Revisiting the Meaning of the Denominator

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Abstract: *Being able to fluently work with fractions as quantities involves reasoning about the denominator as an indicator of size. This view is emphasized in the Common Core State Standards associated with fractions. In this article, the authors examine specific classroom problems that can build students' ability to reason about the denominator as a reference to size.*
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1 Introduction

Developing the ability to reason about the size of different fractional units plays an important role in helping students develop fraction number sense and fraction operation sense. Helping students realize that for any fraction, viewing the denominator as an indicator of the size of a unit of measure is an important aspect of being able to work with fractions as quantities or tangible amounts. The text box on the next page highlights examples of this viewpoint expressed in the Common Core State Standards for Mathematics (CCSSM) (CCSSI, 2010) for grade three. For example, Standard 3.NF.A.1 refers to the denominator "b" as an indicator of size. It says that students need to "understand a fraction a/b as the quantity formed by a parts of size $1/b$." Using fifths as an example, any fraction with a denominator of five is a quantity composed with some number of 1/5-size parts of a whole. In other words, a denominator of five is a reference to a certain size portion of the whole. In this case the size of the portion or unit is 1/5 of the whole. Developing an understanding of the denominator as an indicator of the size of a fractional unit requires more than the use of simplistic counting tasks such as the one in Figure 1 (Kieren, 1988).

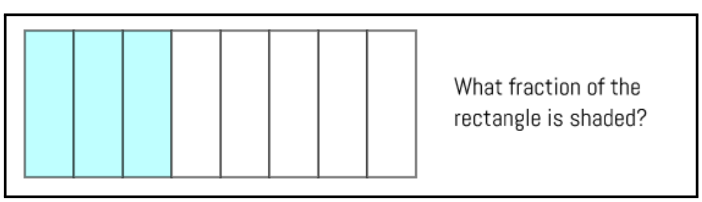


Fig. 1: *What fraction is this?*

A task of this type leads students to count the number of parts in the whole and count the number of parts that are shaded. This task does not focus on reasoning about the size of the fractional parts that make the whole. This article will examine ways to support students' ability to reason about the size of fractional units and view the denominator as an indicator of the size of a fractional unit in addition to the number of equal parts in a whole.

Develop understanding of fractions as numbers - Common Core Standards (Grade 3)

3.NF.A.1
Understand a fraction $1/b$ as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size $1/b$.

3.NF.A.2
Understand a fraction as a number on the number line; represent fractions on a number line diagram.

3.NF.A.2.A
Represent a fraction $1/b$ on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size $1/b$ and that the endpoint of the part based at 0 locates the number $1/b$ on the number line.

3.NF.A.2.B Represent a fraction a/b on a number line diagram by marking off a lengths $1/b$ from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line.

3.NF.A.3
Explain equivalence of fractions in special cases, and compare fractions by reasoning about their size.

3.NF.A.3.A
Understand two fractions as equivalent (equal) if they are the same size, or the same point on a number line.

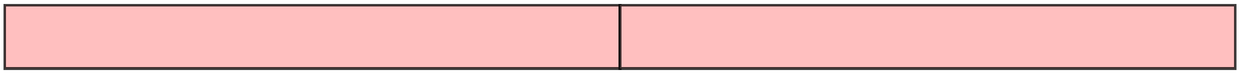
3.NF.A.3.B
Recognize and generate simple equivalent fractions, e.g., $1/2 = 2/4$, $4/6 = 2/3$. Explain why the fractions are equivalent, e.g., by using a visual fraction model.

3.NF.A.3.C
Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. Examples: Express 3 in the form $3 = 3/1$; recognize that $6/1 = 6$; locate $4/4$ and 1 at the same point of a number line diagram.

3.NF.A.3.D
Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols $>$, $=$, or $<$, and justify the conclusions, e.g., by using a visual fraction model.

2 Developing Images for Fractions by Folding Paper Strips

The CCSSM related to fractions mention the importance of using models. Models help students visualize fractions and support an understanding of their size and location (Petit, Laird & Marsden, 2010). An important model emphasized in the third grade CCSSM standards is the number line (Barnett-Clarke, Fisher, Marks & Ross, 2010). The activity in Figure 2 asks students to pretend a paper strip is a piece of licorice and figure out how to fold strips to show halves, fourths, and eighths. As they do this they begin to develop visual images about the size of halves, fourths and eighths in relation to each other and in relation to one whole. These visual images can support understanding of fractions as numbers located on a number line.



1. Above is a long piece of licorice folded into two halves so that you and your friend can share. Start with a strip of paper folded like the licorice.
   a. How can you fold the half-strip so that four people can share the licorice equally?
   b. What would you call the part of the whole piece of licorice that each of the four people will receive?
2. You have another piece of licorice folded in half.
   a. How can you fold it so that eight people can share it equally?
   b. What size would the part of the licorice each person will get be called?
3. When you compare the licorice folded into fourths with the licorice folded into eighths, what do you notice about the size of the pieces?
4. Which fractional part is larger—fourths or eighths? Why?
5. Why are eighths smaller than halves?
6. Why are eighths smaller than fourths?
7. Which will it take more of to make a whole piece of licorice—halves, fourths or eighths?
8. If you eat 3/4 of a strip of licorice, and your friend eats 3/8 of a strip of licorice, who will have eaten more? How do you know?
9. If you eat 3/4 of a strip of licorice, and your friend eats 5/8 of a strip of licorice, who will have eaten more? How do you know?

Fig. 2: *Constructing related fraction strips.*

The questions posed in the activity can also help students understand the denominator as an indicator of size. For example, Question 4 focuses students' attention on the size of fractional parts or units such as fourths and eighths, which unit is larger, and why. Similar problems can be developed to explore relationships between other related fractions such as thirds, sixths and ninths, or fifths and tenths.

3 Developing Images for Fractions by Marking Number Lines

Asking students to partition and mark number lines to show fractional parts can support visualization of the size of various fractional units as indicated by the denominator. Figure 3 illustrates an activity where students partition number lines labeled with zero and one. Posing questions similar to those used with the licorice problem supports reasoning about how the denominator references a size. For instance,

1. Mark the distance between 0 and 1 into sixths. What does the six in the fraction 5/6 tell you?
2. What does the five tell you?
3. How many sixths does it take to make 1/6? 2/6? 3/6? 4/6? 5/6? 6/6?
4. How far is 5/6 from zero?
5. How far is 5/6 from one?
6. How far is 5/6 from 1/6?
7. How far is 5/6 from 1/2?
8. Which fraction is closer to zero: 2/6 or 3/6?
9. Which fraction is closer to one: 2/6 or 3/6?
10. What fraction is 1/6 less than 5/6?
11. Which fraction is closer to zero: 1/6 or 1/8?
12. Which fraction is closer to one: 5/6 or 7/8?

Questions such as these deepen student understanding of fractions. Asking students to count as they label a number line partitioned into sixths (0/6, 1/6, 2/6, . . .) helps focus students' attention on the idea that the denominator tells you that the whole is partitioned into six equal-size parts (CCSSM 3.NF.A.2.A) and that each of the parts is a sixth in size (CCSSM 3.NF.A.2.B). By asking questions such as, *What does the 6 in the fraction tell you?* or *How many sixths does it take to make 5/6?*, students are encouraged to see the denominator as the number of parts the whole is partitioned into and that the size of each of the five parts in 5/6 is one sixth.

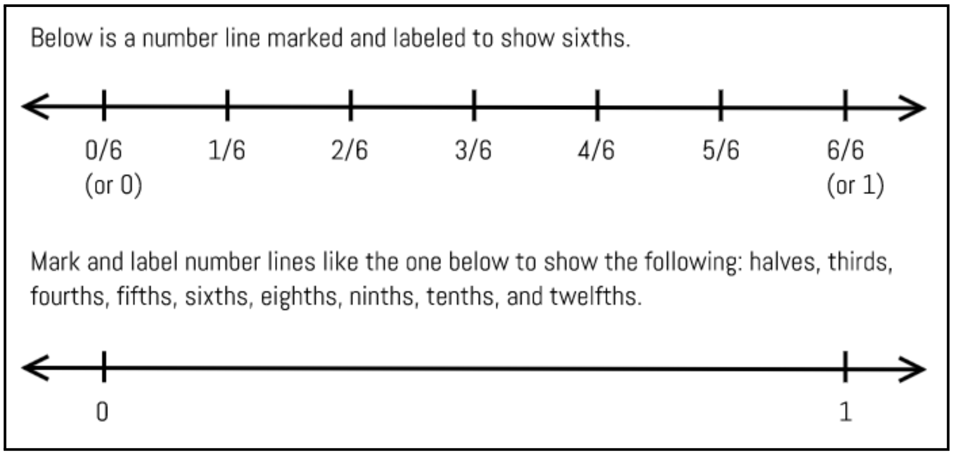


Fig. 3: *Marking fractions on number lines.*

Marking and labeling a number line can support reasoning about how far one fraction is from another fraction (CCSSM 3.NF.A.2.B). For example, asking how far the fraction 5/6 is from zero can help students reason with lengths or measures of size 1/6. There are five 1/6 lengths or parts between zero and 5/6. This reasoning can be extended to include reasoning about a measure of a certain size as a length where the length doesn't necessarily begin at zero on the number line. Asking how far 5/6 is from 1/6 can help students think about the number of 1/6-length portions needed to get from one location to another on a number line. The visual model of a number line that is partitioned and marked helps show that there are four 1/6-length portions between 1/6 and 5/6.

4 Developing Images for Fractions by Placing Them on a Number Line

Asking students to place fractions on a number line and explain their reasoning can provide another opportunity to engage students in reasoning about the denominator as an indicator of size. One approach can be to draw a large number line with markings at 0, 1/2, and 1 on the board. Students can be given sticky notes with various fractions written on them, and asked to go to the board to determine approximately where to place the fraction on the number line. For example, asking where the fraction 1/4 would be located on the number line and how you know this can lead to a discussion about how fourths are smaller than halves. Students can make connections to ideas they developed when folding fraction strips to support their argument. They might explain that if you fold a half strip in half that you will get four parts. Therefore, 1/4 would be located exactly halfway between 0 and 1/2.

Strategically choosing the order fractions are presented to students can lead to opportunities to support equivalence as outlined in CCSSM 3.NF.A.3. For example, asking student to place 3/8 and then 5/8 can help students develop explanations that lead them to compare two fractions with the same denominator. Such reasoning would note that an eighth is a size and when you have three eighth-size parts or lengths that it is less then when you have five eighth-size parts or lengths.

When comparing two fractions with the same numerator a justification would involve recognizing that each quantity is composed of a fractional portion of some size. For example, when placing 3/8 it could be argued that 3/8 is 1/8 less than 4/8 or 1/2. This would lead to placing the 3/8 to the left of 1/2. If 3/7 was posed next, then reasoning about the size of sevenths in comparison to eighths could emerge. Students could revisit their work with fraction strips and think about the end result of a fraction strip folded into eighths and another fraction strip folded into sevenths. Since there are less parts in the whole, sevenths would be larger portions of the whole than eighths. Therefore, if you had three 1/8 and three 1/7, the three 1/7 would be larger. This would indicate that 3/7 would be placed to the right of 3/8 because 3/7 is greater than 3/8.

Another interesting pair to pose would be 3/5 and 6/10. When fifths are partitioned into two parts, the result is tenths because tenths are half the size of fifths. Therefore, two tenth-size portions would be needed to make one fifth-size portion. Each one-fifth piece is equivalent to two one-tenth pieces. And in turn, it would take six one-tenth pieces of the whole to make the same amount as three one-fifth pieces of the whole. Being able to reason about the size of related denominators supports meaningful reasoning about equivalence and comparison as suggested in the Common Core State Standards third grade standards related to fractions. As students become proficient with locating fractions on a number line, they can develop a sense for how big various fractions are based on reasoning about size as suggested by the denominator.

5 Conclusion

Being able to reason about a fraction a/b as the quantities of a parts of size $1/b$ plays an important role in students' developing ability to reason with fractions as tangible quantities. Asking students to discuss strategies and observations about the size of fractions as they fold fraction strips, mark and label number lines, and place fractions on number lines helps them develop a mental map of how big fractions are and where they are located in relation to fractions such as 0, 1/2, or 1 (Petit, Marsden & Laird, 2010). Conversations that focus on the size of fractional units as indicated by the denominator of a fraction, and conversations about how fractional units are related to each other (eg., fourths vs. fifths) supports students' ability to visualize, model and create mathematical arguments about fractions as indicated in the CCSSM (Barnett-Clarke, Fisher, Marks & Ross, 2010). While this article has focused on examples from the third grade Common Core State Standards, the activities and suggested conversational pathways can help students in all grades.

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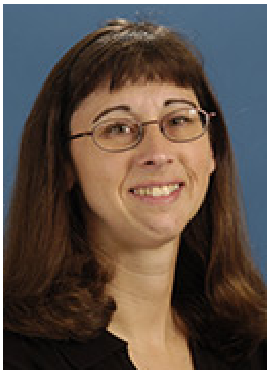

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