Using Toy Animal Ball Poppers to Explore Projectile Motion in Algebra and Calculus Classes

Marsha Nicol Guntharp, *Palm Beach Atlantic University*
Fred Browning, *Palm Beach Atlantic University*
Gloria Royle, *Jupiter Middle School of Technology*

Abstract: *In this article, the authors highlight the use of toy animal ball poppers to provide students with a novel, hands-on experience with projectile motion. Specifically, the authors highlight their use in the modeling of data associated with quadratic functions to generate distance versus time graphs, as well as to motivate the modeling of flight paths with parametric equations.* ***Keywords.*** *Modeling, Algebra, Manipulatives*

1 Introduction

Toy animal ball poppers provide a great way to use projectile motion in the classroom. We used them successfully in algebra and calculus classes to help students model data with quadratic functions, with distance versus time graphs, and also to model the flight path with parametric graphs. Figure 1 shows one of the animal ball poppers we used.

Fig. 1: *An animal ball popper.*

The photo of our dog popper has a soft foam ball in its mouth and a second ball ready to go. When students squeeze the dog popper, the ball shoots out of its mouth. We used iPads to videotape the action, and then we used an app by Vernier called Video Physics to track the flight of the ball. Next we inserted the image into a TI-Nspire graphs page in order to model the data with a graph.

Since the middle school with which we work has 60 iPads loaded with the TI-Nspire iPad app and dedicated to the mathematics department, it was a fun activity that the students could use to learn more about quadratic functions. (Note that, with very few exceptions, the TI-Nspire iPad app has all of the same functionality as the TI-Nspire handheld and the TI-Nspire computer software. The main differences are that the iPad app has a touchscreen, and at this point the iPad app does not accept science probes for data collection.)

Our calculus students own their graphing technology (mostly handhelds), so we used the animal ball poppers in class. Since we have an iPad connected to an LCD projector, we were able to track the ball on our iPad (which has the Vernier Video Physics app, as well as the TI-Nspire app) as the students watched, and then save the tracked ball as an image which could be inserted into the TI-Nspire as shown in Figure 2.

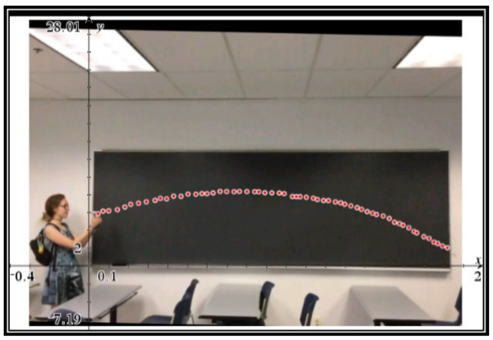

Fig. 2: *Image of tracked ball inserted into TI-Nspire handheld.*

Since the students did not have the Vernier app to be able to track the balls on their own, we emailed them the image for use in their own graphing calculators or iPad Nspire apps. At our university, we also have a computer laboratory loaded with the Nspire software, so students could choose to use the computers to complete the assignment.

2 Parametric Equations in Calculus

In order to model the data with our calculus classes, we had discussions about the use of vectors and parametric equations for modeling projectile motion. We first separated the velocity vector into its component parts, $v_x = v \cdot cos(\theta)$ and $v_y = v \cdot sin(\theta)$, as shown in Figure 3.

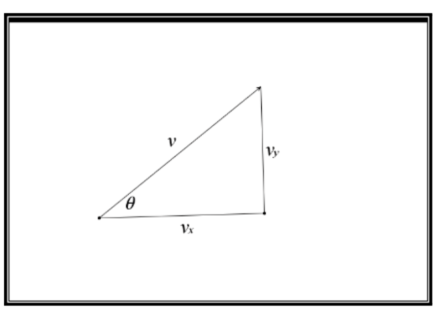

Fig. 3: *Separating velocity vector into components.*

Next, we expressed the parametric equation in the vertical direction, $y = -0.5gt^2 + v \cdot sin(\theta) \cdot t + h$, using the projectile motion equation, $h = -0.5gt^2 + v_0 t + h_0$, where h_0 represents the height of the object from its initial position (we used meters) and g represents the earth's gravity, $-9.8m/s^2$. Finally, we obtained the parametric equation in the horizontal direction, $x = v \cdot cos(\theta) \cdot t$, using the familiar $d = rt$.

We had the students use the sliders feature of the TI-Nspire to explore various values for variables v, θ, and h (or they could have also measured the initial angle of the projectile using the geometry menu of the TI-Nspire). The sliders could be manipulated back and forth, changing the values of the variables, until a graph could be produced that correctly modeled the data. They used the parametric equations $y(t) = -.5gt^2 + v \cdot sin(\theta) \cdot t + h$ and $x(t) = v \cdot cos(\theta) \cdot t$.

Since we have classrooms that have blackboards as well as whiteboards, it seemed like a good option to simply use the blackboard as a backdrop for the ball and then to measure the blackboard to get our correct measurements. The blackboard measured 3.62 meters across, so some students scaled their axes units, as in the following quote from student work.

> The values we found for our initial height, initial velocity, and angle were 3.9, 18, and 19.1, respectively. The distances were not to scale so we converted them to meters by dividing 28.69 (length of the graph in units) into 3.62 (length of the board in meters). That gave us a scaling factor of 0.126 that we used to convert our imaginary units of distance to actual meters.

Figure 4 shows their model prior to scaling.

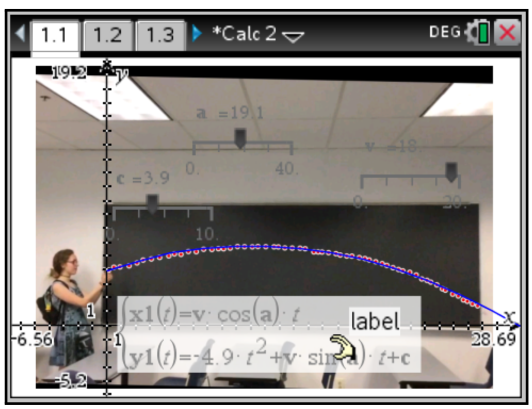

Fig. 4: *Student model prior to scaling.*

After they multiplied 3.9 by 0.126 and 18 by 0.126 to get their initial height of 0.491 meters and their initial velocity of $2.268m/sec$, they gave their parametric equations as $x = 2.268cos(19.1) \cdot t$ and $y = -4.9t^2 + 2.268sin(19.1) \cdot t + 0.491$, however, they made a mistake by using $-9.8m/sec^2$ as gravity in a scaled model that was not in meters. Had they multiplied the -9.8 by the scale $(28.69/3.62)$ to get 77.67, and used that value for gravity in their original equation, they would have obtained $6.5m/sec$ for their initial velocity, rather than $2.268m/sec$. (Note that if they had used the variable g to represent gravity, they could have used a slider to manipulate the value.) Mistakes like the one the students made can be used as teachable moments.

While those students chose to first place their axes and then scale their equations, most students simply changed the x-axis and y-axis to match the measurements of the blackboard. See Figure 5 for a sample of work from other students.

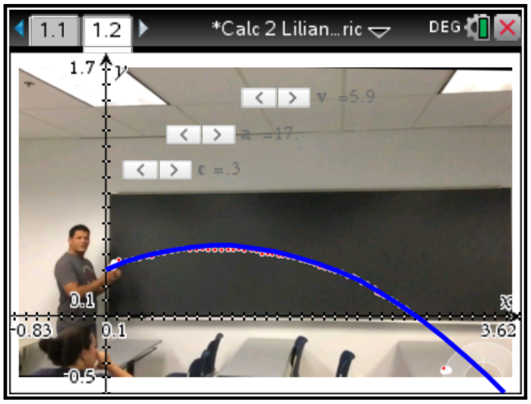

Fig. 5: *Sample student work.*

After students successfully modeled the data, they copied and pasted their resulting graphs into a Word document or simply emailed the TI-Nspire files to us. A typical report from the students included a description of how they scaled their axes or how they figured out the graph.

3 Equations in Algebra

When we used the animal poppers with a middle-school honors algebra class, we had the students each launch the balls outside, as shown in Figure 6.

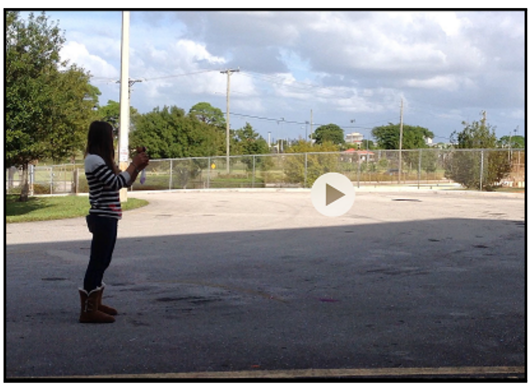

Fig. 6: *Student launch.*

Two of the iPads that were used by the middle school had the Video Physics app installed, so we grouped the students and had them video-tape one another. They were then able to track their own animal popper balls. When we returned to the classroom, the best photos were chosen and emailed to students for selection to be downloaded into the TI-Nspire software on their iPads.

We instructed them how to insert the sliders by using the quadratic function $f(x) = a(x-h)^2 + k$, with (h,k) being the vertex. They then used the TI-Nspire sliders for a, h, and k to manipulate the values of the variables until the equation fit. In this way, students could see what happened when they moved the sliders, making the values for the variables positive and negative. The first question was, "Should the a value be negative or positive? How do you know?" They could also see how the magnitude of the sliders affected their model. Figure 7 shows a function modeling a tracked ball from the video of a middle school student.

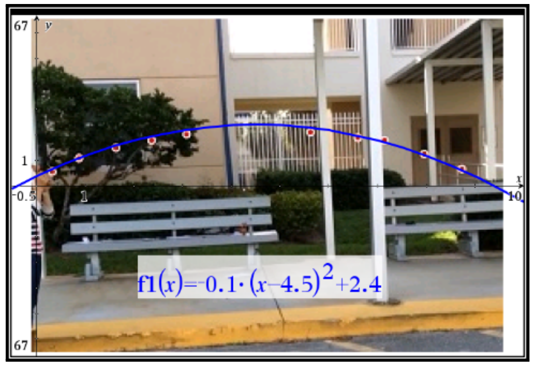

Fig. 7: *Function modeling a tracked ball.*

4 Modeling with the Standard Quadratic Equation

Taking the parametric equations $y = -0.5gt^2 + v \cdot sin(\theta) \cdot t + h$ and $x = v \cdot cos(\theta) \cdot t$, solving for t in terms of x and substituting for the t in terms of y, yields $y = \frac{-0.5g}{(v \cdot cos(\theta))^2} x^2 + tan(\theta) \cdot x + h$. Note that in terms of the standard equation for the quadratic, $y = ax^2 + bx + c$, $\frac{-0.5g}{(v \cdot cos(\theta))^2}$ is the leading coefficient, a; $tan(\theta)$ is the x coefficient (i.e., b); and h is the constant term, c. In this way, students could use real-world data to determine the values for the typical a, b, and c in the standard quadratic equation. Figure 8 shows how this equation models the animal popper ball flight.

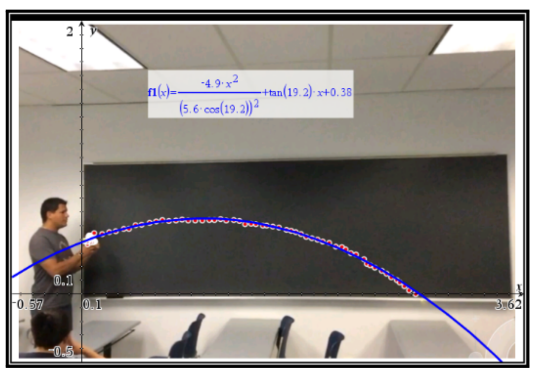

Fig. 8: *Equation modeling flight of animal popper ball.*

5 Conclusion

Comments from students and information from completed assignments have demonstrated the interest in and usefulness of the assignment in engaging students and in aiding their conceptual understanding. The calculus students made comments like, "This was a great assignment. Seeing how the graphs and equations work in the real world was helpful." The middle school students commented that they really enjoyed the freedom of working and learning outside of the traditional classroom setting. Because of the varying levels of mathematics that can be addressed with this activity, we are able to use it with algebra students as well as calculus students. Activities that encourage active participation by allowing students to create their own material provide a unique ownership of their learning. In addition, the usual "why do we need to learn this?" question is answered, encouraging enthusiasm for the learning of mathematics.

Marsha Nicol Guntharp, `marsha_guntharp@pba.edu`, is an associate professor of mathematics at Palm Beach Atlantic University in West Palm Beach, Florida. She is Vice-President-College of the Florida Council of Teachers of Mathematics (FCTM), newsletter editor for Palm Beach County Council of Teachers of Mathematics (PBCCTM), and a regional instructor for Teachers Teaching with Technology (T^3). She lived in Ohio for most of her life and was co-editor of Ohio Journal of School Mathematics for seven years.

Fred Browning, `fred_browning@pba.edu`, is an associate professor of physics at Palm Beach Atlantic University in West Palm Beach, Florida. Fred previously worked in the medical physics department at the University of Wisconsin where he developed a continuous moving table MRI protocol for General Electric MRI machines.

Gloria Royle, `gloria.royle@palmbeachschools.org`, taught middle school and high school mathematics at Conniston Middle School in West Palm Beach, Florida, for the past 15 years. She was the school's mathematics instructional leader as well as their mathematics coach. She is currently a mathematics teacher at Jupiter Middle School of Technology in Jupiter, Florida.

Black Swamp MTC - Bowling Green

The members of the Black Swamp MTC invite math enthusiasts from across the grades to participate in fun and joyful mathematics.

Contact: Marcia Miller, marciamath8@gmail.com

Math Teachers' Circle π