When Scaffolding Is Not Helpful

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Abstract: *During a field placement as pre-service high school teachers, we developed and implemented two versions of a rich mathematical task. When reviewing our students' work, we observed that they had not applied the specific concepts we had intended—due, in part, to the level of scaffolding we had provided. Reflecting on this, we developed a rubric to guide our scaffolding of rich tasks based on task purpose.*
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1 Introduction

As pre-service high school mathematics teachers, we completed a four-week field experience—observing, planning, and teaching lessons in a freshman Algebra I classroom. The field placement opened our eyes to many new perspectives and ideas, and included opportunities to work with students, teach classes, and collaborate with intervention specialists. These experiences gave us a better understanding of the demands, challenges, and rewards associated with the life of a teacher.

Part of our teaching duties involved the development and implementation of rich mathematical tasks for students. According to the NRICH Project (Piggott, 2011), a rich mathematical task is one that actively engages students in constructing their own understanding, challenging their assumptions, and reflecting on their mathematics. Smith and Stein's (1998) Task Analysis Guide (TAG) describes rich tasks as "doing mathematics." Some key attributes of a doing mathematics task are that they:

- Require complex and no algorithmic thinking.
- Require students to analyze the task and actively examine task constraints that may limit possible solutions strategies and solutions.
- Require considerable cognitive effort and may involve some level of anxiety for the student due to unpredictable nature of the solutions process required (Smith & Stein, p. 348).

Following this definition, we created a task that challenged our students to do math while working towards content goals set by our cooperating teacher.

In this paper, we discuss our growth as teachers as we developed and implemented this task. In particular, we discuss our evolving understanding of scaffolding. We had different opinions on what scaffolding looked like and whether scaffolding was a tool for supporting students or a crutch impeding the development of our students' grit and persistence. We address these issues and share a rubric that we developed to help determine the level of scaffolding that was appropriate for our students at various times in the learning process.

2 Student Findings

At the time of our field visits, students were learning about systems of equations. Ultimately, we based our rich tasks on a problem found in the McDougal Littell *Algebra I* textbook, as shown below (Larson, Boswell, Kanold, and Stiff, 2004, p. 407).

Making Garments Task

A storehouse has three kinds of stuff: cotton, floss silk, and raw silk. They take inventory of the materials and wish to cut cost and make garments for the army. As for the cotton, if we use 8 rolls for 6 men, we have a shortage of 160 rolls; if we use 9 rolls for 7 men, there is a surplus of 560 rolls. We wish to know the number of men [that we can clothe] and the amount of cotton [we will use].

$x = \frac{8y}{6} - 160$ $\quad$ $x = -\frac{9y}{7} - 560$ $\quad$ $0 = \frac{8y}{6} - \frac{9y}{7} - 720$

We used this problem to develop a rich mathematical task. First, we increased its difficulty by removing the system of equations that was provided. Next, we enhanced the clarity, accessibility, and cultural relevance. As written, it contained extraneous information and used a context far removed from our students' lived experiences.

From bolts of clothing and sewing, we moved to baking donuts at a local shop. However, when addressing clarity and accessibility, we had conflicting perspectives, so we created two different adaptations of the original task. As we discussed and defended the merits of our revisions, we considered factors such as accessibility, different solution methods, and the concepts that the task covered. Both of our tasks addressed the Common Core Standards for Mathematical Practice 1, 2, and 4: (SMP1) Make sense of problems and persevere in solving them; (SMP2) Reason abstractly and quantitatively; and (SMP4) Model with mathematics. These standards align with our definition of *doing math*, so we worked to consider all three while developing the task. Prior to this task, all the modeling problems that the students had experienced had a single clear path to the solution. In contrast, ours challenged the students to reason quantitatively to discover the solution.

3 A Tale of Two Tasks

In the end, we kept both tasks and agreed to give both to students. The tasks are provided below. We gave these different versions of the tasks to two different classes.

Donut Task (Version 1)

You are making doughnuts for your new job at Ross Bakery. You need to make a lot of doughnuts! You already bought flour, but there are two recipes you could follow.

- The first makes 1 dozen doughnuts with 3 cups of flour, but you are short 7 cups to fill your quota.
- The second makes 3 dozen doughnuts with 5 cups of flour, but you would have 5 cups more than you need once you fill your quota.

Please represent the relationship between the two recipes, however you understand it. Next try to find out how many doughnuts would be made, and how much flour would be used if you wanted to make the same number of doughnuts with the same amount of flour with both recipes. (Please write out your thought process as you go and indicate your final answer. No erasers please!)

> **Donut Task (Version 2)**
>
> You are making doughnuts for your new job at Ross Bakery. You need to make a lot of doughnuts! However you are running low on flour since the shipment is not coming until the afternoon. You only have 29 cups of flour left. You have two recipes you can follow:
>
> - The first makes 12 dozen doughnuts. To make a single batch of 12 dozen, you need 10 cups of flour.
> - The second makes 15 dozen doughnuts. The recipe calls for 12 cups of flour.
>
> You need to make 36 dozen doughnuts. How will you do this? (Please write out your thought process as you go and indicate your final answer. No erasers please!)

Through this assignment, we anticipated that students would learn how to represent variables from the problem to create an algebraic system of equations. The goal of both tasks was to deepen students' knowledge of systems of equations while strengthening their problem solving skills.

3.1 Student performance on Version 1

Student performance varied on the two tasks. Students who were assigned Version 1 used a wide variety of representations including pictures and charts as shown in Figure 1.

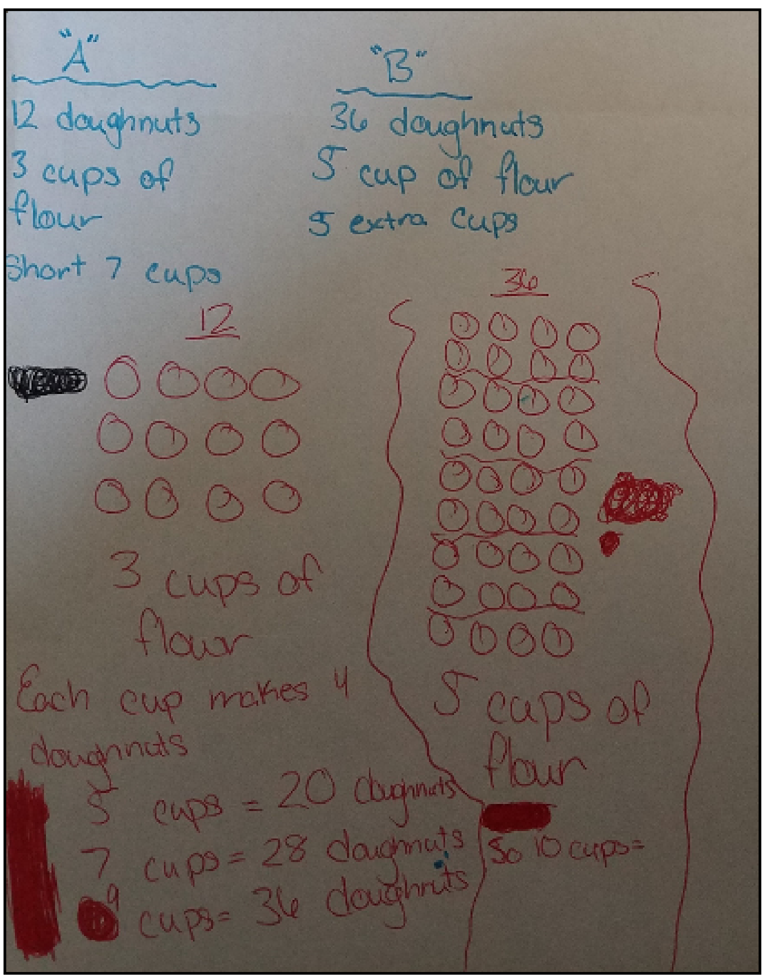

Fig. 1: *Sample student work for Version 1 of the Donut task.*

The few students who represented the task algebraically recognized that they could model the relationship with a linear system. Through their written work, these students showed that they understood what an linear function was in a mathematical context and how it could be applied to real life scenarios. However, these same students struggled to use their system of equations to generate an answer.

3.2 Student Performance on Version 2

Students who were assigned Version 2 of the task were far more successful than their Version 1 counterparts. The average score for solving the problem in Version 2 was 64 percent and in Version 1 was 58 percent. However, the majority of these students did not attempt to create a system of equations to solve the problem. Rather, the most common method they employed was guess and check with differing levels of sophistication. Some stated the answers with a modicum of scratch work. Others wrote the different combinations that they used. Two students attempted to create a system of equations and showed similar struggles when developing the system. Such work is illustrated in Figures 2 and 3.

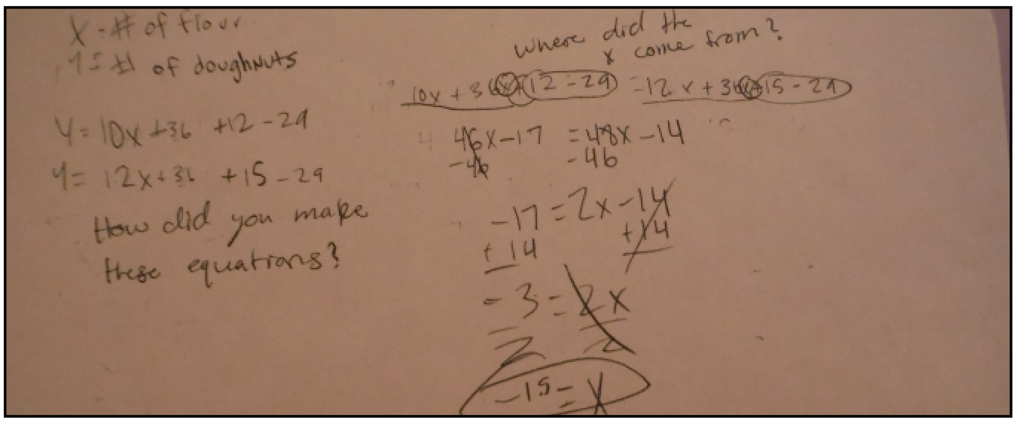

Fig. 2: *Sample student work on Version 2 of the Donut Task.*

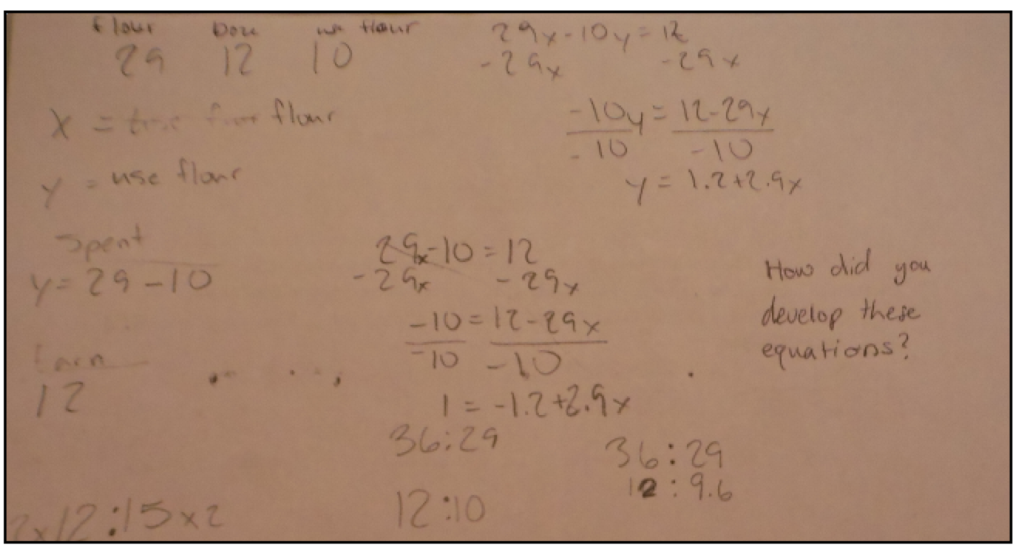

Fig. 3: *Sample student work on Version 2 of the Donut Task.*

4 Interpretation of Student Work

Student struggles often centered around difficulties associated with choice of variables. In Version 2, students assumed that x was the cups of flour used in both recipes, a faulty assumption that resulted in an incorrect answer. Our feedback on both versions of the task encouraged students to analyze how they decided what relationships their equations were representing. This additional feedback helped a number of students recognize their assumptions. Through this process students continue to broaden their thinking and make connections, in particular between systems of equations and the real world.

The purpose of the task was to deepen student understanding of systems of equations. In both cases, students did not fully understand that purpose. In the case of Version 2, the majority of students did not attempt to develop a system of equations although most achieved the answer. In Version 1, students struggled to move beyond representations of the information given. A weakness of both was that we could not provide instruction relating to the task unless students approached us with questions outside of class. This was because our cooperating teacher had limited time to teach material before the next test. This made scaffolding more challenging.

5 Rubric for Scaffolding of Rich Tasks

It was apparent that the wording of the task and the lack of scaffolding resulted in unanticipated outcomes. Reflecting on this, we wanted to find a way to make tasks in such a way that student achievement aligned with the intended outcome. Ultimately, we developed a rubric to provide guidance regarding the amount of scaffolding that should be provided depending on the purpose of a given task.

Types of Scaffolding	**Purpose of the task**		
	Introducing a concept	**Deepening learning of a concept**	**Assessment of a concept**
Initial Verbal Directions	Introduce new terminology and procedures, read through the task and clarify expectations for entire class, check for questions	Read through the task and clarify expectations for entire class, check for questions	Students read task on their own, they can ask questions to clarify directions
Written Directions	Step by step instructions, answers are separate not collective	Builds on students' prior knowledge, phrasing can vary based on the amount of scaffolding given in class	Single question with multiple parts, answers collective
Group or Individual/ Class Time Estimations	Group, ~ 2 days, including introduction and reviewing the task in conclusion	Group, ~1.5 days, introduce and group work for a day, finish as homework, review the next day	Individual, ~1 day, graded, review upon returning papers
Dialogue	Touch base with all students, open-ended questions	Build in as much as you want, open-ended questions	No dialogue besides questions clarifying task
Assessment	Assess for completion, make notes commenting on student ideas	Assess based on a rubric for accuracy of answer, make notes on student ideas	Assess based on a rubric for accuracy of answer

Fig. 4: *Scaffolding rubric.*

The rubric is a guide to determine the amount of scaffolding that needs to be provided based on the purpose of the task. Scaffolding varies based on class level, structure, and the personality of students. Ours is not a be-all end-all guide to scaffolding rich tasks, as interactions in the classroom are very complex.

Reflecting on the Doughnut Task, the purpose was to deepen students' understanding of systems of equations. For both versions, there was an insignificant amount of time to provide scaffolding through dialogue with the students, but the students who worked on Version 2 received more than the students who worked on Version 1. Most of the dialogue was open-ended questioning about students' thought processes. Based on the rubric, Version 1 had multiple parts to the question being asked with no dialogue, as well as very little initial verbal directions. This amount and type of scaffolding is appropriate for a task used for Assessment of a concept. In contrast, Version 2 broke down the information and asked a specific question for the students solve. Additionally, we explained the task in more depth when handing it out, using scaffolding aligned with Introducing a concept. However, both tasks were assessed based on the purpose of the task to Deepen learning.

As a result, more students who completed Doughnut Task Version 2 arrived at the correct answer than those who completed Version 1. Both groups demonstrated similar levels of problem solving as shown in Figures 4, 5, and 6. Since the students were still developing their understanding of the concepts, Version 2 was accessible to them, but did not require students to expand on what they know about systems of equations to solve the task. It could have been made more rich to fall into Deepening learning. The task in Version 1 was presented in a more complex way with little scaffolding through dialogue, classifying it as an assessment. Since the students were not at this level yet, they succeeded at the initial parts of the task but were unable to piece everything together to find a solution.

6 Conclusion and Implications

While our courses for pre-service teachers emphasize how to introduce new concepts to students and assess students, there is also a huge portion of teaching that involves the middle ground where students are deepening learning. This middle ground is where our students will make huge strides in their understanding of mathematics and it is in this struggle that they will come to appreciate the subject. While we engaged in discussion on how to scaffold student learning, our different opinions drove us to understand how scaffolding impacts student learning. Through our rubric we highlight the proper use of different types and amounts of scaffolding in the classroom. Throughout this process, we realized that doing mathematics looks different depending on the purpose of the task and students' readiness for that purpose.

References

Larson, R., Boswell, L., Kanold, T., & Stiff, L. (2004). *Algebra 1: Applications, Equations, and Graphs*. Boston: McDougal Littell.

Piggott, J. (2011). Rich Tasks and Contexts. *NRICH enriching mathematics*. Retrieved from `http://nrich.maths.org/5662`

Smith, M. S., & Stein M. K. (1998). Selecting and Creating Mathematical Tasks: From Research to Practice. *Mathematics Teaching in the Middle School*, *3*, 344-50.

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