Student Math Circles: Building Collaborative Places of Mathematical Inquiry Across Grades

Chris Bolognese, Columbus Academy
Sonam Shahani, Columbus Academy

Abstract: *In this article, the authors provide an alternative to traditional math clubs through the formation and facilitation of a student math circle (SMC). Student math circles are a community of students across different grades who engage in open-ended low floor, high ceiling tasks. The authors provide history and motivation for SMCs as well as a few sample problems that have been used. Additionally, benefits of an SMC model are provided.*

Keywords: *problem solving, math circles, rich tasks*

1 More than Math Clubs

Math clubs are a common extracurricular activity for many students, whether it be participating in MathCounts, Math Olympiad, or another school-run organization. While these opportunities create positive environments for students to learn new mathematics while developing competition-ready students, too often they perpetuate a mindset that mathematics is about speed and accuracy. Moreover, many competitions are individualized without a deliberate focus on collaboration or communication. Lastly, such competitions often prescribe prerequisite math knowledge which limits student participation across grade levels.

In order to supplement the middle school math club environment with one that allowed for deeper exploration, play, collaboration, and discussion across grade levels, we began a Student Math Circle (SMC). Providing such a venue allows students to arrive at meaningful mathematical understandings with little to no prior knowledge. In this article, we contrast math clubs and SMCs while sharing reflections and motivations for establishing your own SMC. It is our hope that this article provokes the reader to consider how SMCs could further enhance the mathematical experiences of their own students.

2 A Brief History of Student Math Circles

So just how old are SMCs? Student math circles first began in Russia and Bulgaria in the 1930s as a reaction to traditional math competitions. These early math circles focused on problem solving and collaboration, quite distinct from more conventional mathematics competitions of the time (Saul, 2006). Also, SMCs became an environment where university mathematics academics and PreK-12th grade students could regularly interact with nonstandard mathematical topics often separate from standard school curricula. While U.S. math competitions date back to the 1950s with the advent of the American Mathematics Competition, SMCs in the United States have formally existed for less than two decades! On a related note, circles for teachers, namely communities of mathematicians and mathematics teachers, have only been in existence since 2006 but now number over one hundred (American Institute of Mathematics, 2017). Again, these communities of students, teachers, and mathematicians were born organically through a desire to engage in meaningful, curious mathematics.

3 Traditional vs. Math Circle Problems

How exactly are the types of problems and activities different between a traditional math club and a math circle? Consider a traditional math problem: *If Lex slices a pie into ten congruent pieces and then eats three pieces, what percent of the pie is left?* In order to solve this problem, students must have prior knowledge of percentages, fractions, and even a linguistic understanding of the word congruent. Such a problem would be rendered impossible for a student who does not have this background. Similar one- right-answer problems rob students of the opportunity to explore and arrive at their own understandings. After solving this problem, would students come away with any real, deep, fun, enjoyable, or enlightening understanding of math? Will they enthusiastically challenge their friends or parents to solve the problem? Furthermore, such a problem is closed-ended in approach and limited in connections or generalizability.

By contrast, consider the following problem explored recently at our SMC:

Insert + or - signs in between digits to make a total of exactly 100: 1 2 3 4 5 6 7 8 9 = 100

Here the problem has less of a content barrier, only requiring the most basic knowledge of addition and subtraction. In this sense, the problem is a "low threshold-high ceiling" task (Boaler, 2016). Further, unlike the percentages problem, this task fosters strategic thinking as students consider advantageous or limiting cases. Specifically, as students generate totals close to 100, such as 99, they have to make decisions as to how their calculation can be amended to reach 100 (Figure 1). Moreover, the task incites collaboration and curiosity as students record different discoveries in a public shared space (Figure 2). As students discover more representations for 100, their peers are eager to analyze and build upon these new representations, often asking "is there another way we could find 100?" We found that starting with a problem in which the rules were limited and easily digestible were useful, especially for students of younger grades. Students as early as fourth grade and as old as eighth grade regularly participate in our circle. Such problems naturally lend themselves to the nature of problem posing: formulating new problems from previous ones (Silver, 1994; Brown & Walter, 2005). Here, students make their own rules, such as "What if we only used addition?" or "What if we could use other operations like multiplication and division? Is this harder or easier with more operations?"

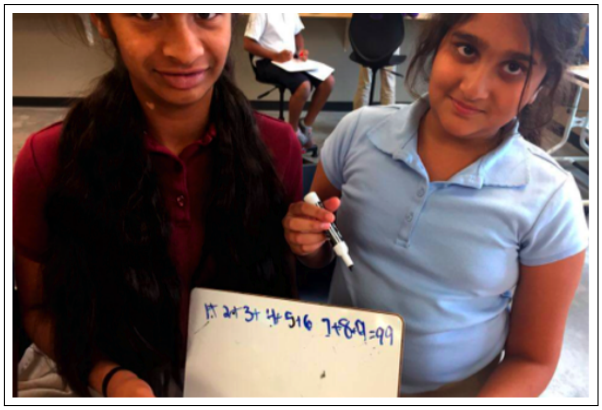

Fig. 1: *Students get a sum of 99.*

Fig. 2: *Recording solutions.*

4 Another Sample Problem

Refer to Figure 3 and consider the following problem: Send the yellow frogs to the right side and red toads to the left side. The frogs can only jump over one other animal of the opposite type if a vacant lily pad is available.

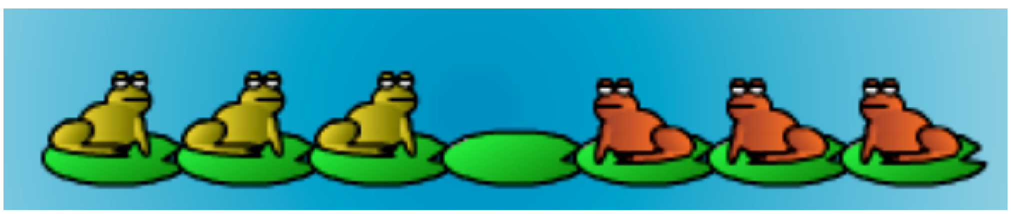

Fig. 3: *Frogs and Toads.*

We began this task by "acting it out," with six students standing on makeshift cardboard lily pads holding either a yellow or red piece of construction paper to denote their role as a frog or toad. The full group directed the frogs and toads where to jump in order to solve the puzzle, counting the number of jumps as they went. In facilitating these problems, we have found a concrete model to always serve as a useful starting point. Students were then directed to a digital model of the problem to explore further (Britton, 2016). While most students began analyzing the problem by considering three frogs, four frogs, five frogs, and so on, some began to see a recursive numerical pattern, but could not express this pattern algebraically.

While students evoked useful problem solving heuristics such as looking at smaller cases (Polya, 1957), only one student recognized the utility in looking at the simplest case of one frog and one toad. The notion of the nth case is not intuitive for some students while younger students are unaware of the use of variables in mathematics. Students not only began to teach each other about unfamiliar terms, but older students served as role models as they processed the patterns aloud. The nature of the "low-threshold, high-ceiling" problems allows even younger students to arrive at sophisticated understandings on their own, equipping them with the ability to mentor older students.

5 Benefits

Game-based learning provides an innovative and effective way to teach. Engaging students in iterative, open-ended activities not only promotes higher-level thinking but also fosters an inquisitive spirit that is not present in traditional math instruction. Students are excited to go home and continue exploring the "What If" questions they generate during Math Circle, thus stimulating a positive attitude about mathematics and a conscious desire to explore difficult concepts.

Multi-age classrooms have long been the structure of learning in Montessori schools, summer camps, and day care agencies. In traditional schools, groups and clubs such as Math Circles foster the benefits of cross-divisional collaboration. Pratt (1983) concluded that both the younger students and the tutors [in multi-age settings] benefited from the experiences. French, Waas, Stright, and Baker (1986) found that in the multi-age/multi-year structures more students had the opportunity to be leaders, including older students who otherwise might not have assumed leadership positions. We witnessed these interactions through various problems that students analyzed in our SMC. In Figure 4, a fifth grade student leads a discussion about an opening puzzle to her older peers.

Fig. 4: *Fifth grader teaching 6th, 7th, and 8th graders.*

6 Summary

If your school currently has a math club for students, consider how this math club environment could be reconceptualized to cultivate a supportive, multi-age, inquiry-based community. Student math circles can provide an alternative to a more conventional math club whose primary foci are tricks and accuracy to solve closed-ended problems. While this traditional format of contest mathematics still promotes engagement and newfound content knowledge, SMCs genuinely allow students to take on the role of a mathematician. Through collaboration, curiosity, communication, and conjecturing, students of all ages can find real joy and success in doing mathematics.

References

American Institute of Mathematics (2017). *Math Teachers? Circle Network*. http://www.mathteacherscircle.org/. Retrieved 1 March 2017.

Boaler, J. (2016). Tasks by grade: Low floor high ceiling. *YouCubed* [website]. https://www.youcubed.org/grade/low-floor-high-ceiling/. Retrieved 2 March 2017.

Britton, J. (2016). *The Frog Puzzle*. Retrieved 2 March 2017 from http://britton.disted.camosun.bc.ca/frog_puzzle.htm/.

Brown, S. I., & Walter, M. I. (2005). *The art of problem posing*. Psychology Press.

French, D. C., Waas, G. A., Straight, A. L., & Baker, J. A. (1986). Leadership asymmetries in mixed-age children's groups. *Child Development*, *57*(**5**), 1277–1283.

Isaacs, S. (2015). The Difference Between Gamification and Game-based Learning. *ASCD in Service*, January, 15.

Polya, G. (1957). *How to Solve it: A New Aspects of Mathematical Methods*. Prentice University Press.

Pratt, D. (1983). Age segregation in schools. Paper presented at *Annual Meeting of the American Educational Research Association*. Montreal, Quebec, Canada.

Saul, Mark (2006). What is a Math Circle? *National Association of Math Circles Wiki*. Mathematical Sciences Research Institute. Retrieved 1 March 2017.

Silver, E. A. (1994). On mathematical problem posing. *For the learning of mathematics, 14*(**1**), 19–28.

Chris Bolognese, bolognesechris@gmail.com, is the PK - 12 mathematics department chair at Columbus Academy. He enjoys problem solving, math circles, and dynamic geometry.

Sonam Shahani, lshahanis@columbusacademy.org, teaches sixth grade language arts at Columbus Academy and coaches the middle school math club.