Proof Without Words: Partial Fraction Decomposition

Hervé Svoboda
Lycée Amiral de Grasse, Provence-Alpes-Côte d'Azur, France

Abstract

This short article presents two visual proofs for classical partial fraction identities: one in two dimensions, the other in three. Rather than relying on algebraic manipulation, each diagram uses geometry to convey the decomposition clearly and intuitively. The visual format invites students to explore the structure of the expressions without getting lost symbolic detail, opening space for meaningful discussion and deeper understanding. These diagrams may also serve as a springboard for students to create their own visual representation of algebraic ideas.

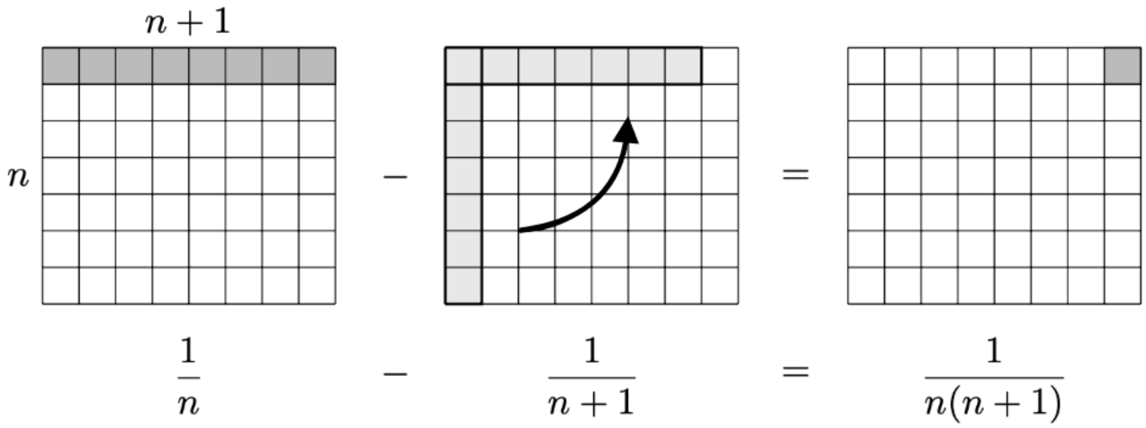
Keywords: Visualization, proofs without words, argumentation

1 Proofs Without Words

Proof.

$$\forall n \in \mathbb{N}^*, \quad \frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1}$$

Figure 1
Visual proof of the partial-fraction identity.



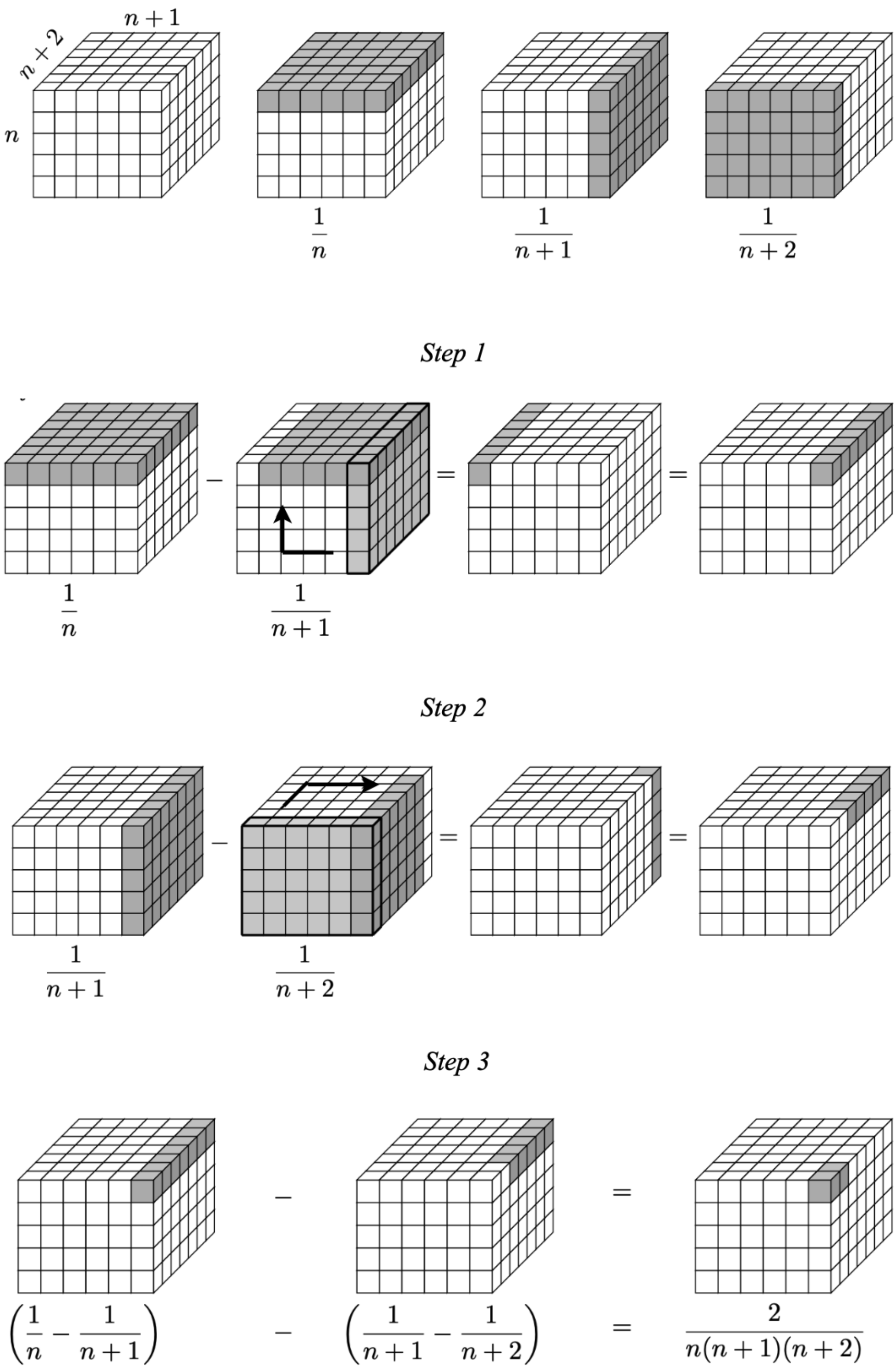
□

Proof.

$$\forall n \in \mathbb{N}^*, \quad \frac{2}{n(n+1)(n+2)} = \frac{1}{n} - \frac{2}{n+1} + \frac{1}{n+2}.$$

Figure 2

Presentation of the key components.



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2 Accompanying Note for Teachers

This visual article offers a geometric interpretation of two well-known identities from partial fraction decomposition:

$$\frac{1}{n(n+1)} = \frac{1}{n} - \frac{1}{n+1} \quad \text{and} \quad \frac{2}{n(n+1)(n+2)} = \frac{1}{n} - \frac{2}{n+1} + \frac{1}{n+2}.$$

These identities often appear in high school courses, especially when working with telescoping sums or simplifying rational expressions. Instead of presenting a traditional algebraic proof, this approach invites students to see the truth of the identity through geometry. In class, this can serve as a great starting point. Simply display the image and ask: “What do you notice? What do you wonder?” (Boaler & Humphries, 2014).

The question often sparks insightful observations and encourages students to connect visual patterns to algebraic reasoning. A few classroom ideas:

- Use the image as a launch point before introducing symbolic decomposition.
- Challenge students to reconstruct the identity by interpreting the areas or volumes.
- Invite them to create their own visual proofs for other identities.

This approach works particularly well for students who are more visually or spatially inclined, or those who find symbolic algebra challenging. It also helps foster conceptual understanding and shows how geometry and algebra can beautifully complement each other. More than just a proof, this is an opportunity to explore the aesthetic side of mathematics and to encourage students to appreciate the ideas behind the formulas.

References

Boaler, J., & Humphreys, C. (2014). Notice & Wonder. In *Mathematical Mindsets: Unleashing Students' Potential through Creative Math, Inspiring Messages and Innovative Teaching*. Jossey-Bass.

Kifowit, S. J. (2005). Proof Without Words: A partial fraction decomposition. *College Mathematics Journal, 36*, 122.

Hervé Svoboda (svoboda.herve@gmail.com) is a senior secondary school teacher (agrégé de mathématiques) and has been teaching since 2010 at Lycée Amiral de Grasse in southeastern France. Hervé is passionate about visual mathematics and ”proofs without words,” which he integrates into his teaching and mathematical explorations. Hervé's work focuses on making abstract concepts more accessible through imagery and intuition.