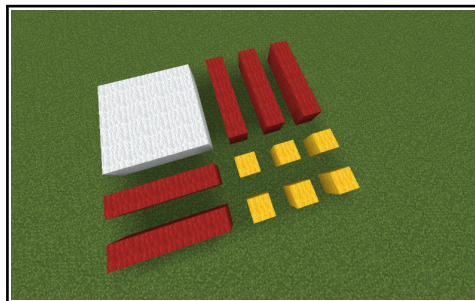

Figure 12: $x^2 + 5x + 6 = (x+3)(x+2)$.

Let's factor a trinomial! Figure 13 represents $2x^2 + 5x + 2$. The large white slabs represent x^2, the long blue rods represent x, and the small yellow blocks represent single units.
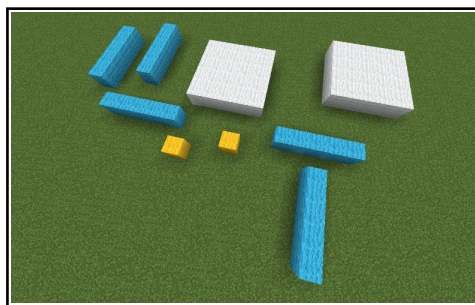

Figure 13: $2x^2 + 5x + 2$.

By arranging these objects into a rectangle, the factored form $(2x+1)(x+2)$ is revealed in the margins. There are two blue rods and a white unit along one side and one blue rod and two units along the other, as depicted in Figure 14. All trinomials with rational roots can be factored by building rectangles.
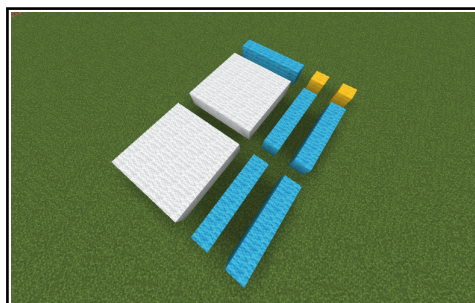

Figure 14: *Factored form of* $2x^2 + 5x + 2 = (2x+1)(x+2)$.

Just as Minecraft provides a virtual environment for grouping and using an area model of multiplication to help students visualize composite numbers, Minecraft can provide a virtual environment to help students visualize the structure within trinomials. Utilizing multiple representations improves learning (NCTM, 2014), and Minecraft as an immersive virtual environment provides a new platform for the traditional algebra tiles model.

As a 9-12 math teacher at a private school in central Florida, I introduced these factoring techniques to my students. This school had a little more than 300 students. The demographic makeup was 38% White, 31% Hispanic, 20% Asian, and 8% Black. Approximately 98% of the graduates of this high school attend four-year colleges. The vast majority of my students were familiar with Minecraft, and all of the students with whom I have introduced these factoring techniques were already highly proficient within Minecraft. For these students, the mention of using Minecraft in math class created great excitement.

I have found it takes time before students can rearrange the objects to form the rectangles that reveal the factors. Once they begin to see the solutions, the benefits are worth it. Some students who struggle with one of the many traditional procedures for factoring trinomials, find this visual approach beneficial. Typically, factoring trinomials involves finding the factors of the coefficients of the bookends and then using guess-and-check the see which numbers add to the center coefficient. After learning to factor within Minecraft, I have had students say things like, "this is so much easier than the way we were taught," or "why hasn't anyone shown me this before." Notice this visualization of quadratic factoring reveals the structure behind the math, and because of this, it improves conceptual understanding.

FACTORING THIRD DEGREE POLYNOMIALS

This is only the beginning. Minecraft is 3D. So far, this representation of factoring is only taking advantage of two of these three dimensions. Just as an area model can be expanded to volume, the factoring of 2nd degree polynomials with rational roots can be expanded to 3rd degree polynomials with rational roots. For factoring 3rd degree polynomials, let a rod be $1 \times 1 \times x$, a slab be $1 \times x \times x$, and a box be $x \times x \times x$, with x being some arbitrary length.

Common Core State Standard `HSA.APR.B.3` requires high school students find the factors of a polynomial when possible. Finding rational factors of higher degree polynomials is traditionally begun by using a rational zeros test. With this test, all possible zeros of a polynomial must be of the form $\frac{p}{q}$ where p is a factor of the constant term and q is a factor of the leading coefficient. After determining all possible zeros, students are taught to use guess-and-check to determine the actual zeros. Although this does reinforce the concept of zeros of a function and this method works when the polynomial has complex factors, these guess-and-check methods do little to reveal the structure within. Notice $2x^3 + 5x^2 + 4x + 1$ is represented in Figure 15a. By playing with structure, the pieces can be rebuilt into a rectangular prism as in Figure 15b. Again, the factors appear in the Margins. So $2x^3 + 5x^2 + 4x + 1 = (2x + 1)(x + 1)(x + 1)$ where $(2x + 1)$ is the length, $(x + 1)$ is the width, and $(x + 1)$ is the height.
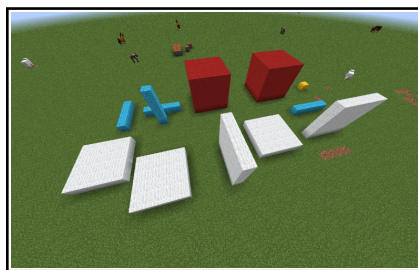
(a) $2x^3 + 5x^2 + 4x + 1$ *in random order.*
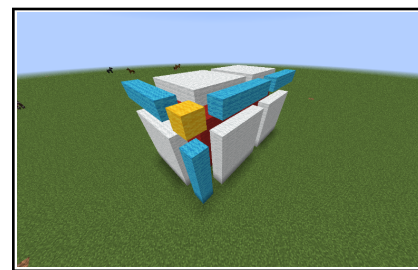
(b) $2x^3 + 5x^2 + 4x + 1 = (2x + 1)(x + 1)(x + 1)$.

Figure 15: *Fractoring trinomials.*

Other visual representations

PRIME NUMBERS

Once my students begin to visualize factors, they have made statements similar to, "you can just see the answer." By visualizing structures, many concepts become apparent. Fourth grade standards have students learning the concept of prime. A prime number is a number that cannot be further factored. In Minecraft 'prime' is a number of blocks that cannot be arranged into a rectangle without having leftovers. Figure 16 illustrates a representation of 13 in Minecraft.
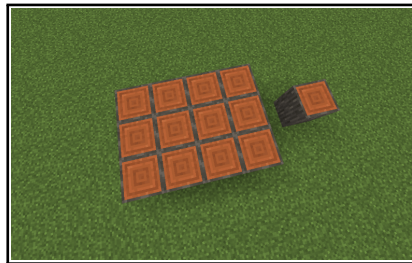

Figure 16: *13 as $3 \times 4 + 1$.*

PERFECT SQUARE TRINOMIALS

High school standards include recognizing perfect square trinomials. A visual representation of a perfect square trinomial reveals there must be a rod on both the length and the width. This reveals why the middle term of a perfect square trinomial is always even, and the leading and trailing coefficients must be squares (see Figure 17).
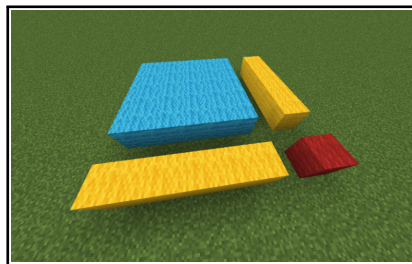

Figure 17: *A perfect square trinomial $(x + 1)^2 = x^2 + 2x + 1$.*

COMPLETING THE SQUARE

This visual representation also makes the structure for completing the square evident. Notice $4x^2 + 12x$ would need 9 units to complete the square (see to Figure 18).
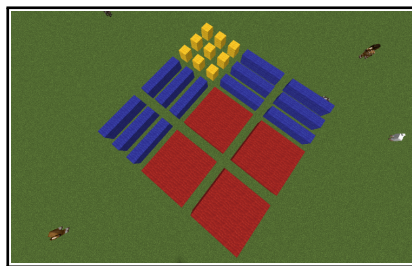

Figure 18: *Completing the square by adding 9 units to $4x^2 + 12x$.*

In eighth grade, students learn the Pythagorean theorem. A spatial representation reveals the structure of Pythagorean triplets. Figure 19 demonstrates how $3^2 = 9$, $9 - 1 = 8$, and $\frac{8}{2} = 4$.
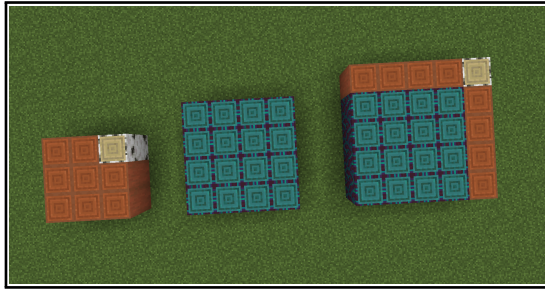

Figure 19: *Pythagorean triplet* $3, 4, 5$.

Thus, a square 3 units wide can be deconstructed to wrap a square 4 units wide to make a 5 unit wide square. Similarly, $5^2 = 25$, $25 - 1 = 24$, and $\frac{24}{2} = 12$. A square of width 5 units can be deconstructed to wrap a 12 unit wide square to make a square 13 units wide. Every even perfect square is also part of a Pythagorean triplet. One pattern is found by subtracting 4 for the corner and dividing the result by 4 for the two rows on each side, e.g., $8^2 = 64$, $64 - 4 = 60$, and $\frac{6}{4} = 15$. An 8×8 square can be deconstructed to wrap a 15-unit square twice to make a 17-unit square. Figure 20 shows a 6×6 square deconstructed to wrap an 8×8 square to make a 10×10 square.
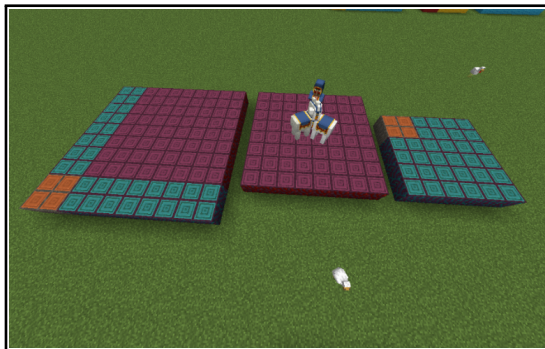

Figure 20: *Pythagorean triplet* $6, 8, 10$.

Notice the corner is $2^2 = 4$ units. There are many interesting patterns for finding Pythagorean triples, and all of them have visual representations.

The next steps

This tour through the grade levels, starting with using blocks to demonstrate place value, to visualizing structure within 3rd degree polynomials can be accomplished outside of Minecraft. Minecraft has some limitations: There are no pre-made forms for the rods, slabs, and boxes that exist in traditional algebra tiles. When simulating algebras tiles, I usually make rods five units long, slabs five units square and boxes five units cubed. This can lead to misconceptions as the length of x should not be a whole number. General construction of these forms can also be difficult in Minecraft as any movement of a rod, a slab, or a box requires breaking it apart and then rebuilding it in a new location. By utilizing the fill command, construction can be completed much quicker. For instance, the command

```
/fill ~1 ~1 ~1 ~5 ~1 ~1 <block name>
```

will create a rod in front of the player. Similarly, the command

```
/fill ~1 ~1 ~1 ~5 ~1 ~5 <block name>
```

will create a slab in front of the player, and the command

```
/fill ~1 ~1 ~1 ~5 ~5 ~5 <block name>
```

will create a box in front of the player. See Figure 21 for the command to a rod.

Figure 21: *Using the fill command to build a rod.*

It would be helpful to have pre-made objects that represent a rod, a slab, and a box. This type of representation for 3rd degree polynomials may become more prevalent as currently there is at least one online demonstration with similar representations for x, x^2, and x^3 (Srebnick, 2014).

Conclusion

Students benefit when teachers incorporate multiple representations into their teaching (NCTM, 2014). Traditional manipulatives such as tiles and blocks are a great way to begin exploring multiplication and factoring. By extending these concepts to the virtual realm, educators are able to add a new representation of an old technique to their repertoire. The benefit being that many students play Minecraft for fun.

Similarly, including cultural and community resources into instruction promotes equity in the classroom and fosters positive mathematical identities among students (Stanford Center for Assessment, Learning, and Equity, 2019). Minecraft is a cultural resource known to most students. This makes it a great motivator for students who already play. Explaining math concepts in an environment associated with fun helps students to see themselves as doers of math. We build positive mathematical identities by fusing math content with cultural resources that students view in a positive light.

When used to complement a student's interest, Minecraft allows explorations into mathematical structures to happen in a fun and relaxing environment. By encouraging relationships between student culture and conceptual math in an environment students find enjoyable, they begin to relate to math in new positive ways. These activities promote, challenge, and require students to be creative with math content. As students gain agency and overcome challenges associated with modeling mathematical structures, they begin to feel the triumph of discovery, and it is this triumph that has the potential to promote a lifelong passion for mathematics.

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