Proof Without Words: Sum of the Squares of the First *n* Integers

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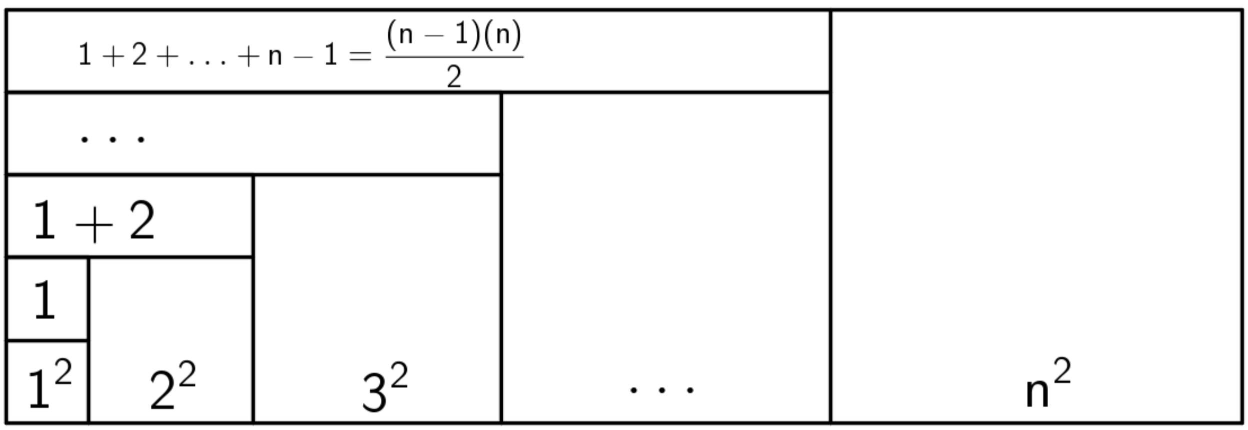

$$AREA = n \cdot \left(\frac{n(n+1)}{2}\right) = \frac{n^3+n^2}{2} \tag{1}$$

$$\begin{aligned} AREA &= \sum_{i=1}^{n} i^2 + \sum_{i=1}^{n-1} \frac{i(i+1)}{2} = \sum_{i=1}^{n} i^2 + \frac{1}{2}\sum_{i}^{n-1} i^2 + \frac{(n-1)n}{4} = \frac{3}{2}\sum_{i}^{n} i^2 - \frac{n^2}{2} + \frac{(n-1)n}{4} \\ &= \frac{3}{2}\sum_{i}^{n} i^2 - \frac{n^2}{4} - \frac{n}{4} \end{aligned} \tag{2}$$

$$(1) = (2) \implies \frac{n^3+n^2}{2} = \frac{3}{2}\sum_{i=1}^{n} i^2 - \frac{n^2}{4} - \frac{n}{4}$$

$$\sum_{i=1}^{n} i^2 = \frac{2}{3}\left(\frac{n^3+n^2}{2} + \frac{n^2}{4} + \frac{n}{4}\right) = \frac{2}{3}\left(\frac{2n^3+3n^2+n}{4}\right) = \frac{n(n+1)(2n+1)}{6}$$

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