Using Soma Cube Puzzles to Master Common Core State Standards

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Abstract: *The Soma cube is a three-dimensional puzzle constructed using seven different three-dimensional puzzle pieces. Despite the heavy mathematics foundation of the Soma cube, the puzzle has been utilized in a wide variety of spaces such as entertainment, psychological research studies, and both elementary and secondary classrooms. This article outlines specific classroom applications in elementary, middle, and high school. Research supporting the classroom activities and specific Common Core State Standards addressed by those applications are included.*

Keywords. *Geometry, history of mathematics, problem-solving, spatial reasoning*

1 Introduction

The Soma cube is a three-dimensional puzzle invented by Piet Hein in 1933. The $3 \times 3 \times 3$ cube is constructed using seven different three-dimensional puzzle pieces, each constructed from either 3 or 4 unit cubes. We illustrate a soma cube and its seven composite shapes in Figure 1.

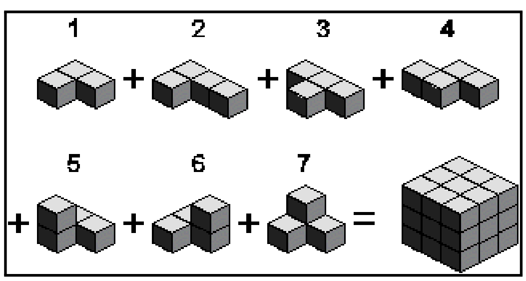

Fig. 1: *A* $3 \times 3 \times 3$ *Soma cube and its seven composite pieces (McFarren, 2016).*

The Soma cube was invented during a lecture on quantum mechanics. Despite the heavy mathematics foundation of the Soma cube, the puzzle has been utilized in a wide variety of spaces such as entertainment, psychological research studies, and both elementary and secondary classrooms.

2 History

Piet Hein was a Danish scientist, mathematician, inventor, author, and poet in the 20th century. He was best known under the pseudonym "Kumbel," meaning "tombstone," after writing a series of short poems titled "Grooks," which were published in a daily Danish newspaper shortly after the Nazi occupation (Hein & Arup, 1969). His anti-Nazi poems used clever symbolism to evade the newspaper censors at the time. While he was a popular poet during the Danish resistance movement, Piet Hein was also an intelligent mathematician. In 1933, he created the Soma cube puzzle during a quantum mechanics lecture. As the lecturer spoke, Piet Hein sketched six figures comprised of four unit cubes and one comprised of three. He believed these could be assembled to form a larger cube. After the lecture, Piet Hein tested his theory by gluing dice together to form the shapes he had sketched (Gardner, 2001). When put together, these shapes created the $3 \times 3 \times 3$ Soma cube puzzle seen in Figure 2.

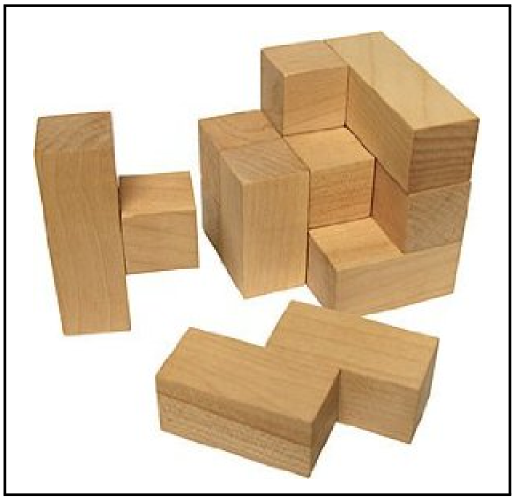

Fig. 2: *A partially assembled Soma cube (McGuire, 2011).*

As the puzzle gained popularity, it began appearing in many different spaces. The U.S. toy and game manufacturer Parker Brothers began mass producing the puzzle in the 1967. Their packaging claimed that the cube had over one-million possible solutions. This included rotations and reflections of every piece. With these transformations accounted for, the puzzle actually has 240 possible solutions (Spector, 1982).

In 1969, Carnegie Mellon University graduate student Edward Deci used the Soma cube in psychological experiments, which led to the founding of Self-Determination Theory. This dissertation work focused on intrinsic and extrinsic motivation. In his research, subjects were asked to solve the Soma cube puzzle under different conditions. First, subjects were simply asked to solve the puzzle. The time spent solving was used to measure intrinsic motivation. Next, subjects were offered a monetary prize for completion of the puzzle. Finally, the extrinsic reward was removed and intrinsic motivation was measured by time again (Deci & Ryan, 1975). Their study found that intrinsic motivation decreases with the introduction of an extrinsic reward, but all subjects were intrinsically motivated to solve the puzzle initially.

The findings of Deci and Ryan's study are important for the classroom, where intrinsic motivation is more easily maintained than extrinsic motivation. The combination of the fun, game-like use of the cube by Parker Brothers and the mathematical foundation created by Piet Hein makes the Soma cube the perfect classroom tool.

3 Classroom Applications

Since Soma cubes address a multitude of math concepts such as spatial reasoning and dimensional analysis, they can be used to address Common Core State Standards from the early elementary grades through high school. Students can learn mathematics through Soma cubes by engaging in measurement exercises and building geometric understanding.

3.1 Elementary school

Many early elementary classrooms work to classify objects in accordance with CCSS K.MD.B.3. This standard states students should be able to classify objects into given categories and then count the number of objects in each category. The seven Soma cube pieces are great for sorting because it is recommended that the number of objects for sorting does not exceed ten. The teacher can conduct a closed sort in which the categories are defined or allow students to sort the pieces in their own way. Possibilities of categories include by color, pieces made of three unit cubes versus four unit cubes, and pieces than can lay flat. Students will most likely come up with additional categories. The same activity can be expanded upon in second and third grade. Students can sort the pieces then represent their sort through a bar graph in accordance with CCSS 2.MD.D.10 and 3.MD.B.3. These standards state that students be able to represent data with a picture graph or bar graph (CCSSI). This activity helps elementary students experience mathematics in a fun, interactive, and hands-on way.

Elementary students can also practice their measuring skills using completed Soma cubes and Soma cube pieces. Second graders are expected to measure and estimate lengths in standard units using tools like rulers, yardsticks, meter sticks, and measuring tape in accordance with CCSS 2.MD.A.1 (CCSS). Students can select the most appropriate measurement tool and determine the length of each piece and the dimensions of the entire puzzle.

3.2 Middle school

Moving further into elementary school and middle school, Soma cubes can be best utilized when addressing CCSS 6.G.A.4 and introducing CCSS 5.MD.C.3 regarding surface area and volume of three-dimensional shapes. When focusing on surface area, students are expected to represent three-dimensional figures using nets made up of rectangles. These nets should then be used to find the surface area of these figures (CCSSI). Middle schoolers can begin by exploring nets by focusing on single cubes. Many students have difficulty visualizing how nets fold into three-dimensional figures, so it is important to build spatial reasoning prior to working with Soma cube nets. This can be done through hands-on manipulatives or technology such as the National Council of Teachers of Mathematics Illuminations page in which students navigate through a series of nets and determine whether or not it can be folded into a cube (Cube Nets). Teachers may also print this web page and allow students to cut the nets and attempt to fold them.

Moving forward, students can be provided with the nets of Soma cube pieces and find the surface area of each piece. They can then fold them together to create their own puzzles. This helps create fluency between three-dimensional objects and their two-dimensional representations, a topic that many middle school students find challenging (Cooper & Sweller, 1989).

After finding the surface area of each piece, students can move to volume. According to the CCSSM, students should recognize volume as an attribute of solid figures and understand concepts of volume measurement (CCSSI). They should also be able to measure volumes by counting unit cubes. After recording the volume and surface area of each piece, students can share their findings in a whole class setting. This engaging discussion should focus on the often difficult concept that two figures may have the same volume, but different surface areas. Standards 6.G.A.4 and 5.MD.C.3 are just two of seven middle school standards specifically addressed by the geometry of Soma cube puzzles (Leonard & Tracy, 1993).

To address statistical standards, middle school students can use Soma cubes to conduct a simple experiment and record and analyze the data collected. Standard 6.SP.B.5 states that sixth graders should be able to summarize and describe distributions in numerical data sets (CCSSI). To do this, students can use a stopwatch to time how long it takes classmates to solve a Soma puzzle.

Name	Relation to Designer	Time Taken to Solve Cube
Mariana Martinez	Fellow student	27.9 seconds
Madison Kemeny	Fellow student	61.9 seconds
Haley Hubner	Fellow student	137.7 seconds
Mr. Robinson	Teacher	25.9 seconds
Scott Talkington	Fellow student	74.5 seconds

Fig. 3: *A sample data set for time taken to solve Soma cube puzzle (McGuire, 2016).*

Students can use the above data to calculate quantitative measures of center like the median or mean and variability measures like range. The mean for this data set would be 65.6 seconds and the median would be 61.9 seconds. The range is 111.8 seconds. The class can then interpret this data to determine the most difficult puzzles or the fastest solvers.

3.3 High school and beyond

At the high school level, activities to develop spatial reasoning can help students understand the maximum of 240 solutions due to the rotation and revolution of pieces. To introduce this concept, teachers can challenge small groups of students to sketch as many unique Soma pieces as possible using unit cubes as seen in Figure 4. Unit cube manipulatives should be available to aid visual-

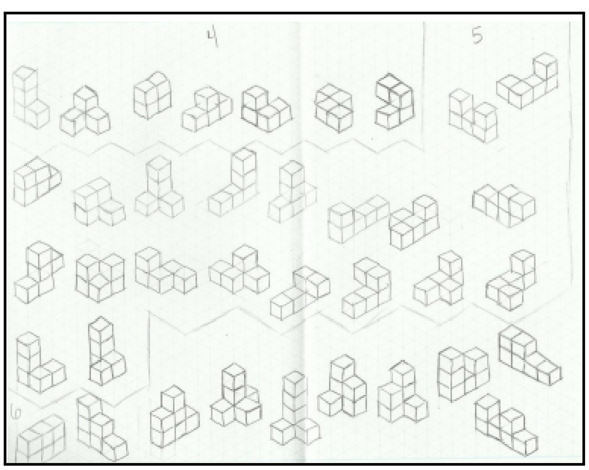

Fig. 4: *Isometric sketches of possible Soma cube pieces (McGuire, 2011).*

ization. The class can then share their sketches, discuss, and delete pieces that are rotations or reflections of one another, which requires the skills addressed in CCSS.HSG-GMD.B.4, identifying three-dimensional generated by rotations of two dimensional objects (CCSSI). Students can also eliminate pieces that do not meet constrictions of the $3 \times 3 \times 3$ puzzle.

Project Lead the Way, a high school pre-engineering program, uses Soma cubes to introduce complicated technologies such as automatic Computer Aided Design (AutoCAD). AutoCAD is used to virtually design two- and three-dimensional objects and products. Students can use self-created Soma cubes to produce technical drawings and exploded views on the AutoCAD software as seen in Figure 5. The drawing shows the front, top, and right view of the five Soma cube pieces used to generate this puzzle. The exploded view on the right displays how the cube should be assembled (Rawding, 2015).

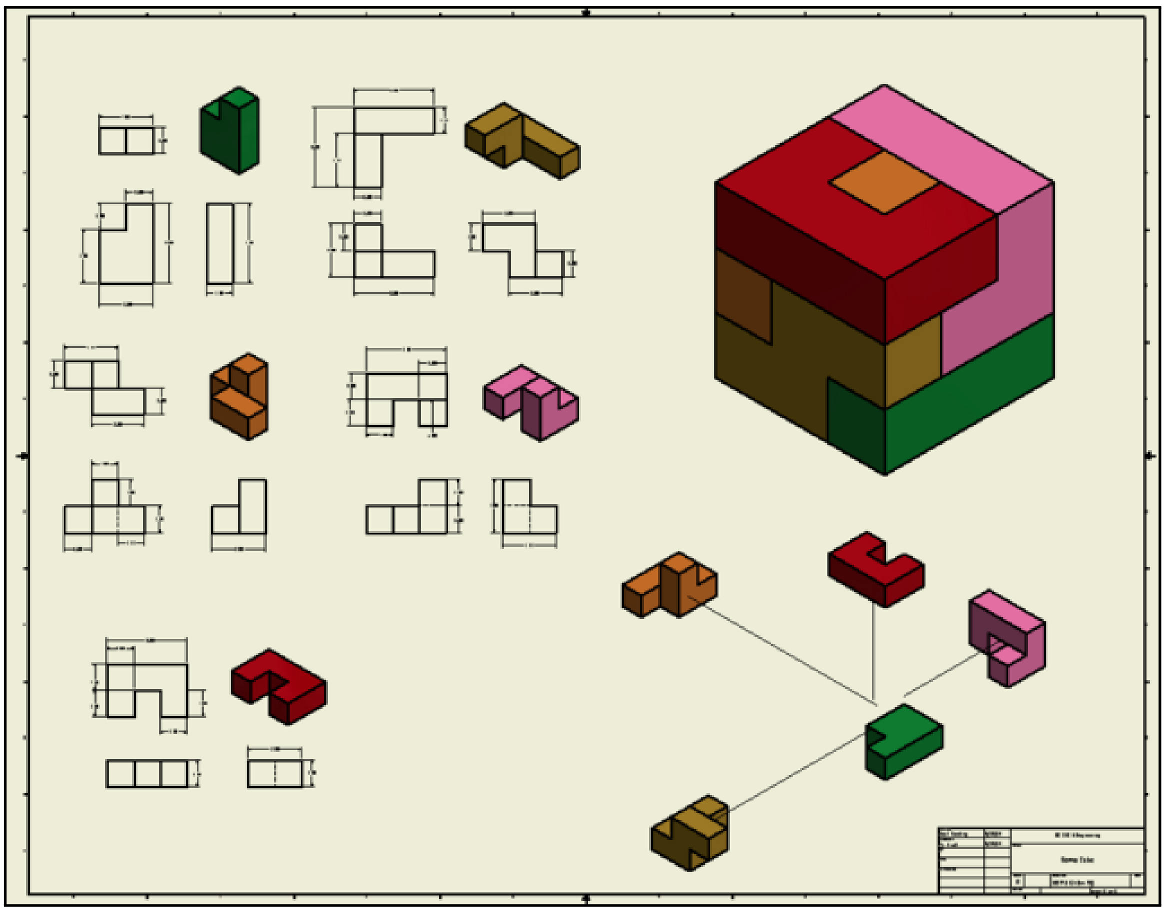

Fig. 5: *AutoCAD rendering of Soma cubes (Rawding, 2015).*

This task further develops the fluency between three-dimensional objects and their two-dimensional representations while tackling CCSS HSG-GMD.B.4: Identify three-dimensional objects generated by rotations of two-dimensional objects (CCSSI). Spatial reasoning is increased as students must consider images from different perspectives and angles (Izard, 1990). At the postsecondary level, the smooth edges, flat planes, and ninety-degree angles of the Soma cube make it an excellent introductory product for 3D printing software.

4 Conclusion

From counting and cardinality to spatial reasoning and geometric understanding, the Soma cube has a place in every classroom. Research suggests that the puzzle is naturally engaging and intrinsically rewarding - attributes that make them valuable in today's mathematics classrooms. Whether it is kindergarteners in an elementary school or college students in an engineering laboratory, every student can benefit from the interactive properties of the Soma cube.

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