Helping Students Hit the Target on Probability

Ahmad M. Alhammouri[1], **Bre Winningham**[2], **Steve Phelps**[3], **& Gregory D. Foley**[4]
[1]Jacksonville State University, [2]Heard County HS, [3]Mariemont SD, [4]Ohio University

Abstract

We use a Desmos simulation of a dartboard game to engage middle school students in geometric probability, the concept of randomness, and proportional reasoning to help them connect experimental probability to theoretical probability.

Keywords: probability, middle school, Desmos, simulation, experimental probability, theoretical probability, geometric probability

The Common Core State Standards for Mathematics (CCSSI, 2010) emphasize probability in Grade 7. According to the standards, students must be able to "investigate chance processes and develop, use, and evaluate probability models" (p. 50). Moreover, students should be able to link experimental probability to theoretical probability and be able to "compare probabilities from a model to observe frequencies" (CCSSI, 2010, p. 51).

To link theoretical and experimental probability in the minds of middle school students, we engage them in a conjecture–simulation feedback loop to develop critical concepts prior to theoretical calculation. We first ask them to make a conjecture about a probability value within the setting of a dartboard game and then engage them in a simulation to test their conjecture. Next, we repeat the process multiple times, each time giving them an opportunity to revise their conjecture followed by another simulation. In this way, we allow students to learn the rules of the dartboard game while forming their understanding of the meaning of a *random* event and related probability concepts. We do this because "effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding" (NCTM, 2014, p. 10). The result is students who understand both the context of the activity and key probability concepts as a basis for engaging in meaningful theoretical probability calculations.

Like Bloom, Kurz, and Yanik (2020), we connect experimental and theoretical probability using a contextual situation. Whereas Bloom and colleagues employ *mathematical modeling*, that is, using mathematics to shed light on a real-world situation; we engage students in *modeling mathematics*, using a realistic situation—a dartboard game—to develop understanding of a mathematical topic—geometric probability (Felton-Koestler, 2017).

1 The Dartboard Activity

To engage students in a virtual game of darts, we created a series of simulations in Desmos. We encourage you to use the following link to explore the activity:

https://teacher.desmos.com/activitybuilder/custom/60ca0b8034947b5c5aa2f6d0

The link includes a teacher's guide with a checklist and a place to write notes during the student engagement. This activity is designed for two days.

1.1 Purpose of the Two-Day Structure

Day 1 focuses on experimental probability through conjecturing and simulation, allowing students to surface misconceptions and notice variability in results. Reflection between days centers on why outcomes differ and what stabilizes as trials increase. Day 2 builds on this experience by introducing theoretical probability through area, positioning it as an explanation for the experimental results rather than a standalone formula. Below we will walk you through the activity. Along the way, we share screenshots from this activity.

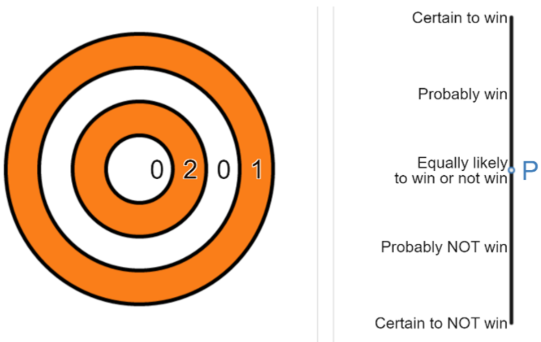
1.2 Guessing, Checking, and Revising

Figure 1 shows a color-coded dartboard that serves as the basis for this activity. The dartboard has regions labeled 0, 1, and 2, colored white and orange. Students are told that the dartboard in this activity is not like a normal dartboard that they might know: The dart will always land on the board, but where it lands will be completely random. If the dart lands within region 0, 1, or 2, the player wins 0, 1, or 2 free games, respectively.

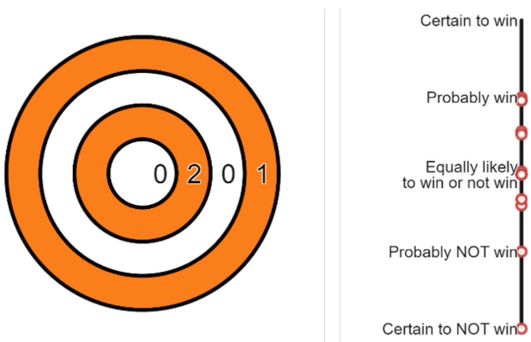
Figure 1
Color-coded dartboard.

First, based only on observing the dartboard (Fig. 2a), students are asked to use a slider to show their guess of how likely they are to win a game. Then, they get to compare their guesses with the rest of the class, as shown in Figure 2b.

Figure 2
Guessing the probability of winning a game and comparing with the rest of the class.



(a)



(b)

Next, the teacher engages the students in a discussion of how they made their guesses and why. This discussion will help students to understand terms such as *probably*, *equally likely*, *certain*, and *random*.

Understanding these terms will prepare the students for the next two phases of the activities (i.e., experimental and theoretical probabilities).

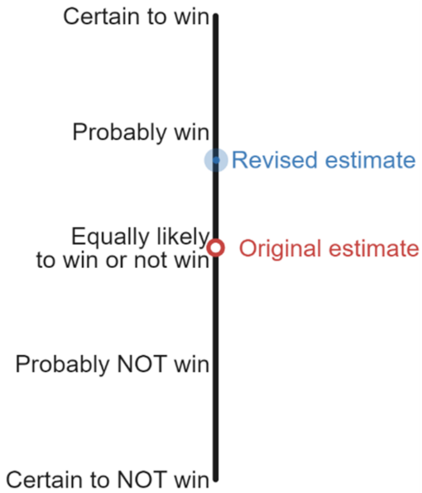
As the activity continues, students simulate throwing one dart, 10 times, with the results being recorded in a table. Each student repeats this process three times. A typical table of results is shown in Figure 3.

Figure 3
Simulating 10 single attempts and recording the results in table.

Throw	Score	Free game?
1	0	✗
2	1	✓
3	1	✓
4	0	✗
5	1	✓
6	1	✓
7	1	✓
8	1	✓
9	0	✗
10	0	✗

Next, as shown in Figure 4, students are given an opportunity to revise their guesses made at the beginning of the activity. The Desmos simulations provide data to inform the students' decisions about the probability of winning free games.

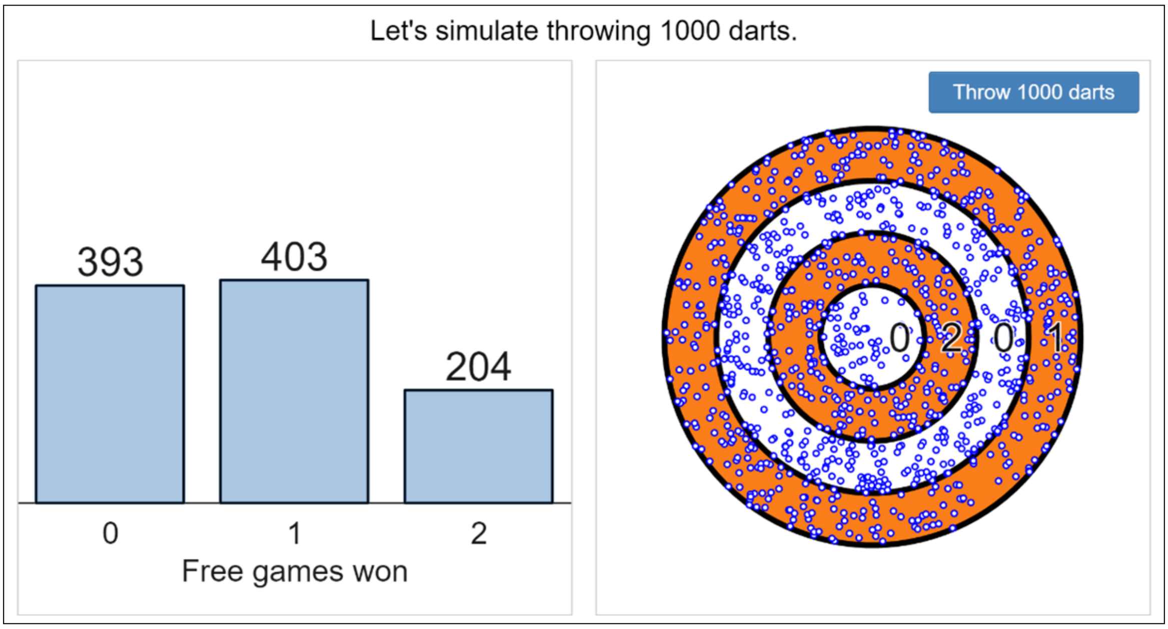
Figure 4
Revising the initial guess after engaging in repeated simulations.



The students then engage in two additional simulations: throwing 50 darts at a time and throwing 1000 darts at a time. As before, the results are automatically displayed, but this time using a bar chart, as shown in Figure 5.

Figure 5

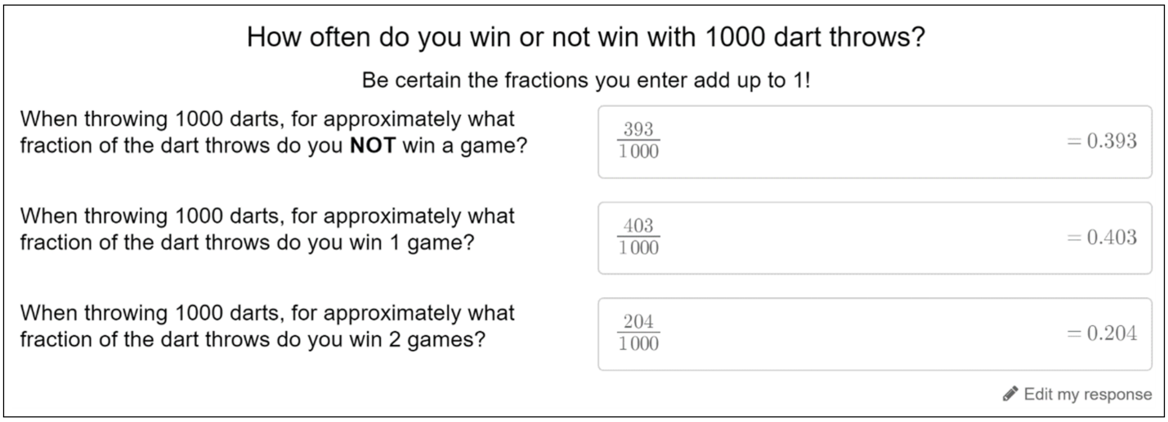
Simulating 1000 random dart throws.



After students complete the simulations of 50 and 1000 attempts, they are asked to find the fraction for winning 0, 1, and 2 games, as shown in Figure 6. That is, they are asked for the experimental probabilities of these events.

Figure 6

Finding the experimental probability.



How often do you win or not win with 1000 dart throws?

Be certain the fractions you enter add up to 1!

When throwing 1000 darts, for approximately what fraction of the dart throws do you **NOT** win a game? $\frac{393}{1000} = 0.393$

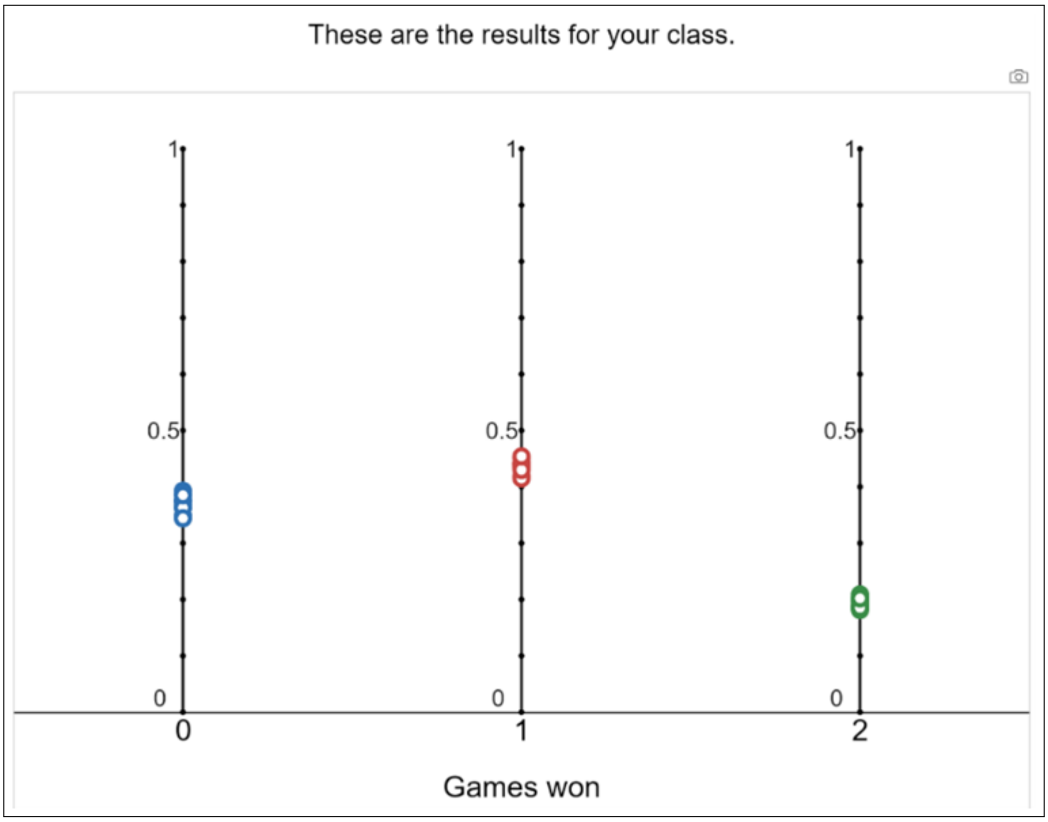
When throwing 1000 darts, for approximately what fraction of the dart throws do you win 1 game? $\frac{403}{1000} = 0.403$

When throwing 1000 darts, for approximately what fraction of the dart throws do you win 2 games? $\frac{204}{1000} = 0.204$

Edit my response

Then, students compare their findings with the rest of the class, as shown in Figure 7. Here, the teacher can orchestrate a reflection discussion, asking, for example: What have you learned? Will all simulations yield the same results? Why do the answers vary? By doing so, students will be able to deepen their understanding of experimental probability. Because the results are based on attempts, there is not a one correct answer; however, the answers will be close. After the students complete the experimental probability portion of the activity on Day 1, the next day they will calculate the theoretical probabilities for the same events.

Figure 7
Comparing experimental probability findings with the rest of the class.



2 Theoretical Probability

At this stage, students find the theoretical probability of winning 0 games, 1 game, and 2 games. Students make sense of the geometric regions within the dartboard. They define the regions in terms of the circles in the dartboard and find the outer and inner areas, region areas, and then the probabilities of the dart landing within the various regions.

Figures 8–10 provide an example of how to compute the theoretical probability of winning 1 free game. When students record their findings for the outer area, the inner area, the area of the region (the difference), and the theoretical probability, Desmos helps them to self-evaluate their responses by providing a green mark if the answer is correct or a red X if the answer is not, as shown in the table included in Figures 8 and 9. In addition, students will be able to compare their findings from the experimental probability for the same event based on the simulation of throwing 1000 darts. This comparison provides insight to the students about the relationship between experimental and theoretical probability. Understanding this relationship is a key foundation for work in statistics (Langrall et al., 2017).

Figure 8

Finding the area of the dartboard.

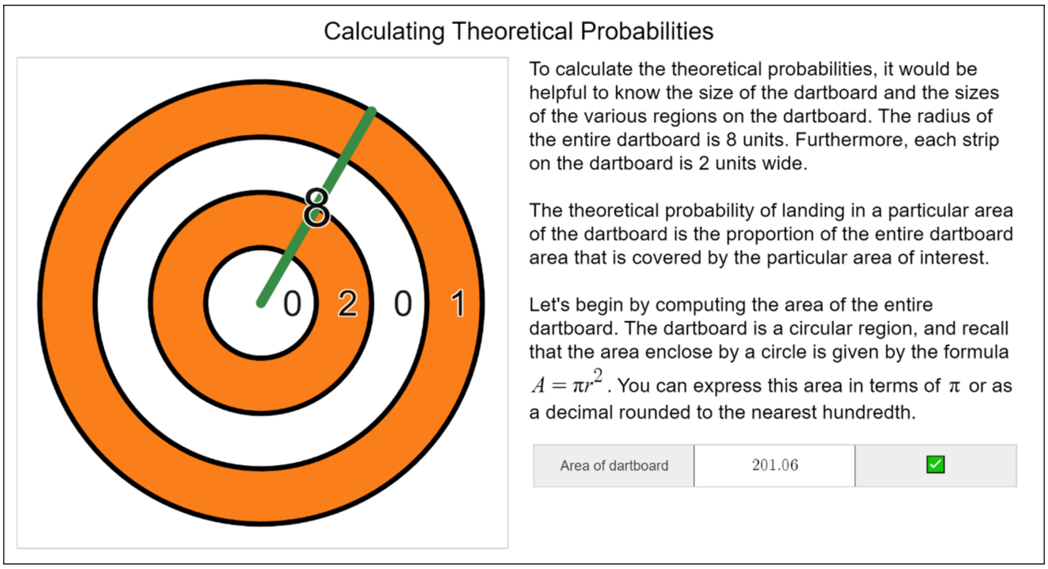


Figure 9

Finding the area of outer region 1.

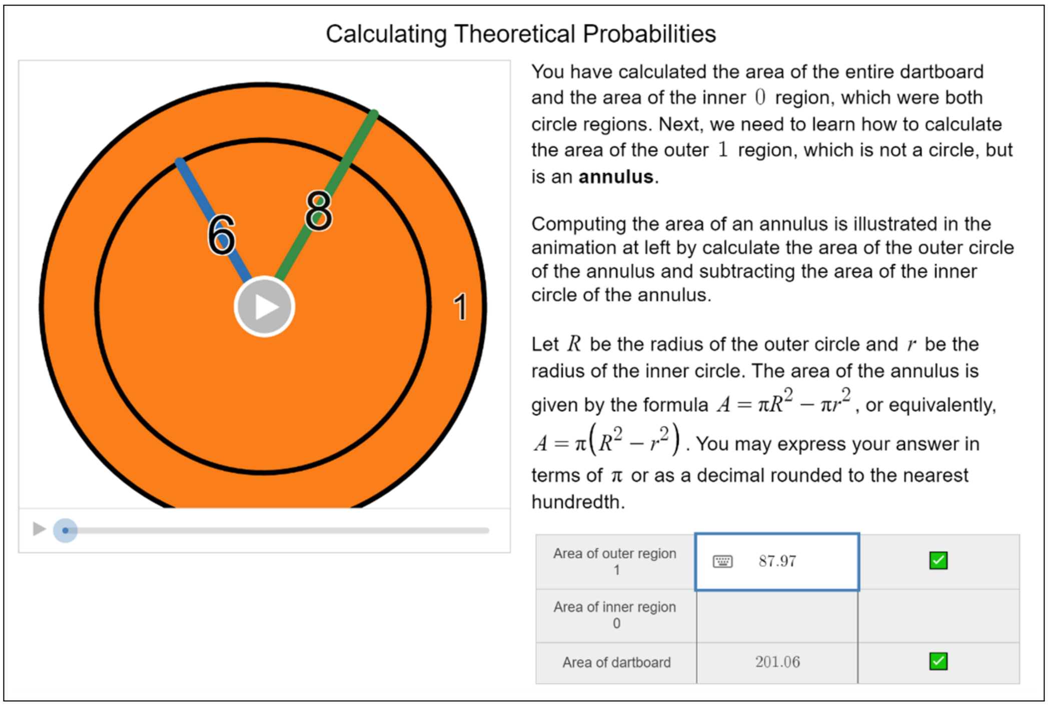
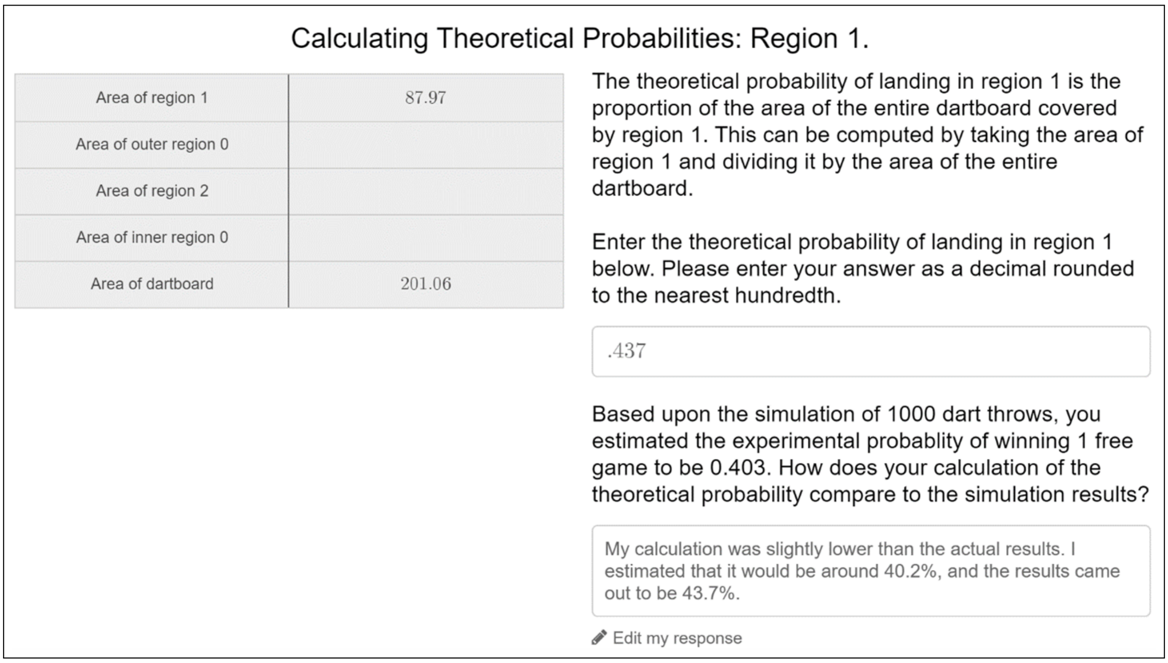


Figure 10
Computing theoretical probability and comparing it to experimental probability.



To close the activity, the teacher can orchestrate a final discussion on the relationship between experimental and theoretical probability. Wheeler and Champion (2016) stated that "experimental probability values determined by a small number of trials are expected to vary more widely from theoretical probabilities than experimental probability values determined by a larger number of trials" (p. 337). This is an important takeaway from this activity.

3 Implementation Notes

The dartboard activity was piloted with seventh-grade students during instruction focused on probability concepts, including randomness, experimental probability, and theoretical probability. The lesson was implemented over two class periods. The first day emphasized conjecturing and experimental probability through repeated simulations, while the second day focused on theoretical probability using geometric area. Students worked individually during initial conjectures before engaging in whole-class discussion, allowing authentic intuitions and misconceptions to surface prior to refinement.

During the conjecture and early simulation phases, students relied heavily on visual intuition and symmetry. A common misconception was that randomness implies equal likelihood, with several students arguing that outcomes should be "the same" because the dartboard could be "cut in half." Other students initially used additive reasoning, focusing on the number of regions or colors rather than their relative areas. As students engaged in simulations of 10, 30, and larger numbers of trials, their reasoning began to shift toward frequency-based and proportional reasoning. For example, students justified revised conjectures using statements such as winning "more than half" of the trials, while also noting that results varied across simulations. These observations created natural opportunities to discuss variability and the role of sample size in experimental probability.

Later discussions revealed important progress in students' understanding of randomness and independence. Students debated whether a sequence of wins would influence future outcomes, initially suggesting that prior results might affect subsequent trials. Through guided discussion, students articulated that each dart throw is independent and that previous outcomes do not determine future results in a random process. Informal indicators of learning included students revising conjectures based on simulation data, distinguishing between experimental and theoretical probability, and reasoning about independence, area, and relative frequency. Teachers implementing this activity should plan time for discussion after each simulation phase and anticipate misconceptions related to symmetry, equal likelihood, and the influence of prior outcomes.

Although student work during the theoretical probability phase was not formally recorded, our experience implementing this activity with preservice teachers suggests that students may struggle with applying the area formula for circles and annuli. In prior implementations, preservice teachers often needed additional support connecting the formula for the area of a circle to the geometric structure of the dartboard, particularly when reasoning about the difference of areas. Teachers implementing this activity with middle or early secondary students may therefore consider reviewing area formulas in advance, providing visual scaffolds, or allowing students to estimate relative areas before computing exact values.

4 Practical Classroom Guidance and Discussion Support

This activity is flexible and can be implemented across a range of classroom contexts. When individual devices are available, students can engage directly with the Desmos simulations independently or in pairs. In classrooms with limited technology, the simulations can be projected and run as a whole-class experience, with students recording results, making predictions, and discussing outcomes collectively. Small-group setups using shared devices also work well, particularly during the conjecture and simulation phases.

During early simulations, students often rely on visual intuition or symmetry and may attribute differences in results to luck, balance, or skill. Teachers should plan regular discussion pauses after each simulation phase to surface these ideas and press students to justify their reasoning using observed frequencies. Prompts such as *Why didn't everyone get the same results?*, *What changes as the number of trials increases?*, and *Does a previous win affect the next outcome?* help students distinguish randomness from fairness and develop an understanding of independence.

As students transition to theoretical probability, some may need additional support connecting area formulas to probability. Based on prior implementations with preservice teachers, students may struggle with applying the area formula for circles and annuli or with reasoning proportionally about regions. Teachers can scaffold this phase by reviewing area formulas in advance, providing visual representations of regions, or allowing students to estimate relative areas before computing exact values. Formative checkpoints include asking students to explain why one outcome is more likely than another, compare experimental and theoretical results, or predict how increasing the number of trials would affect outcomes.

For students ready to extend their thinking, teachers may invite students to design their own dartboards, modify region shapes, or experiment with alternate geometric configurations. These extensions deepen understanding of geometric probability while reinforcing proportional reasoning.

5 Conclusion

The dartboard activity connects experimental and theoretical probability within a geometric setting. It helps students to understand how these two types of probability are alike and how they are different. Students explore experimental probability through repeated simulations. They find the corresponding theoretical probabilities by computing areas of the regions on the dartboard and then dividing these areas by the area of the entire dartboard. When computing these theoretical probabilities, students have an opportunity to apply the formula for the area of a circle, and they learn about an annulus and its area. They also engage in proportional reasoning to compute the probabilities of landing in the various regions once they have found the circular and annular areas for the dartboard game.

As this activity demonstrates, technology can enhance student engagement significantly and can provide immediate feedback for students to test and refine their conjectures. This (admittedly contrived) experiment would be impossible to do physically. (In real life, we sometimes miss the dartboard and do not throw randomly; we aim to earn points.) The Desmos simulations allow students to generate 1000 trials instantly, to provide direct feedback to students.

As noted earlier, our goal here is not to model a real-world situation, but rather to use the realistic and inviting dartboard context to develop student understanding. We believe that the dartboard game will deepen student understanding of experimental probability, theoretical probabilities, and the relationship between these two. The activity connects modeling, geometric probability, the concept of randomness, and proportional reasoning. Technology is used strategically to build and support these connections. The activity offers opportunities for class discussions to help students "hit the target" on critical concepts of probability.

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Ahmad M. Alhammouri (aalhammouri@jsu.edu) is an Associate Professor and Associate Department Head in the Department of Secondary Education at Jacksonville State University in Jacksonville, Alabama. He obtained his Ph.D. in Curriculum and Instruction, specializing in Mathematics Education, from Ohio University. His research interests focus on mathematical modeling and quantitative reasoning, including how to advance mathematics teachers' capacity to address these topics in their instruction. Dr. Alhammouri is a member of several professional organizations, including the Mathematical Association of America, the National Council of Teachers of Mathematics, and the Society of Industrial and Applied Mathematics.

Bre Winningham (bre.winningham@heard.k12.ga.us) is a certified high school mathematics educator at Heard County High School, where she teaches Algebra, Geometry, and advanced mathematics courses. She holds a Master's degree in Secondary Education with a concentration in Mathematics from Jacksonville State University. Her professional interests include algebraic reasoning, real-world applications of mathematics, and instructional approaches that promote deep conceptual understanding and student-centered learning.

Steve Phelps (sphelps@mariemontschools.org) is a Mathematics Coach, Digital Coach, and Computer Science teacher at Mariemont City Schools in Cincinnati, Ohio. He is Co-Founder and Co-Facilitator of the Cincinnati Math Teacher's Circle, Co-Director and Trainer at the GeoGebra Institute of Ohio, and a National Instructor for Teachers Teaching with Technology. Steve has been an active leader in mathematics education technology integration, serving as a former Co-Editor of Technology Tips for NCTM and Website Editor and Social Media Manager for the Ohio Council of Teachers of Mathematics.

Gregory D. Foley (foleyg@ohio.edu) received B.A. and M.A. degrees in Mathematics and the Ph.D. in Mathematics Education from The University of Texas at Austin. He currently serves as the Robert L. Morton Professor of Mathematics Education in The Patton College at Ohio University and has a courtesy appointment in the Department of Mathematics. His research, curriculum development, and professional development for teachers focus on the mathematical transition from secondary to postsecondary education. His scholarly interests include cognitively demanding tasks, inquiry-based instruction, tools and technology to enhance student construction of knowledge, spatial thinking, quantitative literacy, financial modeling, and teacher professional development in statistics and modeling.