Follow the Signs to Promote Accurate Geometric Shape Knowledge

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Abstract: *This article will help readers see shape-related children's books in new ways to help PreK-12 learners, as well as future teachers, to develop mathematically accurate concepts. The printable templates and guidelines help preschool, elementary, middle school, and high school teachers select and use children's books about shapes. Librarians, parents, authors, illustrators, and publishers will also find this resource helpful for selecting or creating accurate books about shapes.*

Keywords: *Geometry, children's books, misconceptions*

1 Introduction

How often have you heard students insist that a square cannot be a rectangle? Have you noticed that many say a square turned one way is a square, but turned another way is a "diamond"? Have you had students explain that some pictures of triangles are "real triangles," while other three-sided polygons are not (Schifter, 1999, p. 363)? How often have you heard students call balls "circles" and pyramids or party hats "triangles?" As an educator for more than 20 years, teaching math to students of all ages—children as young as 3 years old to adolescents and adults—I've heard educators and researchers comment that such mistakes occur because students look at images and see similarities. If this were the case, such situations could be remedied by helping students see shapes differently through comparing and contrasting these shapes. However, as a teacher, I've often suspected there was more to their misconceptions than meets the eye. I've looked for other explanations for the difficulties that geometric shape categorization poses for children and adults, alike. From my research, I've discovered sources from our environment that explicitly and implicitly misinform. For instance, most children's books "mis(teach) shapes" which puts unnecessary obstacles in students' paths (Nurnberger-Haag, 2017). Such findings have implications for teachers in the later grades—namely, we need to realize that our students have learned inaccurate information about shapes for countless years before we meet them.

Many resources advise elementary and preschool teachers to use children's books to teach children mathematical concepts; however, most books that explore 2D shapes reinforce popular misconceptions (Nurnberger-Haag, 2017; Resnick, et al., 2016). In this paper, I share a Traffic Light Rating Scale for evaluating the mathematics of two-dimensional shapes in children's books to help students

[*]Special thanks to Katherine Bryk who made the figures I envisioned possible. I also want to thank the hundreds of teachers and future teachers who, over the years have analyzed children's books with different versions of the rating scale, which makes it possible that we might make a difference for other teachers and students we'll never meet.

and teachers avoid sources of inaccurate information. Books rated green are recommended for children and older students for independent reading and in our math lessons. Yellow and red ratings signal to proceed with caution when selecting children's books. Figure 1 displays the quality characteristics for each level. To the side are recommendations of how books of each rating should be used to foster accurate shape concepts with young learners.

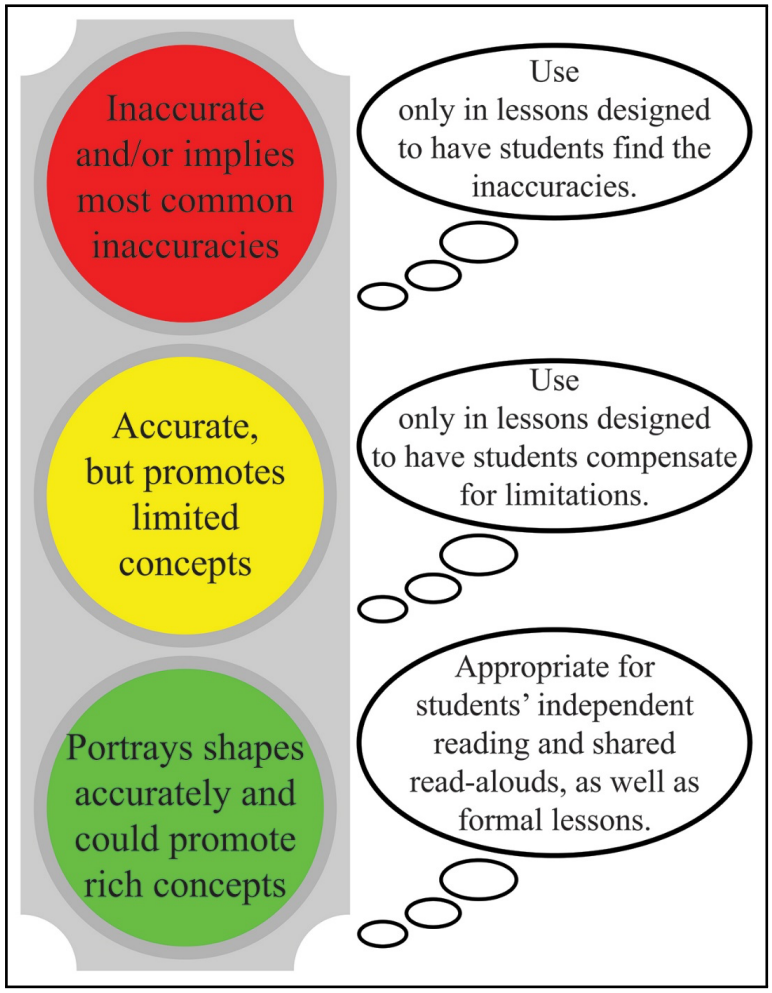

Fig. 1: *Rating scale for presentation of 2D shapes in children's books.*

Building on my teaching experience and research on children's books and with the input of hundreds of pre- and in-service teachers in several states, I have developed, tested, and refined the rating scale. Recent feedback suggests that the scale has helped teachers recognize how portrayals of mathematics in children's books can positively or negatively influence geometric learning. The scale has helped some adults recognize their need for more detailed knowledge of shape. The following feedback is typical.

- *It gave me specific things to look for in a "good book about shapes" [emphasis in original]*
- *I used this book with my toddlers last semester, but looking back now, I wish I would have done things differently.*
- *It actually made me realize I didn't know some of the characteristics [of shapes].*
- *This showed me how to distinguish between different shapes, and all the problems with the way I was taught as a kid.*

2 Characteristics of Good Books about Shapes

Green-rated books are good resources for learning about shapes. Because the mathematical accuracy of children's shape books are often not held to the same standard as textbooks, we must verify that any definitions or properties presented are accurate and that pictures of shapes have the following characteristics:

- Polygon pictures must have straight sides that intersect to form angles/corners (not curves).
- 3D shapes must be labeled with only 3D names and 2D shapes with only 2D names.
- Pictures should not violate the properties of the intended shape outline or approximate the shape for artistic purposes.
- Pictures should portray several varied examples (e.g., if a book uses the term "triangle," pictures of triangles with no congruent sides as well as triangles with two or three sides the same length should be portrayed.).
- Pictures of the shape should be shown in such a way that they are not just horizontal to the bottom of the page—that is, the shape should be portrayed in varied orientations (e.g., if a book uses the term "triangle" and shows triangles, these should not all be pointing down or up, but instead shown in several orientations) (Author, 2017; Sarama & Clements, 2009).

The first three bullets and the rating system will be explained more thoroughly in the remainder of this paper. Figure 2 provides more information about the last two bullets using rectangles as an example.

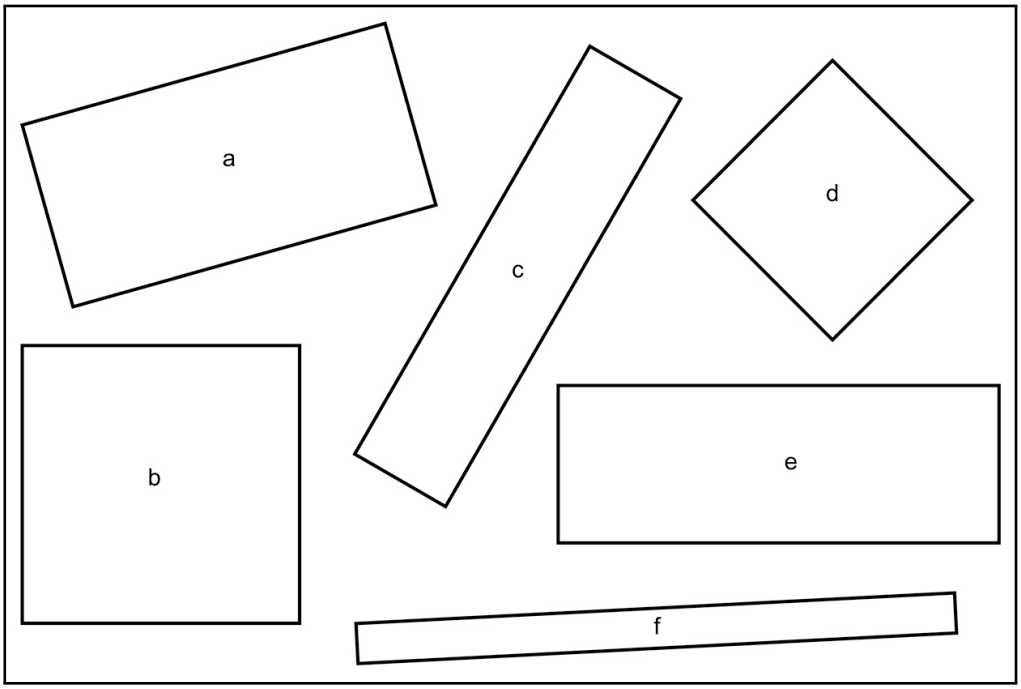

Fig. 2: *Rectangles should be displayed with a variety of orientations and dimensions (including all sides congruent).*

Notice that Figure 2 provides several varied examples of rectangles, including those with all sides the same length (squares in 2b and d) as well as long, skinny rectangles (2c and f). Moreover, more of the rectangles are rotated (2a, c, d, and f) than have a horizontal orientation (2b and e) to show that no matter how the shape is turned, it is still a rectangle (i.e., orientation is not a defining property).

Figure 3 illustrates similar ideas with parallelograms.

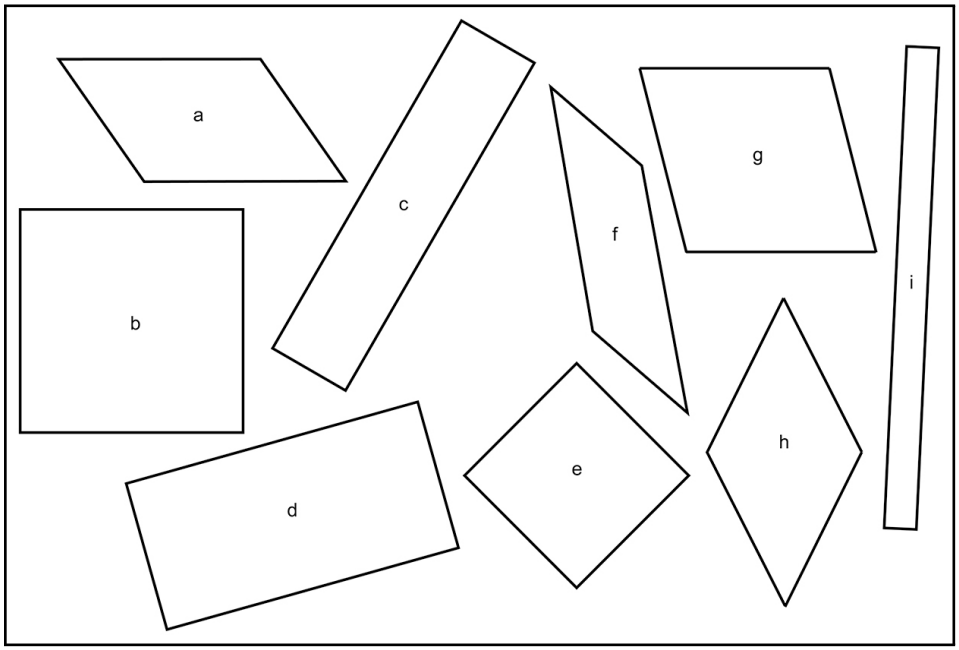

Fig. 3: *Parallelograms should be displayed using the full variety (all subtypes) with a variety of orientations.*

A variety with all subtypes from our rating scale is shown: parallelograms with congruent sides (3b, g, and h) and parallelograms with right angles (3b, c, d, e, and i). Examples such as these visually reinforce that parallelograms can, but do not have to, have slanted sides. Moreover, more of the parallelograms are rotated (3c, d, e, f, and h) than have a horizontal orientation (3a, b, g and i), illustrating that orientation is not a defining property.

Additional characteristics, although infrequently found in books, further enhance a book's potential as an instructional tool. Inclusion of these characteristics can lead a book to be rated "Super-Green" for a particular shape. These include helping readers accurately compare two-dimensional and three-dimensional shape properties, compose and decompose shapes, and/or provide non-examples of shapes. Books like *Muddy, Muddy Mess* (Burnett & Irons, 2010) help students notice 2D faces as parts of a 3D shape. Non-examples are important to build student understanding (Clements, 1999). A book that helps children use non-examples might say "find the shapes that are not triangles" or show a chevron or pie slice with the caption "these are not triangles."

3 Let's Decide How You'll Use the Scale

Let's revisit the Traffic Light Rating Scale in Figure 1 to decide what your goal is for using a given book, consider how much time to devote to evaluation, and plan how to use the book with students. The flowchart in Appendix A can help you choose which parts of the rating scale to use based on your available time and goals. The first column of Appendix A focuses on choosing your goal:

- Get rid of inaccurate books;
- Fully rate the quality as RED, YELLOW, GREEN/SUPER-GREEN; or
- Only select GREEN/SUPER-GREEN quality books.

The second column summarizes what to do to accomplish the goal and how much time is required. The third column provides a flow chart that identifies which parts of the rating scale to use based on your goal and time. When you encounter the END signs, you've come to the end of the road to rate that color.

3.1 Goal: Eliminate Inaccurate Books

If you have just 2 to 5 minutes to dedicate to evaluating a book, then you have time to use the No-Sign to decide if the book has inaccuracies (i.e., a red quality rating) (see Row 1 of Appendix A). If you check any of the boxes in the No-Sign, the book is a red book, and you can stop evaluating it, avoid using it, and discourage use for children and parents. Please note, however, that even if a book does not have any red qualities, it may not be recommended for use with children. At this point, if you think you would like to use the book with students, continue evaluating it, as suggested by the U-turn icon directing you to the full Traffic Light Rating Scale.

3.2 Goal: Use My Books Even if They are Imperfect

If you wish to plan inquiry-based lessons that compensate for a book's imperfections or perhaps capitalize on inaccuracies, the full Traffic Light Rating Scale helps you carefully critique a book to determine if it has red, yellow, or green ratings for each shape. For example, the book may have all green-rated qualities for triangles and circles, but red qualities for rhombuses and rectangles. Row 2 of Appendix A shows the steps to do this. Additional time would be needed to plan effective lessons with these books.

3.3 Goal: Use Only Completely Accurate Books

If, however, you are interested in only using completely accurate books that reflect the variations within and between geometric shape categories, then in less than 20 minutes you could determine if the book has only green qualities. As soon as you find any red or yellow characteristics, you will have determined that the book is not green, so you can stop. For those who have strong knowledge of geometric shapes and have used the full rating scale (Appendix B) several times to know what nuances to look for, the one-page Short-Cut Form in Appendix C could be used to identify Green/Super-Green books.

4 Lets Get Started: How to Rate Books with the Full Rating Scale

Before using the rating scale, first decide if the entire book is about shapes or if only part of it is. Some focus entirely on a single shape, while others explore several shapes. Some discuss numbers, shapes, and other mathematical concepts all within the same book. Read the parts that are about shapes to gain overall familiarity with the book and to identify the shapes that the book names. The Overview of Directions page in Appendix B explains the purpose of each page of the rating scale. The top of each page explains how to use that page.

The following sections provide additional tips. You can also access "How To" videos at the *Math For All Ages* YouTube channel (`https://www.youtube.com/mathforallages`). These videos were created by teachers who are passionate about making geometric shape learning easier for students and use the rating scale when choosing books for their classrooms. They created these "How To" videos to help other teachers.

4.1 Start with the No-Sign

Regardless of whether a teacher has two minutes or 30 minutes to evaluate a book, the first step is to look at the No-Sign in Appendix B. I used the universal "no-sign" to create a checklist of the characteristics that should be avoided. The No-Sign in Appendix B provides a checklist of errors common in children's books (Nurnberger-Haag, 2017). Use a red marker or pen on the No-Sign to check off instances of errors. If a book has any of the characteristics listed in the No-Sign, think of it as a red light. Some of the characteristics in the checklist are inaccuracies, whereas others are treatments of shape that could indirectly lead to student misconceptions. OJSM readers whose primary expertise is mathematics may not need the commentary on the outside of the circle, but others who use this rating scale have found these explanations helpful.

Notice that the left-most part inside the no-sign is a list of "inaccurate properties." Look at the words in the book to check for these issues. For example, if a book shows pictures of rectangles that are only oblong and uses the word "rectangle," but avoids stating the inaccurate property about lengths of sides, we would not mark the box, "Says a rectangle has two sides longer than the others," in the properties section. For the "Misleading Pictures" section, we focus on critiquing pictures. For instance, if you recognize a triangle in a picture with rounded corners, but the book never uses the word "triangle," then we would not check the box, "Corners or sides of shapes labeled as triangle . . . are . . . curved."

4.2 Look for Yellow and Green Qualities

The next two pages of the template (Look for Yellow and Green Qualities) help us determine if a book avoids all other negative criteria (yellow) while identifying positive characteristics (green). A book can be green even if it does not have every positive characteristic for a shape.

It is common for books, especially those for children from about birth to age 5, to present labeled shapes with no additional text or to use rhymes and phrases to tell a story about the shape as a character in the story. For such books that provide no properties of shapes, much of the Look for Yellow and Green Qualities pages of the template in Appendix B are not applicable. For these books, complete the Orientation section at the top, skip the "Properties That All Polygons Share" section, turn the page and skip the left-most column "What other properties of each shape do the WORDS say?" Because the words in the book do not say anything about properties, leave each of these sections blank. Check the characteristics in the right-most column about the pictures entitled "What variety do the PICTURES of each shape show?"

Some books will teach certain shapes, but not others. This is fine. What is important is to critique the images and properties of the shapes that the book does name. For example, if you notice a polygon that you recognize as a rhombus, but the book does not use the word rhombus, then the rhombus section of the rubric is still NA or not applicable, because the book did not intend to teach the concept of the term rhombus.

On the second page of Look for YELLOW and GREEN Qualities. If you highlight a negative characteristic, do not highlight the related green characteristic. For example, the more types of triangles a book includes makes it better for concept development. However, if the text only shows acute triangles, this negates any positive impact. So, if all triangles in a given book are acute, highlight in yellow "- only acute," but do not highlight "+ acute." See the how-to-videos for more explanation.

4.3 Look for SUPER-GREEN Qualities

On a positive note, at least two recently published children's books qualify as super-green, meaning they include non-examples and/or actively involve the reader. Refer to the Look for SUPER-GREEN Qualities page for more specific characteristics. The super-green books I have found so far encourage the reader to discuss properties of shapes without naming or labeling them.

Muddy, Muddy Mess (Burnett & Irons, 2010) is a super-green big book appropriate for preschool through second grade that encourages readers to notice and compare faces of three-dimensional shapes. In the story, four personified three-dimensional shapes play in a muddy puddle. This book encourages the reader to determine which shape made the muddy prints by looking at the two-dimensional marks that each leaves behind: "So each one crept out of the muddy puddle. What prints do you see? What shape could it be?" (Burnett & Irons, 2010, p. 8-9).

Which One Doesn't Belong? A Shapes Book (Danielson, 2015) shows four two-dimensional shapes on every page without labeling the shapes. The text encourages readers to justify why they think a particular shape doesn't belong with the other three. A reader could exclude any of the four shapes using a different explanation. Adults reading the books with children will need to help children understand that even when they are correct that one of the pictures doesn't belong with the other three on the page because of color or orientation, that these attributes are not mathematical properties that define shapes.

Neither of these two books teach readers the names of shapes, so we skip the No-Sign and Look for YELLOW and GREEN Qualities pages of the template in Appendix B. On the Look for SUPER-GREEN Qualities page, we mark positive qualities. For instance, for the book *Muddy, Muddy Mess* (Burnett & Irons, 2010), we mark "The WORDS and PICTURES in the book show children how the footprints or outlines of a 3D object or shape are 2D shapes" in the section "Compares 2D and 3D Shapes." The book *Which One Doesn't Belong? A Shapes Book* (Danielson, 2015) has two SUPER-GREEN qualities: (1) The very first positive characteristic about properties: "The WORDS encourage the reader to discuss, compare . . . " and (2) the non-example characteristic, because the text encourages readers to identify non-examples as they determine which shape is not like the others.

5 Let's Plan: How to Use Each Color-Rating

Broadly, think of green as appropriate for all readers. Red and yellow lights signal that guidance from a knowledgeable adult is needed.

5.1 Using GREEN/SUPER-GREEN

Parents, preschool educators, elementary teachers, librarians, and other caregivers can make green/super-green books available for shared read-alouds as well as independent reading. Green-rated books could also be used to supplement inquiry-based explorations of geometric shapes involving discussions of shape properties, sorting and classifying, and composing/decomposing shapes (Sarama & Clements, 2009). High school geometry teachers could have students distinguish between properties and definitions in green childrens books as they consider necessary and sufficient conditions for shape definitions.

5.2 Using YELLOW or RED

Adults might read pages with green-rated portrayals aloud and skip those pages that portray a particular shape in less than accurate ways (yellow or red). Alternatively, yellow and red portrayals of shapes might be used in carefully planned lessons that encourage students to critically analyze books. When trusted sources include incomplete and inaccurate ideas, teachers need to intervene. Future teachers can explore yellow or red books in methods courses as a powerful method for exploring and possibly improving their understanding of geometric shapes.

To compensate for limitations of yellow-rated books, we can read these with students to point out the misleading images or words. We can use a shared reading activity like this to launch a lesson in which we challenge students to create representations that more accurately reflect the full range of examples that fit given properties and definitions. For example, a teacher might ask students to make as many different looking polygons as possible that retain the properties of a rectangle. Students could use software tools, physical straws, snap-together materials, or geoboards to create their own better example pages for the book or rewrite misleading property statements.

Another way to use imperfect books (i.e., red and yellow) is as part of a proofreading project to find and correct errors in books. We know how much pleasure children and adolescents take in proving adults wrong, so a project like this can capitalize on that. This work could also serve as a performance assessment instead of a unit test to determine a student's understanding of particular shape concepts.

6 Let's Get Real

So that you don't feel blind-sided as I did when I began looking at children's books more than ten years ago, it is important to share with you the current status of available books. The reality is that most books about two-dimensional shapes we find will be rated red or yellow based on the guide in Appendix B. Even though such books were created to help children learn, they contribute to a cycle of misinformation. It seems that on the road toward geometric understanding, many children have gotten stuck on a roundabout and become adults who continue to circulate misinformation to children.

In a study of 66 children's books about shapes, 76% of the books explicitly gave at least one inaccurate idea, in words, pictures, or both (Nurnberger-Haag, 2017). Even more books used images and words that could lead students to have limited concepts of 2D shapes. The inaccuracies and limitations found in red and yellow-rated books are the same geometric shape misconceptions reported in research with people of all ages, which suggests that people have learned what these books taught them (Nurnberger-Haag, 2017). More than ten years after my initial data collection, it is disheartening to see the same problems persist. I'll share some of these issues to help show the reasons that these characteristics are in the rating scale. If, after reading these, you would like more information about this study, a free audio podcast is available at `https://www.podomatic.com/podcasts/mathed/episodes/2016-11-23T06_26_27-08_00`. In the section that follows after this, I'll discuss how we can work together to change the status quo.

First, even the youngest students are capable of learning more than many shape book authors assume. Most books only teach some combination of four shapes: circles, triangles, squares, and rectangles (Nurnberger-Haag, 2017). Fewer than 25% of all books use other shape names, such as octagons or pentagons, and only about 10% give readers a chance to learn terms like quadrilateral, parallelogram, rhombus, or trapezoid (Nurnberger-Haag, 2017). Second, books teach these shapes in

very limited ways, primarily by showing readers pictures with labels rather than explaining explicit properties such as the number of sides, number of angles, or sizes of angles (Nurnberger-Haag, 2017). For example, even though "triangle" literally means "three angles," only 39% of books about triangles describe any properties of triangles, and those that do typically only describe the number of sides, not angles (Nurnberger-Haag, 2017).

6.1 Pictures are Worth a Thousand Words

Suppose students read all 66 children's books from the study. They would see about 1,000 images of squares and rectangles and about 1,200 images of triangles, most of which are limited variations of the shapes (e.g., triangles with two sides of the same length; rectangles shown only as oblong; parallelograms only shown without right angles), shown with primarily horizontal orientations, and inaccurately presented with 3D pictures (Nurnberger-Haag, 2017). We already discussed how important it is to show all variations of shapes and orientations. The primary inaccuracy in children's books about 2D shapes is that these books inaccurately show real, three-dimensional objects and claim they are two-dimensional shapes. In my study (Nurnberger-Haag, 2017), I uncovered three main ways books misrepresent 2D and 3D shapes:

- 3D mathematical shapes are named inaccurately (e.g., a party hat that is a mathematical cone was called a triangle; a ball that is a mathematical sphere was called a circle, etc.)
- The outline or footprint of a face of a 3D mathematical shape is 2D, but the book called the entire 3D object by the 2D name (e.g., an object like a can that is a cylinder was called a circle)
- Objects that only somewhat resemble the named 2D shape were used to represent the shape (e.g., clothes hangers labeled as triangles, spiral suckers labeled as circles, or rolls of clay placed next to each other with uneven edges labeled as rectangles)

In addition, curved sides or rounded corners mislead students about concepts of polygons. Adult illustrators often draw polygons to look like a child drew them or cut them out of cheese, so the sides are wavy rather than straight line segments. It is no wonder that some students think polygons can have curved sides. Similarly, straight line segments should meet at a vertex to form an angle rather than continuing on a curve with rounded corners. Yet depending on the term used, about 30 to 40% of books named a polygon but then showed pictures with curved corners or sides (Nurnberger-Haag, 2017).

To better explain the inaccuracies of pictures, let's look at a real-world object found in many children's books: a musical triangle such as the one shown in Figure 4. A musical triangle is a 3D object, and if we trace around it, the outline would approximate a triangle; however, it is not a mathematical triangle. It has curved corners. The gap that allows the metal to resonate means it is not a closed polygon—thus a *musical triangle* is not a *mathematical triangle*.

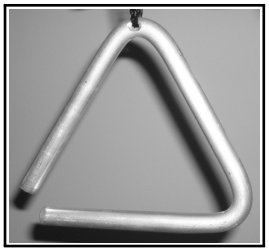

Fig. 4: *The rounded corners, gap, and 3D nature of a musical triangle make it a poor example of a mathematical triangle.*

6.2 Words in the Book Describe Incorrect Ideas

When books describe at least one property of a shape in words, they may provide inaccurate properties or definitions. Some examples include:

- 43% of the books that gave some property of rectangles inaccurately told readers that two sides of a rectangle have to be longer than the other two sides (even though a square is a rectangle)
- Some books said that some [rectangles, triangles, circles, etc.] are flat. This is inaccurate because *all* polygons must be 2D, not just some of them.
- Books commonly note that rhombuses cannot have right angles (Nurnberger-Haag, 2017).

The rating scale helps us identify these inaccuracies as well as other incomplete ideas that authors communicate in words.

7 Let's Change the Status Quo

Experts used to believe that mathematics was not developmentally appropriate for young children, but we now know better (Erikson Institute, 2014). Children as young as 2 or 3 should start to learn, for example, the meaning of "triangle" by seeing many valid variations of triangles and discussing the parts or properties (e.g., trace the sides with your finger and count the sides). In Summer 2017, when I taught a math camp for preschool children (ages 3 to 6), I offered snap-together tools for children to create closed shapes (outlines of polygons). In the following year, after the 2018 camp, I realized how much I had underestimated these preschoolers. I upped the level of challenge by having the children make outlines of polygons and then sort their creations by the number of sides and corners. The three- and four-year-olds were so excited to trace and count the sides and angles with their fingers and accurately sort them. I helped the children develop conceptual understanding before assigning formal mathematics language by waiting to introduce the names of polygons until after they had significant time (i.e., three days) to focus exclusively on shape properties. The children loved using these new big words like "quadrilateral" to name their shapes. In light of this experience, it's noteworthy that only 10 to 20% of children's books give readers a chance to think about such terms (Nurnberger-Haag, 2017). Young children are capable of traveling on the road to accurate geometric shape knowledge if we give them access to it and avoid putting obstacles like inaccurate resources in their path. Specific guidelines help all of us—even experts—better evaluate children's books (van den Heuvel-Panhuizen & Elia, 2012). Indeed, many of the books "experts" recommend to integrate mathematics with literacy have mathematical inaccuracies that have gone unnoticed that are revealed by the rating scale in Appendix B. Once a book is identified as green, yellow, or red, teachers are empowered to use it in a manner that is appropriate to help children learn correct mathematics.

As busy teachers, we would all like a quick and efficient system. After using the rating system, a future teacher explained that the number of pages and the details of the Traffic Light rating scale seemed overwhelming at first, but that "it is actually just systematic in helping teachers thoroughly assess the accuracy and quality of shape books." The full Traffic Light Rating Scale has to be detailed in order to be helpful. A condensed, Short-Form version of the scale, one that only identifies green-rated books, is provided in Appendix C. It can used by those who have used the full Traffic Light Rating Scale several times and have strong knowledge of geometric shapes.

7.1 Supporting our Youngest Learners

As teachers, it is our responsibility to select and use resources in ways that support students' learning. In spite of the pervasive availability of books, apps, and toys about shapes, for children

younger than kindergarten, it is probably best to provide only hands-on experiences like the activity I used at the math camp or the van Hiele puzzle (1999) published for teachers. If books are used with our youngest learners in daycare facilities and preschools, then they should be green quality.

7.2 Ways to Use the Rating Scale Beyond Children's Books

Textbook selection committees and individual teachers could use the Traffic Light rating system to assess the quality of textbooks' treatments of shapes. Even if the definitions are technically accurate in textbooks (which is not guaranteed), a study found that 76% of high school geometry textbooks only showed prototypical shapes (e.g., horizontally oriented oblong rectangles) despite the fact that students typically are more influenced by pictures than textual definitions (Usiskin & Griffin, 2008).

In addition, the Traffic Light Rating Scale can be used to evaluate the displays on classroom walls and student desks. Once we realize our resources give off red or yellow "signals," we can have our students create more accurate displays. Just as red or yellow traffic lights signal us to proceed with caution, inaccurate or incomplete resources must be used carefully, in ways that compensate for or capitalize on imperfections to foster greater student learning. By offering the opportunities described in this article, together we can pave paths for students to experience and eventually arrive at accurate and rich geometric shape concepts.

7.3 Final Thoughts

When I first began critiquing shape books with other teachers more than a decade ago, my efforts were met with a mixed response. Some teachers continued to use their favorite books because they liked their rhymes and illustrations and because they had been taught that rigorous mathematics was not developmentally appropriate, even though the books taught their students inaccurate information. This was also before mathematics standards recognized that our youngest school children can use properties of shapes meaningfully. However, other teachers with whom I have collaborated in professional development workshops have immediately recognized that their calendar math kits promoted mathematical misconceptions and taken action to change the books they use. They began to pose inquiry-based tasks to help students overcome limitations promoted by faulty posters and books. Today, I'm more optimistic that we can change the status quo. A recent pre-service teacher left me the following comments after using the Traffic Light Rating Scale in her methods course.

> This activity (the Traffic Light Rating Scale) showed that using the correct terms from the beginning is vital to teaching children without confusion. When children create schemas about shapes, we want those schemas to be correct from the beginning. If they are not, this can cause misunderstanding about what it takes to be a shape.

Imagine the change you can make by sharing this article with at least two teachers or librarians. What if several of us created projects in which students critique the books and contact publishers, authors, illustrators, or educational product companies to explain what is incorrect about their products and demand that they sell accurate information for future students? Imagine a route of geometric learning students could travel without these unnecessary roundabouts.

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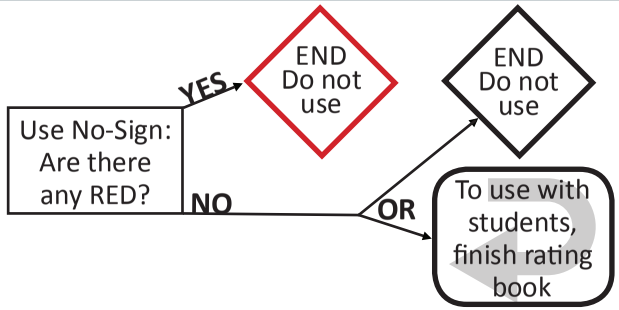
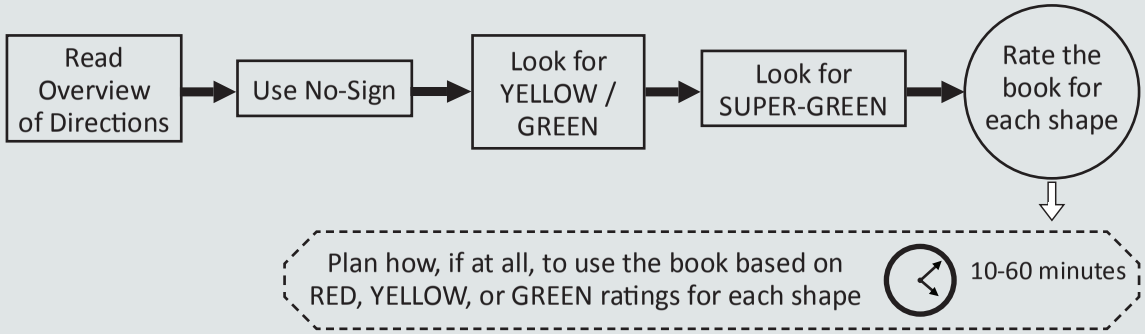
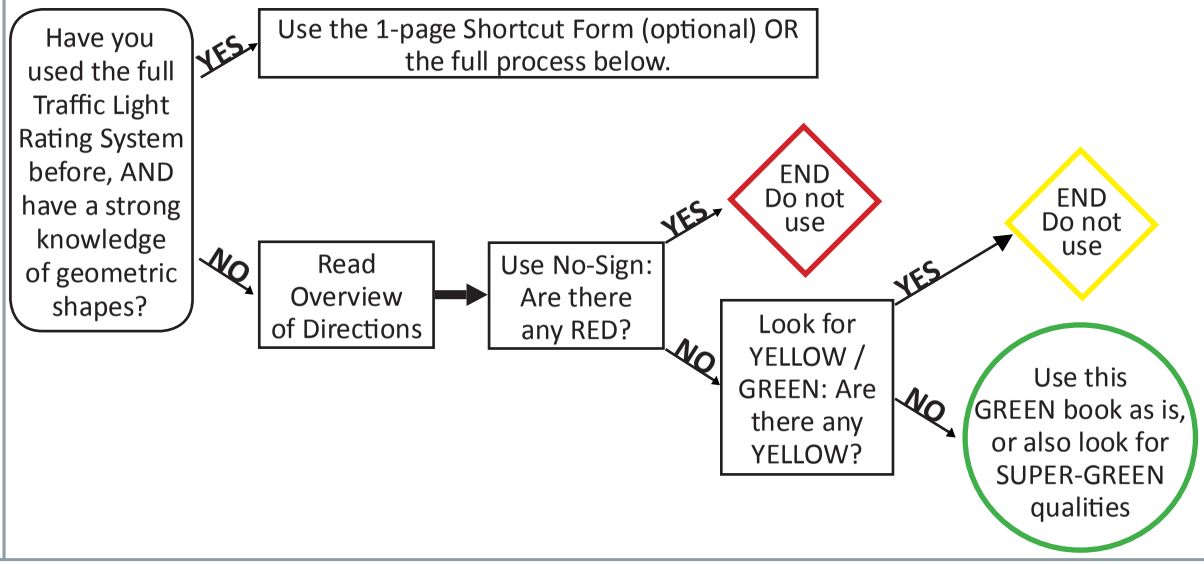
Appendix A

I want to...	What to do	How to do it
Get rid of inaccurate books	Recognize and avoid red lights 2-5 minutes	
Use my books even if they are imperfect	Use the full Traffic Light Rating System to recognize whether the book is RED, YELLOW, or GREEN for each shape 20-30 minutes	
Only use completely accurate books that promote the varied conceptions my students need	Look for GREEN lights (Stop if you see any YELLOW or RED lights.) 2-5 minutes (Shortcut Form) OR 2-20 minutes (full process)	

Appendix B

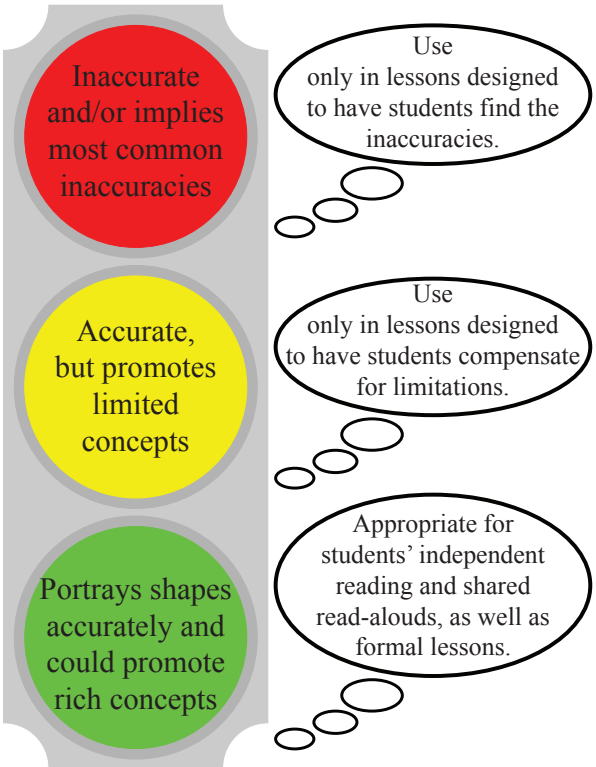
Traffic Light Rating Scale:
How to Evaluate Children's Books about 2-D Shapes

Overview of Directions

Only mark what you find in the book. If you read a characteristic on the checklist, but you don't see it in your book, leave it blank. If you do see it, follow the directions to mark it RED, YELLOW, or GREEN. Remember, the shape only counts as the book teaching the shape if the author uses the geometric shape word/name.

Use red, yellow, and green writing utensils to color-code the quality of each shape. Pens, markers, colored pencils, or highlighters will work.

1. **Look for RED qualities:** Use red on the No-Sign checklist to mark common inaccurate or misleading ideas about 2-D shapes that the book says in words or shows with pictures.
2. **Look for YELLOW and GREEN qualities:** Grab yellow and green to bubble in characteristics you find in the book by following the directions on this 2-page checklist. Please note:
   - Only a few books explain **accurate properties of shapes in words** (GREEN), or have **images** that portray shapes in positive ways (GREEN), so most of the checklist may be blank.
   - Most books only have a few shapes such as triangles, squares, rectangles, and circles. Draw a loop around NA for any shape words/names that are not in the book.
3. **Look for SUPER-GREEN qualities:** You'll rarely find a book that shows non-examples or accurately explains how 2-D and 3-D shapes are related; but if you do, these are SUPER-GREEN qualities!
4. **Rate the quality:** Use the last page to rate the quality of the book as RED, YELLOW or GREEN to decide how well it could teach each shape.
5. **Decide how to use the book:** Use the traffic light guidelines to decide how, if at all, to use the book with students.



Look for RED qualities **Name of Book:** _____

Directions: Use a red pen to check off "Misleading Pictures" or "Inaccurate Properties" that the book said in words.

Misleading Pictures

☐ Labels pictures of real-objects (e.g., box, can, party hat, pizza slice, cookie) or 3-dimensional shapes (cone, cylinder, sphere, rectangular prism) with 2-D shape names (square, rectangle, triangle, circle)

This is wrong, because squares, rectangles, triangles, circles, hexagons, and other polygons can only be 2-D.

☐ Treats a diamond as a geometric shape

This is wrong, because "diamond" should be taught alongside nonmathematical concepts (star, heart, crescent), not with mathematical shapes (square, circle, triangle).

☐ Shows only 1 image for any shape named

☐ Images labeled as triangles are only equilateral or acute-isosceles

These are problems, because if children only see a single example of any shape, or triangles with 3 equal sides or 2 equal sides and all angles less than 90°, these images become the only prototype that children and even many older students think of as triangles or other shapes.

Inaccurate Properties

☐ Says a rectangle has two sides longer than the others

This is wrong, because a rectangle can have all sides the same length (which makes it a square). A rectangle is a quadrilateral (4-sided polygon) with right angles. Rectangle literally means "right-angled."

☐ Says a parallelogram or rhombus has to have slanted sides or can't have right angles

This is wrong, because parallelograms can have right angles, they just don't have to.
If a parallelogram or a rhombus has right angles, then it is also a rectangle.

☐ N-gons (e.g., hexagons, octagons,…) described as having all sides the same length

☐ Other inaccurate properties (describe)

☐ Corners or sides of shapes labeled as triangle, rectangle or other polygon are open or curved (this includes sides that are not a straight line segment)

☐ If N-gons are named (e.g., hexagons, octagons …), the images only show all sides the same length

This is wrong, because polygons must have straight sides (line segments), so images with any round sides are not polygons.

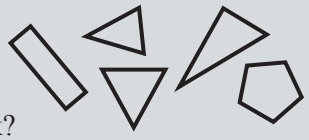
N-gons are defined by the **number of sides**, not the lengths of those sides. All of these images **are hexagons**, because they have 6 sides.

Look for YELLOW and GREEN qualities on these two pages.

Orientation

Most books show images of shapes only in a horizontal orientation. Because of this most children think, for example, that a triangle turned some other way than the "point on top" isn't a triangle. We need to ensure children see a variety of orientations of every shape.

Examples of other orientations that are not horizontal could look like this:



How varied are the orientations of the PICTURES of each shape in this book?

1. If an image of a **square is shown oriented 45° from horizontal** like this: ◇ and the word "square" labels the picture in words, this is a good thing, so use a green marker to fill in the positive bubble. If the book does not do this, then leave this bubble blank: ⊕

2. Write the name of each shape in the column that best describes how the images of that named shape were oriented.

All of the images of the shape are **horizontal** **(rated YELLOW)**	Some of the images of the shape are horizontal and **some** are **not horizontal** **(rated No Color)**	**Most** of the images of the shape are **not horizontal** **(rated GREEN)**

Properties that All Polygons Share

Use a green marker to fill in the positive bubble that applies, if the WORDS in the book

- State the number of sides or number of angles, or ask the reader "How many sides, vertices, corners/angles does the shape have?" Also write the name(s) of the shape(s) below.
- Or invite the reader to discuss the shapes so they might talk about properties (such as number of sides, length of sides, size of angles, concavity, etc.) Also write the name(s) of the shape(s) below.

⊕ Number of sides: _____

⊕ Number of angles: _____

⊕ Number of vertices: _____

All polygons by definition are two-dimensional (2-D) and closed. If the book uses words to tell the reader that a shape is 2-D or "flat," use a green marker to fill in the positive bubble.

⊕ Write the names of the shape(s) that the book said were 2-D or flat: _____

_____

⊕ Write the names of the shape(s) that the book said were closed: _____

_____

Directions:

- Use YELLOW to bubble ⊖ negative characteristics found in the book. If not found, leave blank.
- Use GREEN to bubble ⊕ positive characteristics found in the book. If not found, leave blank.

Named Shape	What other properties of each shape do the WORDS say?	What variety do the PICTURES of each shape show?
Triangle NA		⊖Only Isosceles or Equilateral ⊖Only acute *If not yellow above, use green for all that apply below:* ⊕ Scalene ⊕ Obtuse ⊕ Isosceles ⊕ Right ⊕ Equilateral ⊕ Acute
Square NA	⊖4 sides or 4 angles only property stated ⊖Fails to say right angles necessary ⊕Right angles ⊕Opposite sides parallel ⊕All sides same length	
Rectangle NA	⊖Claims opposite sides same length, but implies not adjacent sides ⊖Fails to say right angles necessary ⊖4 sides or 4 angles only property stated ⊕Right angles ⊕Opposite sides parallel ⊕Opposite sides same length	⊖Only oblong rectangles shown ⊕At least 1 picture of a square shown (along with oblong rectangles) to show a square is a rectangle (does <u>not</u> have to say this in words) ⊕Many sizes of rectangles shown, including long skinny rectangles
Rhombus NA	⊖4 sides or 4 angles only property stated ⊕All sides same length ⊕Opposite sides parallel	⊖Only slanted or non-right angles shown ⊕Square picture labeled as rhombus
Parallelogram NA	⊖4 sides or 4 angles only property stated ⊕Opposite sides parallel ⊕Opposite sides same length	⊖Only slanted or non-right angles shown ⊕Square image labeled as parallelogram ⊕Rectangle image labeled as parallelogram
Trapezoid NA	⊖4 sides or 4 angles only property stated ⊖Says two sides have to be same length ⊕Exactly (or at least) 1 pair parallel sides	⊖Only isosceles trapezoids ⊕Trapezoid with right angle included ⊕More images that are not isosceles (no sides the same length) than isosceles
Kite NA	⊕Two sets of adjacent sides (sides that touch or share a vertex) the same length	
N-gon(s): _____, _____ NA	⊕Avoids saying all sides the same length unless specifying regular polygons compared to irregular polygons	⊖Regular only (or if pentagon, only regular and a shape like baseball home plate) ⊖Convex images only ⊕Irregular polygons ⊕Concave images found
Circle NA	⊕Edge is fixed distance from the center	

⊕If the book said any <u>other accurate properties</u> of shapes, fill this bubble with green and describe here:

Look for SUPER-GREEN qualities
(A book can be SUPER-GREEN even if it does <u>not</u> label shapes with geometric shape names.)

<u>Encourages Reader to Notice Properties or Compose/Decompose Shapes</u>

⊕ The WORDS encourage the reader to discuss, compare, or contrast parts of the shapes in potentially accurate ways. Even if the book does not specifically name properties such as sides or angles, some SUPER-GREEN books encourage the reader to do this thinking.

⊕ The book uses prefixes and/or root words to teach polygon vocabulary and properties (e.g., *triangle* means "3 angles", *rectangle* means "right-angle", *polygon* means "many angles", etc.)

⊕ The WORDS and/or PICTURES show or explain how shapes can be put together or broken apart into other shapes (but DO NOT check this if the book shows two right triangles making a rectangle as the only example in the entire book).

<u>Shows Non-Examples</u>

We learn best by comparing examples with non-examples. **Non-example** means the book shows an <u>image</u> that is <u>not</u> the shape and the <u>words</u> tell the reader something like "this is not a [named shape]" or "find the one that doesn't belong" and so forth. If you find a non-example in the book, use a green marker to fill in the positive bubble below. If not, leave it blank.

⊕ Write the name(s) of the shape(s) for which the book gave a non-example: _____

_____

<u>Compares 2D and 3D Shapes</u>

Does the book try to accurately clarify the relationship between 2D and 3D shapes by doing any of the following?

We rarely find books that do the following positive things. However, if it does, use a green marker to fill in the appropriate bubble. If not, leave it blank.

⊕ The WORDS in the book explain to children that the triangle, square, rectangle, etc. is **part of** the 3D shape or **on** the 3D shape.

[e.g., "I spy a rectangle on the building." Or "I spy a rectangle that makes the front of the building." Or "A hexagonal prism is made of 6 rectangles and 2 hexagons."]

⊕ The WORDS in the book encourage children to find the **2D faces** of 3D shapes.

[e.g., "How many triangles and rectangles can you find on the pyramid?" Or "A hexagonal prism has 6 faces that are rectangles and 2 faces are hexagons."]

⊕ The WORDS in the book encourage children to **trace** the outline of [2D named shape, such as triangle, square, rectangle, etc.] **on** a 3D image.

[e.g., "Trace the outline of a rectangle on the prism with your finger."]

⊕ The WORDS and PICTURES in the book show children how the **footprints** or **outlines** of a 3D object or shape **are 2D named shapes**, such as triangle, square, rectangle, etc.

[e.g., A cone, a square pyramid and a rectangular prism are shown as well as a square and triangles with the words: "Can you guess which shape made these prints?"]

Rate the quality: RED, YELLOW, or GREEN
How well does the book teach each two-dimensional shape?

Name of Book: _____

Directions:

- Look at the No-Sign results again. For each shape below, if it earned any red marks on the No-Sign sheet, then draw a loop around RED for that shape.
- Look at the rest of the checklist. If a book had no RED and no YELLOW, and at least some GREEN, then the quality for that shape is GREEN. Otherwise it should be treated with caution as a YELLOW or RED book.

Quality of Book for Teaching Triangles:

RED **YELLOW** **GREEN / SUPER-GREEN** or **NA**

Quality of Book for Teaching Each Type of Quadrilateral:

- Square **RED** **YELLOW** **GREEN / SUPER-GREEN** or **NA**
- Rectangle **RED** **YELLOW** **GREEN / SUPER-GREEN** or **NA**
- Rhombus **RED** **YELLOW** **GREEN / SUPER-GREEN** or **NA**
- Parallelogram **RED** **YELLOW** **GREEN / SUPER-GREEN** or **NA**
- Trapezoid **RED** **YELLOW** **GREEN / SUPER-GREEN** or **NA**
- Kite **RED** **YELLOW** **GREEN / SUPER-GREEN** or **NA**

Quality of Book for Teaching Each N-gon:

- Polygon or N-gon by name

_____ **RED** **YELLOW** **GREEN / SUPER-GREEN** **or NA**

_____ **RED** **YELLOW** **GREEN / SUPER-GREEN** **or NA**

_____ **RED** **YELLOW** **GREEN / SUPER-GREEN** **or NA**

Quality of Book for Teaching Circles:

RED **YELLOW** **GREEN / SUPER-GREEN** **or NA**

Appendix C

Short-Cut Expert Form to Identify a GREEN/SUPER-GREEN Book about Polygons

Directions: Those who have used the full Traffic Light Rating system many times AND have strong knowledge of geometric properties and classifications (e.g., know which properties are necessary or sufficient) could use this short-cut form to identify a GREEN book. Note that to keep it short, this form has stricter criteria to be GREEN/SUPER-GREEN than the full rating system. **To be a GREEN book, every checkbox in NEVER and MUST below needs to be marked.**

Things the book should NEVER do:

- ❑ NEVER label 3-D pictures (real-life objects or polyhedra) with 2-D shape names
- ❑ NEVER treat "diamond" as a geometric shape term
- ❑ NEVER say or imply that a rectangle or parallelogram has two sides longer than the others
- ❑ NEVER say that an n-gon (i.e., hexagon, octagon, etc.) has all sides the same length or all angles the same size (unless specifying regular polygons compared to irregular polygons)
- ❑ NEVER say that a trapezoid has two sides the same length
- ❑ NEVER say any other inaccurate property. If you find one, list it here:

Things the book MUST do:

- ❑ Show MORE THAN ONE image for any named polygon
- ❑ For EACH named polygon, MOST images are oriented not horizontal
- ❑ The set of images for EACH named polygon MUST give children the opportunity to see that some shapes are not mutually exclusive from each other; however, the book does not have to explain this in words:
    - Triangle images include at least 1 image of each type that you as an adult know to be obtuse- scalene and right-scalene. [The book does not need to label these subtypes, but children need the chance to see these variations labeled with the word "triangle."]
    - Whenever a quadrilateral is named, images of every subclass are shown to help children as well as adolescents understand the valid variations of the shapes. (e.g., the parallelogram label MUST include at least 1 image that is a rectangle, at least 1 image that is also a square, and at least 1 other image that is also a rhombus; similarly, the rectangle label MUST show at least 1 square; and so forth)
    - For any n-gon named, the book MUST show images of concave AND irregular versions
- ❑ State the number of sides OR angles of every polygon (this includes asking the reader to count the sides/angles or asking "how many sides" etc.)
- ❑ Words in the book say defining properties of any polygon:
    - Parallelograms must have opposite parallel sides (or another defining property)
    - Rectangles (and so also squares) must have right angles (or another defining property)
    - Trapezoids must have at least (or exactly) one pair of parallel sides
    - States closed and/or 2-D unless defines the named shape as a type of other polygon (because this means it already includes this defining property)
- ❑ Angles of EVERY polygon image are formed by two straight line segments that meet at a vertex (the vertex does not need to be emphasized by a drawn point, but the corner cannot be rounded). Illustrations should not show wavy sides as though a child drew it.

Things the book SHOULD do (If already GREEN, these qualities could make it SUPER-GREEN):

- ❑ Use prefixes and/or root words to help readers understand meaning of polygon terms
- ❑ State the number of sides AND angles of every polygon (or ask the reader to count the angles or ask "how many corners"). [Too often books focus only on sides]
- ❑ Correctly explain in words how 2-D shapes relate to 3-D shapes (e.g., rectangles *on* the building)
- ❑ Explain or have reader find at least 1 non-example (e.g., find the shape that is not a rectangle, or ask reader to compare/contrast shapes or "Which shape is not like the others?")