The calculator provided the "wrong" answer. Now what?

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Abstract: *Graphing technology in the mathematics classroom can, at times, lead to frustration. Students expect the technology to provide "correct" answers, but such is not always the case. Teachers need to help students interpret the output generated by technology. Moreover, teachers need to consider idiosyncrasies of various technology-oriented tools to support their students. Solving homework problems in multiple ways with multiple tools can help teachers anticipate student questions and trouble spots. The authors shed light on such issues through the careful analysis of two seemingly straightforward classroom tasks.*

Keywords: *Technology, discourse, problem solving*

1 Introduction

Technology has the *potential* to transform teaching and learning, particularly in mathematics classrooms. The *Standards for Mathematical Practice* recognize that today's students need to "make sense of problems and persevere in solving them." Technology has the potential to play an important role in the Algebra classroom. As the authors of the Common Core note,

> Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need ... Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" (CCSSM.MP1).

My students love to use technology. They use Instagram, Snapchat, and Facebook every day. I teach at a trade school where much of the students' work is "hands on," and students are encouraged to use whatever resources they have available to solve problems. We use *MyMathLab*, *GeoGebra*, *Desmos*, and graphing calculators to assist our work and our learning. In such an environment, students soon discover that technology can be a useful tool but one that presents its own set of challenges. For instance, when a calculator provides an answer that's not in the correct form, or has been rounded when the instructor has asked for the exact answer, what is the student to do? Technology, although powerful, has its limits, and these limitations need to be explored with students. In this paper, I share several examples that I've encountered in my classroom that highlight the need for students to be savvy users of technology. Teachers who know technology's limitations can harness them as opportunities for discussion and debate.

2 Student Challenges with Technology-Generated Answers

Consider the following tasks from *MyMathLab*, an online learning management software.

MyMathLab tasks

(a) Find an equation of a line that passes through the points (4,5) and (5,6).

(b) Find an equation of a line that passes through the point (-4,4) and has a slope of 4.

(c) Find a point that lies on both lines from steps a) and b). Express your answer as an ordered pair. Simplify your answer.

(*Editor's note: These tasks seem so innocuous, so standard. What could possibly go unexpectedly when using technology to solve them? Continue reading to see!*)

Students generate lines for parts (a) and (b) using a variety of strategies including by-hand manipulation and line drawing tools. Figures 1-3 illustrate plots in *Desmos*, the *TI-83 Plus*, and *GeoGebra* respectively, using the default settings of each technology.

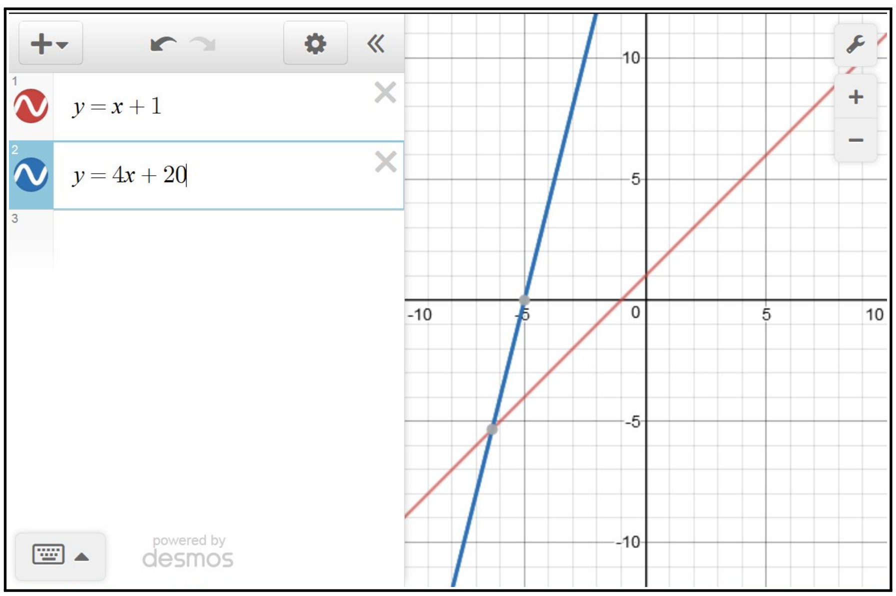

Fig. 1: *Desmos initial screen after entering equations.*

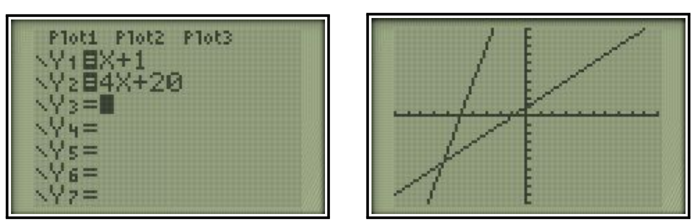

Fig. 2: *(Left) Equations entered into TI-83 Plus;(Right) Plot of equations in a standard viewing window.*

Each technology has its own set of limitations, demanding different user behaviors to yield useful output. For instance, in *Desmos* (see Figure 1) and the *TI-83 Plus* (see Figure 2), the intersection is visible. In *GeoGebra* (see Figure 3), it is not.

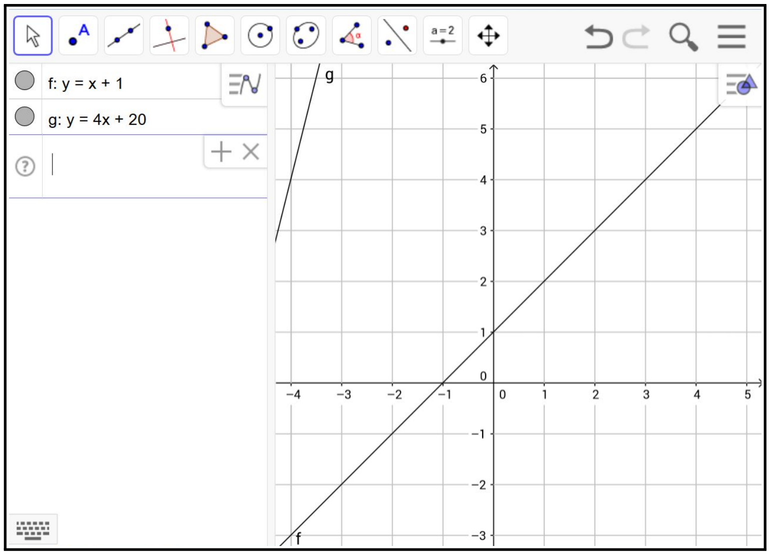

Fig. 3: *GeoGebra initial screen after entering equations.*

To make coordinates visible in *Desmos*, students need to click on the intersection. The result is shown in Figure 4. TI-83 Plus users use the `Calc > Intersect` feature to generate coordinates. This is shown in Figure 5. In *GeoGebra*, the student will need to reposition the window in order to see the intersection and then use the `Intersect` tool, as suggested in Figure 6.

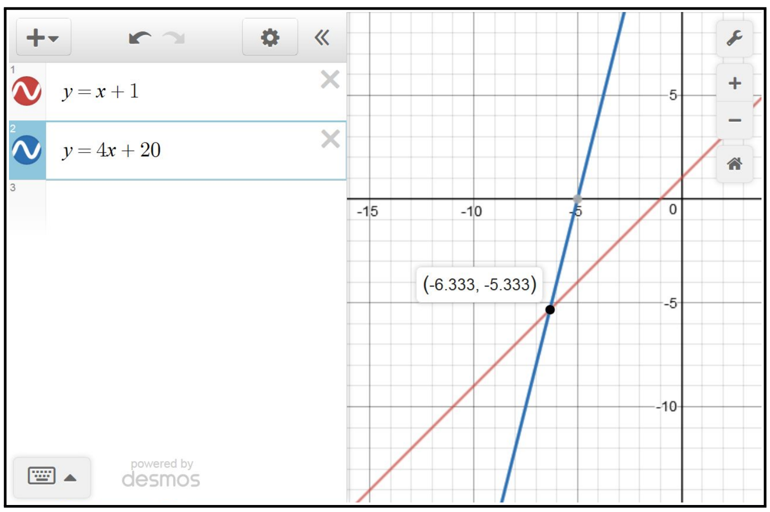

Fig. 4: *Desmos screen with intersection and coordinates visible.*

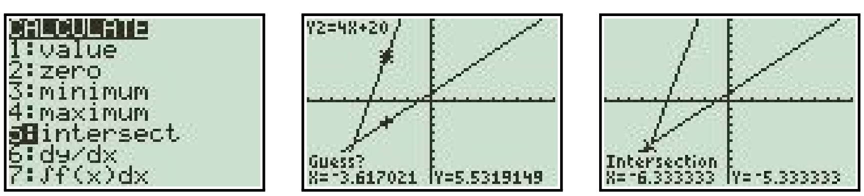

Fig. 5: *Steps to make intersection and coordinates visible on TI-83 Plus.*

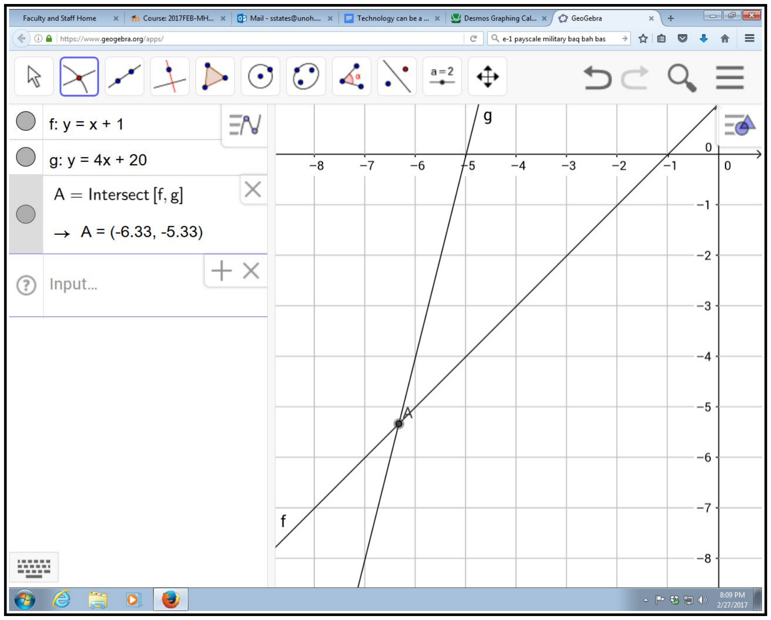

Fig. 6: *GeoGebra screen with intersection and coordinates visible.*

Depending on students' familiarity with the tools, they can arrive at answers relatively quickly, certainly faster than doing the problem by hand; however, none of these examples provides the exact answer, $(\frac{-19}{3}, \frac{-16}{3})$. In each , the tool provides a slightly different answer. *Desmos* displays 3 decimal places. *GeoGebra* provides 2, and the *TI-83 Plus* gives an answer with 6 decimal places. Why? Are these default settings? Students using the *TI-83 Plus* will likely conclude that the decimal repeats, whereas in *Desmos* and *GeoGebra* repetition is less clear due to fewer displayed decimal places. Students using these technologies with *MyMathLab* (MML) will have their answers marked as incorrect, because, MML requires each coordinate to be a reduced fraction on this question.

3 Student Challenges with Context

In the previous example, we explored a situation in which technology provides an answer, but in a format that is not acceptable in MML. What should a student do when technology fails to provide the desired format of the result?What do students do? Consider the following example.

MyMathLab tasks

On the basis of data from 1990 to 2006, the median income y in year x for men and women is approximated by the equations given below, where x equals 0 corresponds to 1990 and y is in constant 2006 dollars. If these equations remain valid in the future, in what year will the median income of men and women be the same?
Men: -264x + 2y = 59106
Women: -865x + 3y = 44405

Again, we compare and contrast answers generated by *Desmos*, *GeoGebra*, and the *TI-83 Plus*.

3.1 Finding an Intersection in Desmos

Figure 7 illustrates the student's initial screen after equations are entered in *Desmos*. Note that no lines are initially plotted. Why?

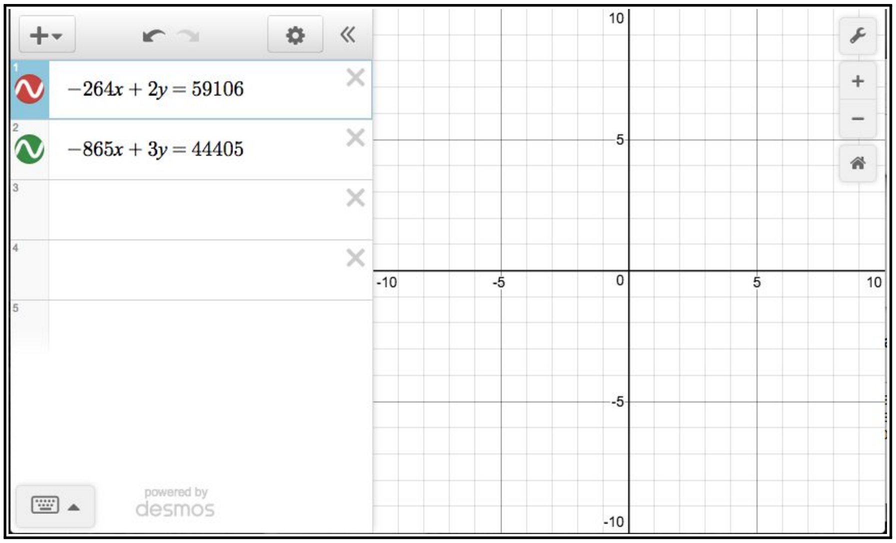

Fig. 7: *Initial viewing window in Desmos.*

To find the intersection in *Desmos*, students need to change the window viewing parameters either by using zoom buttons ($\pm$) in the upper right corner of the screen or by entering minimum and maximum values for x and y within the `Graph Settings` tool, as suggested in Figure 8.

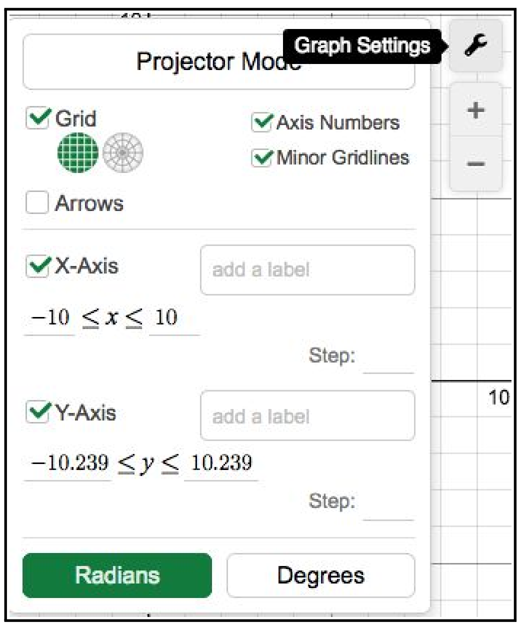

Fig. 8: *Default window settings in Desmos.*

With either approach, students must manipulate viewing settings until the intersection is visible then click on the point to see the coordinates as shown in Figure 9.

3.2 Finding an Intersection in *GeoGebra*

The process for finding the intersection in *GeoGebra* is similar. Again, users begin by entering each linear equation in the left portion of the screen (i.e., in this case, in *GeoGebra*'s `Algebra View`). Note that *GeoGebra* automatically factors a 2 from each term of the first equation: a potentially confusing move for novice students. As was the case in *Desmos*, *GeoGebra* users are initially greeted with a blank screen.

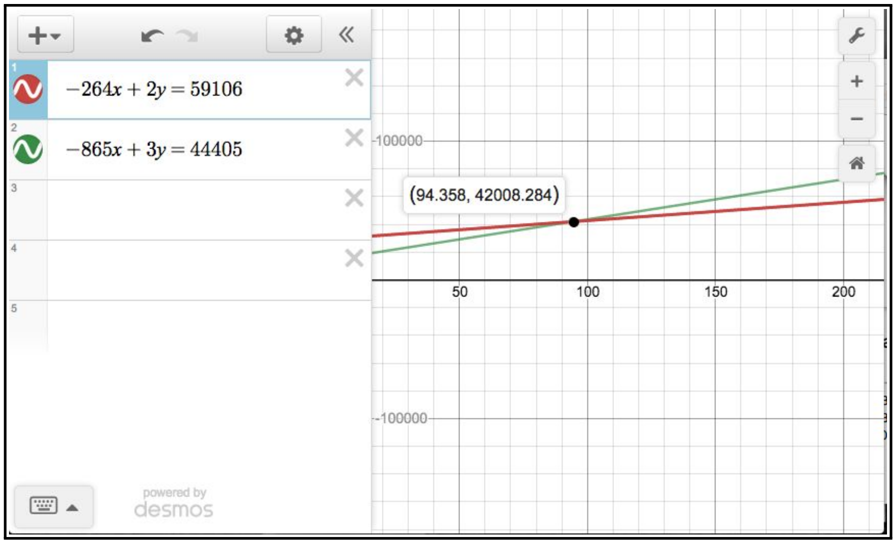

Fig. 9: *Desmos Final window after zoom applied and selecting intersection (available online at `https://tinyurl.com/example2-desmos`).*

From there, students must determine a suitable viewing window, one revealing the intersection of the two linear graphs. In *GeoGebra*, students hold down the shift key then drag on the $x-$ and y-axes to resize each scale, quickly revealing an intersection. An extra step is required to determine the intersection, however. Students need to invoke *GeoGebra*s `Intersect` tool (either by clicking or from the `Input Bar`). Results of such an approach are illustrated in Figure 10.

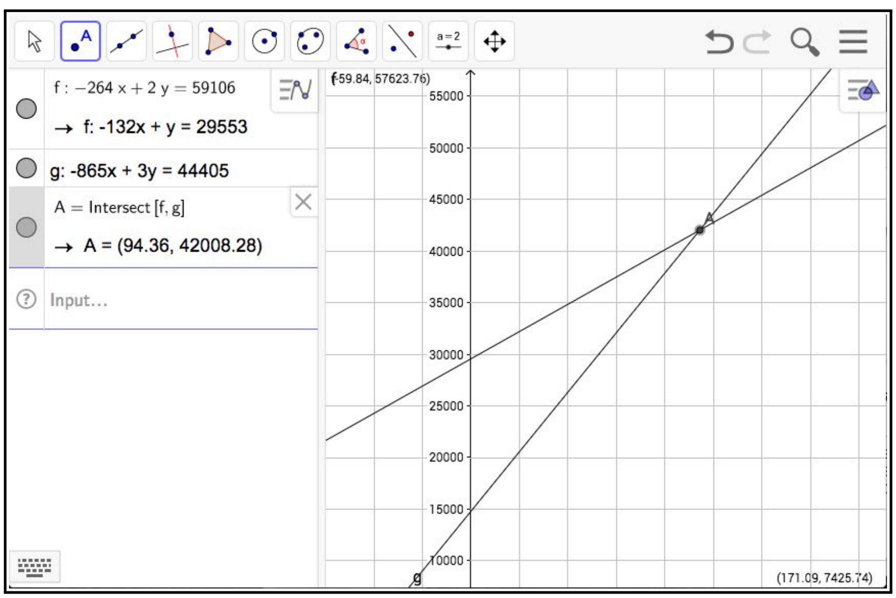

Fig. 10: *GeoGebra screen with found intersection.*

Both *Desmos* and *GeoGebra* generate some interesting questions for my students. First, *is this the fastest way to get an answer in GeoGebra? in Desmos?* Second, *is the intersection point we found with technology the correct answer?* Third, *when I type the ordered pair in MyMathLab, Im told that the answer is incorrect. Whats up with that?*

3.3 Finding the intersection on the TI-83 Plus

Figures 11 through 13 illustrate steps students commonly use to determine the intersection point on the *TI-83 Plus* graphing calculator. Unlike with *Desmos* or *GeoGebra*, one must solve each equation for y before entering them into the calculator (More work!). Typically, students use the `Zoom > Fit` tool to automatically find a suitable viewing window then use the `CALC > Intersect` tool to determine the intersection point (as discussed in our first example). These steps are illustrated in Figure 11.

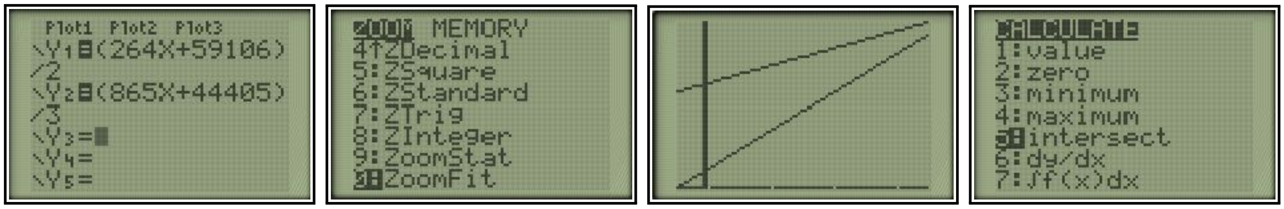

Fig. 11: *Initial steps invoked by students to determine an intersection with the TI-83 Plus calculator.*

Note that this strategy yields unexpected results. Specifically, `Zoom > Fit` yields a graph with the two lines not yet intersecting (see third screen from left in Figure 11). More often than not, this leaves students baffled. *The calculator didn't find an answer, so there must not be one, right?*

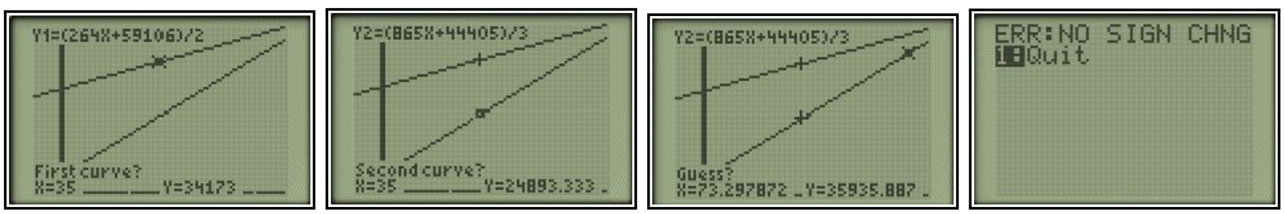

Fig. 12: *Using stock settings, the TI-83 Plus fails to find an intersection.*

To find an intersection point, students need to edit the screen settings generated by `Zoom > Fit`, then repeat their work with the `Intersection` tool. These steps are illustrated in Figure 13.

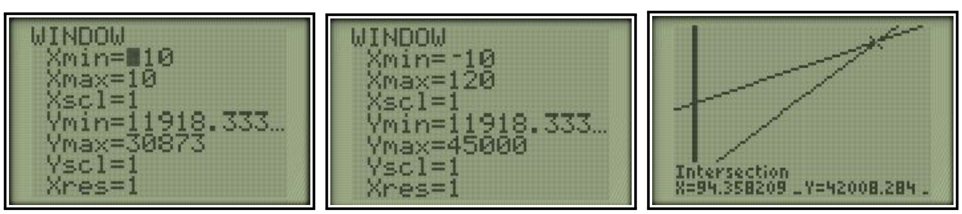

Fig. 13: *(Left) Initial settings generated by* `Zoom > Fit`*; (Middle) Revised settings; (Right) Graph with intersection.*

Note that no two tools produce the exact same answer. Moreover, students typically identify the point of intersection (the ordered pair) as the answer to the original prompt. When using technology, students need to make sense of the output in the context of the problem. In this example, the x value of the coordinate represents years after 1990. In each of the 3 examples, the x value is between 94 and 95 which indicates that median incomes intersect during the 94th year after 1990. Thus students need to perform the addition $1990 + 94 = 2084$ to answer the question correctly.

Beyond interpreting the output of the calculator successfully, it's instructive for students and teachers to consider the usefulness of the calculator in solving the problem to begin with. Recall that in order to use the *TI-83 Plus* successfully in this problem, students need to solve the original equations for y. That requires a significant amount of by-hand work. It's not clear that jumping to the calculator is any faster than continuing the by-hand work that students started as they solve each equation for y.

4 Conclusion

In this article, we've explored the need for students to be savvy users of technology. Each of the two tasks explored (both routine problems familiar to anyone who has taught entry-level or remedial algebra) revealed the inability of popular graphing technologies to generate correct answers for students. We leave it to the reader to decideif the use of the technology presented students with more obstacles than more traditional paper-and-pencil methods. Given the fact that our students live in a technology-oriented world, it seems unrealistic to eliminate the use of graphing technology in the mathematics classroom. However, as each example illustrates, in this world students need to be savvy users of technology when they choose to avoid strictly paper and pencil methods. Technology doesn't eliminate the need for careful thinking in the mathematics classroom: if anything, it necessitates MORE thinking (*What does this screen mean? What was the original question? Is my solution reasonable?*). Without question, as our examples suggest, teachers need to engage students in discussions of strengths and limitations of technology. At a minimum, we need to understand the problems we plan to present to our students, and we need to employ multiple methods as we solve problems to gain an appreciation of struggles that arise when technology is used. We will be better prepared to field student questions if we do!

References

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