Chewing Your Way to Understanding

Dittika Gupta, *Midwestern State University*
Suzanne F Lindt, *Midwestern State University*
Diana Moss, *Appalachian State University*

Abstract: *Helping upper elementary students to connect mathematical concepts to real-world ideas is critical to their long-term conceptual understanding but can be challenging for teachers. This paper explains how 4th and 5th grade students were presented with a real-world mathematical task that was engaging, hands-on, and promoted reasoning and problem solving. The authors share details of the activity and student struggles, reasoning, and understanding resulting from mathematical problem solving from the activity.*

Keywords: *student discourse, rich tasks, problem solving*

1 Introduction

The National Council of Teachers of Mathematics (2000) considers problem solving to be the cornerstone for all mathematics. To foster problem solving, teachers use mathematical tasks that challenge students' mathematical thinking and learning and promote their conceptual understanding of mathematics (NCTM, 2000). Problem solving does not involve a list of exercises to be solved procedurally, although procedures may come into play as students solve rich tasks. Mathematical tasks that build on curiosity and experience in the real world can help students develop mathematical connections in a meaningful way. Children should not only be given multiple opportunities to reason with problems, they should continually reflect on their current mathematical understanding in order to strengthen their mathematical reasoning and explanation skills (Lampert, 1990; Yackel & Cobb, 1994). *Principles to Action* (2014) calls for teachers to engage students in analyzing, discussing, and critiquing each other's ideas and results to build shared understanding of mathematical ideas. Setting tasks in contexts that are relatable to students and providing opportunities for them to collaborate, communicate, and discuss helps develop confidence and a community of collaboration in the classroom. The purpose of this article is to provide an example of a task that engaged our students in problem solving, reasoning, communication, and collaboration in a manner that makes connections to students' lives.

2 The Task

A group of fifteen fourth and fifth grade students were presented with four types of bubblegum described in four different colors: pink, orange, dark green (looks like watermelon with green and red in between), and light green. All identifiers, such as wrapping paper, were removed, so the

students did not know what type of gum they had. Students were given two tasks: 1) Determine which brand of bubblegum has the most sugar, and 2) Determine how much more sugar your gum has than the other types of gum. This task was adapted from an AIMS Educational Foundation activity (By Golly, By Gum 2006). The mathematical intent of this lesson was to give students an opportunity to make sense of the concepts of amount, mass, and estimation along with practicing mathematical skills, including operations with decimals, percent, graphing, and an introduction to the concept of ratio. The purpose was to help students predict, explain, verify, and then justify their understanding based on evidence. The students were to use the scientific process as they reasoned through the problem. Additionally, the goal was to practice concepts such as operations with decimals, percentages, graphing, and also develop understanding of amount, mass, and ratios.

2.1 Common Core Standards

The bubble-gum sweetness activity engaged 4th and 5th graders in mathematical practices recommended by the *Common Core State Standards for Mathematics* [CCSSM] (2010). Specifically, as students engage in the task, they "make sense of problems and persevere in solving them; reason abstractly and quantitatively; and construct viable arguments and critique the reasoning of others" (CCSSM, 2010, p. 6-8). The task of problem solving through the process of predicting 'sweetness,' testing, and connecting the findings with initial estimations helped challenge students' thinking, reason, and construct viable arguments. In the following sections, we describe the activity given to students and the students' discussions, struggles, reasoning, and understanding that resulted from the mathematical activity.

2.2 Authentic Problem Solving

Though understanding data and estimations can be taught in numerous ways, we believe that students learn best when they are asked to incorporate a real-world problem about a topic that is exciting to them. In the activity with gum, students were encouraged to reason about the problem in small groups, make predictions about the possible results, communicate their thinking with others, share results with the other groups, and ultimately discuss results as a whole class.

3 Making Sense of the Problem

The task was presented to the whole class, then students broke into small groups. The question posed was, "Which brand of chewing gum has more sugar?" Students related mass to sugar but were unable to comprehend the proportional relationship between mass and sugar. Initial confusion and misunderstanding was experienced by students, as evidenced through students comments such as, "[I think] the green and red kind has the most sugar (referring to the dark green) because they are big and look like they have a lot of sugar." One student reasoned, "[I think] watermelon has the most sugar because it is the thickest."

Pre-service teachers (PSTs) used higher-level questioning to create additional discussion, such as, "If I give an equal amount of each gum, which one would have more sugar" or "So how much more sugar do you think it has?" or "How can we find out?" These questions shifted students' thinking from choosing sugar content based on the largest type of gum to discussing different flavors and 'sweetness' in terms of sugar content in the gum. The question, "How much sugar do you think it has?" stumped the students for some time as they were still talking about the size of the unchewed bubblegum in this real-world scenario. PSTs had to ask questions such as, "Do you see the sugar?" and "How do you know that it has sugar?" to prompt students to talk with each other to solve the problem.

Some students suggested placing the gum in water to dissolve the sugar and then analyzing the remaining mass to determine which bubblegum had the most sugar content. This strategy was rejected as students reasoned and discussed that this method would dissolve the sugar but would not address the question "how much more your gum has than the other types of gum?" By thinking through the scenario, students reasoned that chewing the gum might be a better alternative to soaking it in water.

4 Developing a Plan to Solve the Problem

Students first chose one of the four types of gum and used a triple beam balance to record the initial mass of the unchewed gum. As they chewed for ten minutes, they communicated with each other about the "sugar" in each brand. Students made predictions about the mass of the chewed gum, the sugar content of the gum, and the percentage of sugar in the gum. Questions such as, *How can we say which one has the most sugar?*; *You say it has the most sugar, why?*; *How much more sugar?*; and *As you are chewing it, what do you think is happening?* promoted students' thinking and reasoning about sweetness, sourness, and flavor of the gum. The following are excerpts of some of the students' discussions regarding sugar content:

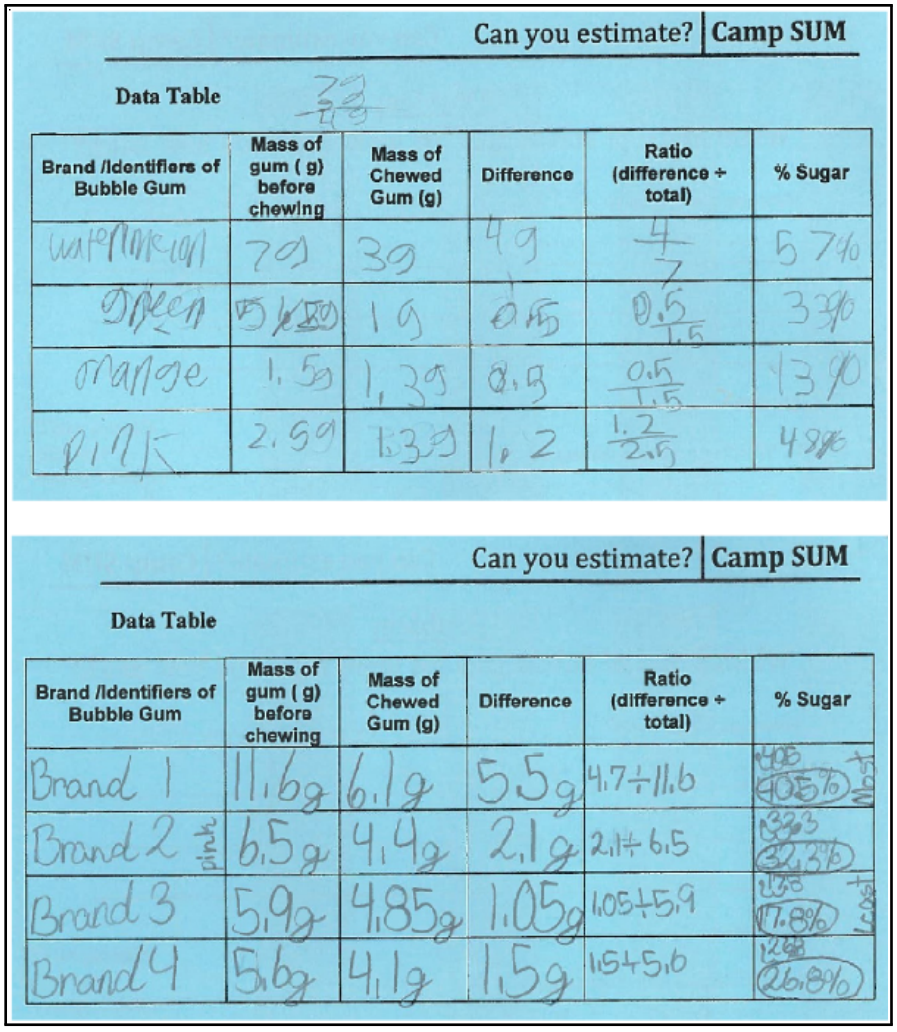
- I don't think mine has much sugar, as it has a bit of sourness in it.
- The pink one [has the most sugar] because the orange was sour, the other was minty, and the watermelon is like fruit, so [it] doesnt have that much sugar.
- [Dark] Green [has the most sugar] because you still have flavor after 3 minutes.

5 Using Quantitative Reasoning

As students finished chewing their gum, they were ready to measure the mass using balances. Specifically, they calculated the mass of their original unchewed gum and their chewed gum (see Figure 1). There was confusion with weight and mass as students used these terms interchangeably, but students learned through the activity that they were measuring mass using the balances.

Students showed understanding about the units of measurement and could quickly explain that the unit measured would be grams. The students practiced using skills of decimal operations as they calculated the difference between the original and chewed mass. As students reasoned quantitatively, they used the mass of the original gum and chewed gum to find the ratio to explain the change in sugar, which led them to calculate the percentage of sugar change. Ratio was a new concept for many of them, and this concept helped students focus on the proportionality of the sugar in the gum instead of the size of the piece of gum.

Students calculated percentage of sugar and were surprised by the sugar percentage results, as several had made a wrong prediction. One student even commented, "No, [I was wrong]; the size threw me off," while another said, "You can't determine the sugar in the gum by taste or look." The activity prompted students to reason and make connections between the concept of ratio and percentage by analyzing more than the size of the object. They practiced their skills with decimal number operations as they calculated the differences in mass.



Can you estimate? | Camp SUM

Data Table

Brand /Identifiers of Bubble Gum	Mass of gum (g) before chewing	Mass of Chewed Gum (g)	Difference	Ratio (difference ÷ total)	% Sugar
watermelon	7g	3g	4g	4/7	57%
Green	5 1/2 g	1g	0.5	0.5/1.5	33%
mango	1.5g	1.3g	0.2	0.5/1.5	13%
pink	2.5g	1.3g	1.2	1.2/2.5	48%

Can you estimate? | Camp SUM

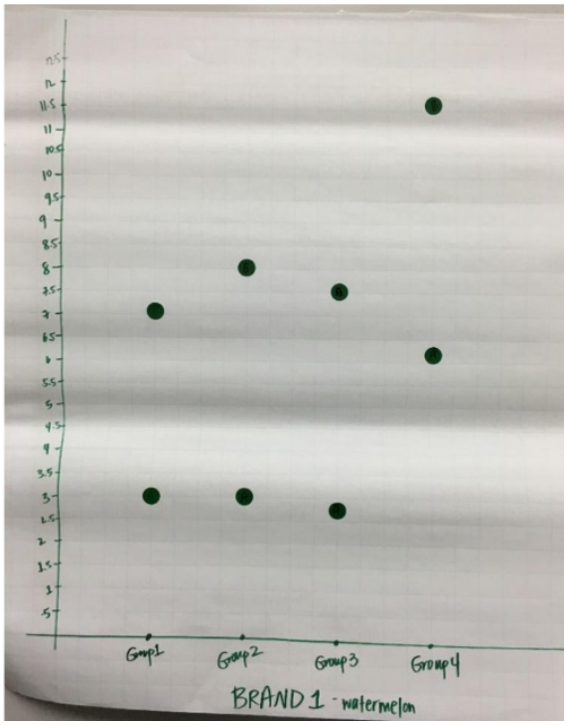
Data Table

Brand /Identifiers of Bubble Gum	Mass of gum (g) before chewing	Mass of Chewed Gum (g)	Difference	Ratio (difference ÷ total)	% Sugar
Brand 1	11.6g	6.1g	5.5g	4.7÷11.6	40.5% Most
Brand 2 pink	6.5g	4.4g	2.1g	2.1÷6.5	32.3%
Brand 3	5.9g	4.85g	1.05g	1.05÷5.9	17.8% Least
Brand 4	5.6g	4.1g	1.5g	1.5÷5.6	26.8%

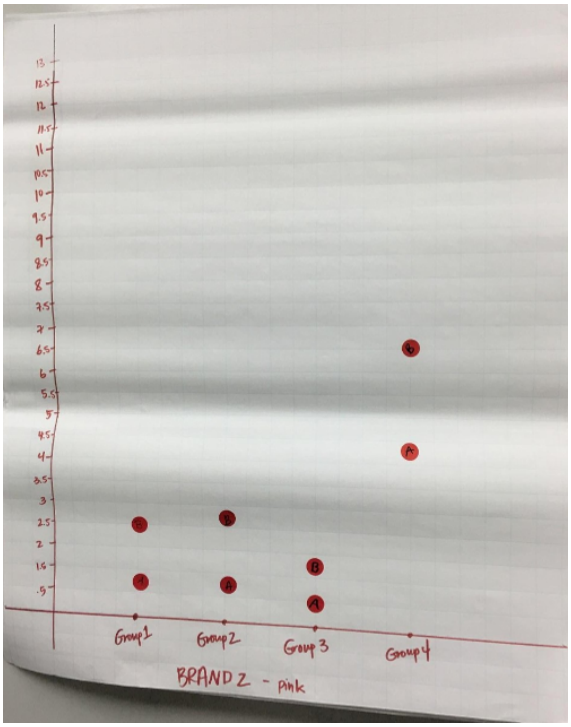
Fig. 1: *Student work samples as they worked on finding the bubblegum with most or least amount of sugar.*

6 Constructing Arguments and Critiquing Each Other's Reasoning

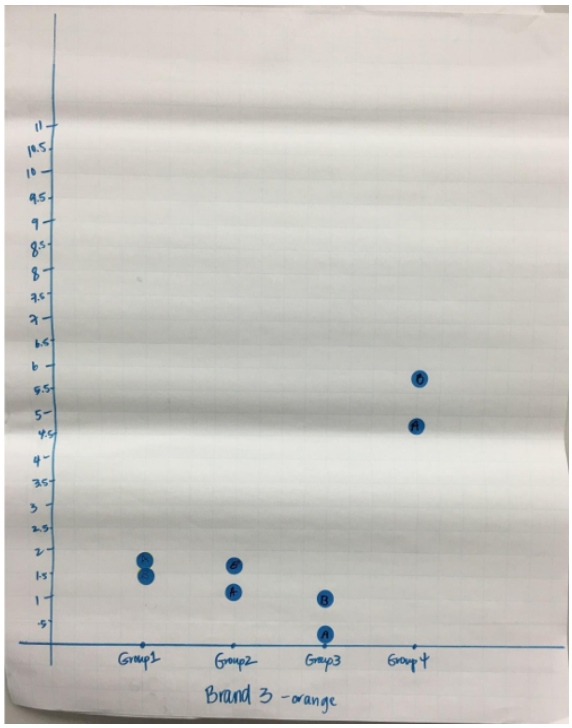
The activity continued with each group member noting the initial mass and the chewed mass for each brand of bubblegum on graph paper (see Figure 2). As a whole class, the students discussed and tried to make sense of the results from various groups. One group had very different results from the other three groups (a much greater mass for both unchewed and chewed gum) and students brainstormed possible reasons for this difference. Though initially students were confused and reasoned that one group had bigger pieces of bubblegum, they eventually realized that the difference was due to the fact that one particular group did not subtract the mass of the paper towel on which the bubblegum was weighed. The "aha" moment of the day occurred when a few students understood that the difference across the groups in mass of each type of chewed gum resulted from differences in chewing. Students explained this as, "We chew gum differently" and, "One may have chewed for a longer time." The students were able to relate the results and make a justifiable rationale.



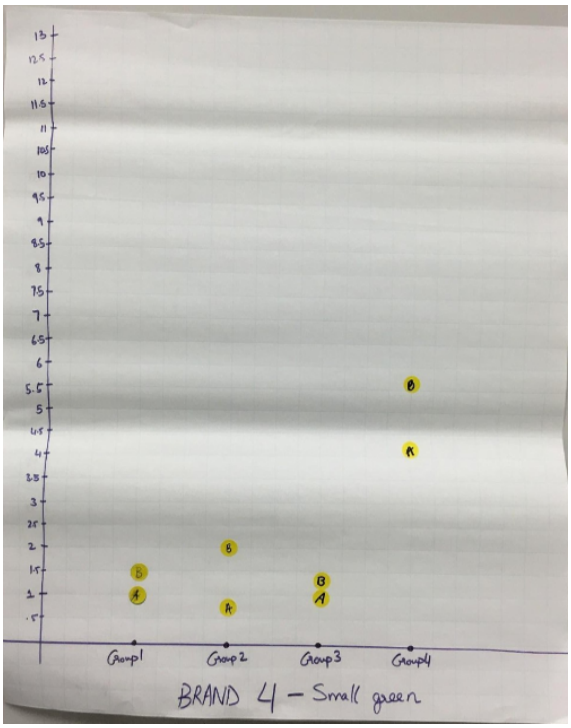
(a) *Student work for Brand 1 (Dark Green (Watermelon))*



(b) *Student work for Brand 2 (Pink)*



(c) *Student work for Brand 3 (Orange)*



(d) *Student work for Brand 4 (Light Green)*

Fig. 2: *Graphs as different groups shared their results (A represents mass AFTER chewing and B represents mass BEFORE chewing).*

As the brands of bubblegum were revealed, students' commented, "I knew it!" indicating that their predictions were accurate in guessing each type of bubblegum. Students were excited to work through these problem-solving techniques to arrive at a justifiable conclusion for the activity. The

students discussed the package information about the grams of sugar in the various brands of gum, compared that to the different brands and their own findings. They enjoyed the activity and conversations ensued about the gum problem, even after completion of the activity. In addition, students also reflected on their learning by responding to questions regarding usefulness of the activities and newly acquired learning. Students shared that they learned about grams, scale, using balances, measuring, finding percentages, and—something new—ratios.

7 Conclusion

The goal of the activity was to foster mathematical reasoning in students and to help them make predictions about a real-world problem. The bubblegum task promoted reasoning about several different parts of a complex task to solve the problem. The purpose of the task was for students to be able to justify their thinking, understand their reasoning after testing their predictions, and to develop an acceptable explanation of the problem solving process. The structure of small group discussions and explorations, sharing of results, and a whole class discussion helped students to reason and communicate their reasoning with peers. Additionally, working on a problem that was relevant to students and one that required them to talk with each other to communicate their ideas, helped students to develop social skills through interaction with new peers in a new setting.

Problem-solving is an integral part of mathematics (NCTM, 2000) and is a skill that incorporates many mathematical components. As mathematics educators seek to integrate various standards into the classroom, developing real world problem solving activities will help students to build their shared understanding of mathematics concepts (NCTM, 2014). The bubblegum activity described provides an example for educators to use in the classroom to assist students in developing a plan to solve a problem, predicting the results of a problem, and eventually justifying the results. NCTM (2000) calls for reasoning to be developed continually throughout the curriculum and making it a habit of mind. The middle school students who participated in this activity were able to reason, communicate, and connect to the problem and results. The activity fulfilled its goal of guiding students to reason through the problem solving process, communicate with their peers in developing understanding of the problem and the results, and to have fun while making sense of mathematics that included practice with number operations, percentages, ratios, and graphing. Extensions could have been provided to encourage a discussion of sugar versus sweetness and to encourage students to justify their thoughts on comparing the package information about grams of sugar to their findings. By using this activity and other real-world activities to solve mathematical problems, teachers can help their students to reach a greater understanding of mathematical concepts.

References

Common Core State Standards Initiative [CCSSI] (2010). *Common Core State Standards for Mathematics* (CCSSM). Washington, DC: National Governors Association Center for Best Practices and the Council of Chief State School Officers. Retrieved from `http://www.corestandards.org/wp-content/uploads/Math_Standards.pdf`

Lampert, M. (1990). When the problem is not the question and the solution is not the answer?: Mathematical knowing and teaching. *American Educational Research Journal, 27*(**1**), 29–63.

National Council of Teachers of Mathematics [NCTM] (2000). *Principles and Standards for School Mathematics*. Reston, VA: Author.

National Council of Teachers of Mathematics [NCTM] (2014). *Principles to Action: Ensuring Mathematical Success for All*. Reston, VA: Author.

Yackel, E., & Cobb, P. (1994). The development of young children's understanding of mathematical argumentation. Paper presented at the *Annual Meeting of the American Educational Research Association*, New Orleans.

Dittika Gupta, `dittika.gupta@mwsu.edu`, is an Assistant Professor of Mathematics Education in the Department of Curriculum and Learning at Midwestern State University. Dr. Dittika teaches methods courses to preservice elementary, middle, and high school teachers. Her research interests focus on teacher preparation, mathematical modeling, technology integration and interdisciplinary teaching including STEM fields and children's literature connections.

Suzanne F. Lindt, `suzanne.lindt@mwsu.edu`, is an Associate Professor in curriculum and learning at Midwestern State University and a former middle school teacher. Her research interests are in motivation and in teaching strategies to improve student engagement and learning in the K-12 classroom.

Diana Moss, `mossdl@appstate.edu`, is an Assistant Professor in Mathematics Education in the Department of Curriculum and Instruction at Appalachian State University. Her teaching and research interests include children's mathematical thinking and teacher education. She is particularly interested in understanding how to support teachers to teach math so that their students will learn. Her work involves using design research in classroom settings to support learning through the perspective of social constructivism.