Mondrian Math: An Artful Exploration of Area and Perimeter

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Abstract: Playing games in the math classroom is an excellent way for students to develop fluency and practice skills. This paper explains Mondrian Math, a game based on the artwork of Piet Mondrian, which gives students the opportunity to develop fluency with the concepts of area and perimeter. The authors share details of the game, including connections and extensions that integrate English language arts and art standards.

Keywords: Mathematics and art, area, perimeter, connections

1 Introduction

Children love to play games. Providing children opportunities to play games that relate to mathematical ideas gives students a context for learning the concepts (Burton, 2010). In *Principles and Standards for School Mathematics*, the National Council of Teachers of Mathematics (NCTM, 2000) discusses the importance of providing young children with a variety of materials, including games that allow them to explore various mathematical concepts. The game that we describe in this paper was created as part of a series of games which focus on integrating mathematics across content areas. In particular, *Mondrian Math* allows students to actively construct knowledge of the concepts of area and perimeter as they explore the artwork of Piet Mondrian.

2 Art Connection

Before playing *Mondrian Math*, children should have the opportunity to learn about the artist, Piet Mondrian. There are many ways to acquaint children to Mondrian's art—more specifically to his work using squares, rectangles, and primary colors, the basis of this game. Books, such as *Coppernickel Goes Mondrian* (Van Reek, 2012) provide children with a glimpse of Mondrian's work through the eyes of a fictional character. Another way to explore Mondrian's work is to provide students with opportunities to identify geometric concepts within examples of his art (see Figure 1).

The book *Piet Mondrian Masterpieces of Art* (Hodge, 2015) is an excellent resource that provides samples across Mondrian's career while providing thorough background of Mondrian for teachers and students.



Fig. 1: Original artwork inspired by Mondrian.

3 Mondrian Math

After introducing the children to the works of Mondrian, we play the game *Mondrian Math*. We've provided a complete list of game materials, rules, and the scoring sheet on the following page. As previously mentioned, a key feature of *Mondrian Math* is the focus on the relationship between area and perimeter. For students to fully appreciate this relationship, teachers should provide students with opportunities to discover that rectangles with equal area may have different perimeters and, conversely, that rectangles with equal perimeter may have different areas. As an example, the teacher could ask the students to determine what rectangles can be created with an area of 12 units. Once students have created their lists, the teacher can encourage further exploration by asking questions such as "How can you be sure that you have all of the possible rectangles?" and "If all of these rectangles have the same area, does that mean that they also have the same perimeter?" Students should then determine the perimeter of each rectangle with an area of 12 (See Table 1 for possible solutions).

Table 1: Rectangles with Area of 12 square units			
Dimensions	Area	Perimeter	
$\begin{array}{c} 1\times12\\ 12\times1\end{array}$	12	26	
$2 imes 6 \\ 6 imes 2$	12	16	
$3 imes 4 \\ 4 imes 3$	12	14	

Mondrian Math

Materials: The game board (one sheet of 1 cm graph paper), four traditional dice, four crayons (yellow, red, blue, and black), and a score sheet for each player.

Rules:

- This game is for 2-4 players.
- To begin play, each group needs one game board and a score sheet for each person. Each player chooses their color (red, blue, yellow, or black). The group should determine a game goal (least area, greatest area, least perimeter, greatest perimeter).
- Taking turns, each player will roll the dice and create a rectangle that has either that perimeter OR area of the sum of the four dice. All four dice must be rolled on each turn. The player should draw a rectangle on the game board and label it with the number of the round (1, 2, 3, 4, or 5). The player then records the sum of the dice roll, and the area and perimeter of the drawn shape on their score sheet.
- At the end of 5 rounds, the players should sum all the columns on their score sheet. The player that has the game goal after 5 rounds wins.

Area and Perimeter Scoring Sheet

Player Color:	Red	Yellow	Blue	Black
Target Score:	Least Area	Greatest Area	Least Perimeter	Greatest Perimeter

	Rolled Number	Area	Perimeter
Round 1			
Round 2			
Round 3			
Round 4			
Round 5			
Total:			

Once the students finish exploring rectangles with equal area, the teacher could facilitate the same discussion for rectangles with equal perimeter, say 12 units (See Table 2 for possible solutions).

Table 2: Rectangles with Area of 12 square units			
Dimensions	Perimeter	Area	
$\begin{array}{c} 1\times 5\\ 5\times 1\end{array}$	12	5	
2×4 4×2	12	8	
3×3	12	9	

After such discussions, children have the prior knowledge necessary to play *Mondrian Math*. Before the start of the game, players select a game goal (least area, least perimeter, greatest area, or greatest perimeter). Based on the selected goal, players engage in higher order thinking skills to determine which rectangle to create during their turn.

To play, students roll four die and sum the total of the roll. The purpose of the die roll is to generate a number used as the perimeter or area of a rectangle that players ultimately construct on the game board. Individual die values do not suggest dimensions of the shape. Players write the sum of their role in the first column of their scoring sheet. Next, they decide if the total will be used for the rectangle's perimeter or area. Players keep the goal of the game in mind (i.e., least perimeter, least area, greatest perimeter, greatest area) when making this decision. Next, players draw a rectangle corresponding to the roll on 1 cm graph paper. After drawing the shape, players determine the missing measurement (area or perimeter) and record this information to their scoring sheets. The winner of the game is the player whose total score after five rounds is the lowest or highest area or perimeter based on the game goal. Scoring sheets and rectangles from a completed game are provided in Figure 2.



Fig. 2: Completed gameboard and student scoring sheets.

As Figure 2 illustrates, players label the shapes they create with the number of each round. The rolled number should match either the area or the perimeter and correspond to the drawn shape. For instance, the player who created the yellow shapes rolled a 15 on the first round and created a rectangle with an area of 15 square units and a perimeter of 16 units. In this example, the game goal selected was greatest area and after five rounds of play, the red player and the blue player had the greatest area.

4 Student Expectations

Mondrian Math aligns to several Common Core State Standards related to children's understanding of area and perimeter (Table 3). The game provides third graders with opportunities to determine the area of a shape by counting the number of squares on graph paper (3.MD.C.6). Children also have opportunities to consider concepts such as "rectangles with the same perimeter and different areas or with the same area and different perimeters" (3.MD.D.8) as they determine which of several rectangles can be generated on a particular roll. Furthermore, the game provides opportunities for children to explore the connection between area and multiplication (3.MD.C.7). In order to adequately consider the various rectangles that could be created during a game round, players need to be fluent in the various factors (rectangles dimensions) that can generate the same product (area).

Table 3: Th	ird Grade M	easurement &	Data Star	ndards (NGA	A Center &	CCSSO 2010)
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CCSS.MATH.CONTENT .3.MD.C.6	Measure areas by counting unit squares (square cm, square m, square in, square ft, and improvised units).
CCSS.MATH.CONTENT .3.MD.C.7	Relate area to the operations of multiplication and addition.
CCSS.MATH.CONTENT .3.MD.D.8	Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length, and exhibit- ing rectangles with the same perimeter and different areas or with the same area and different perimeters.

The gameboard and score sheet can be used by the teacher to assess student proficency calculating area and perimeter of rectangles. Teachers can also use the game score sheet as an informal assessment of student learning or create a rubric to assign a grade.

5 Discovering Initial Game Limitations

When children first play *Mondrian Math*, we encourage them to explore the goal of constructing rectangles with the least perimeter. This allows students to focus on one concept and begin to develop a strategy for this goal. As children become experienced with the game, we encourage them to vary the game goal with each round of play. After playing the game several times, children recognize that the only possible play with an odd sum is drawing a rectangle with the sum as its area. This is because a rectangle created with whole number units cannot have an odd perimeter. This constraint may generate classroom discussion of possible game modifications. For instance, students may wish to give players the option of creating a shape with either the area or perimeter based on the sum of the die roll on each turn. Considering alternative rules provides children with

opportunities to engage in a number of Standards for Mathematical Practice—namely, (a) To reason abstractly and quantitatively, (b) To construct viable arguments and critical the reasoning of others, (c) To look for and make use of structure, and (d) To look for and express regularity in repeated reasoning (NGA Center & CCSO, 2010). A simple modification that resolves issues associated with odd rolls is to allow the player the option of adding or subtracting one from the sum of the die roll. This adds another layer of strategy to the game as a player then needs to consider how adding or subtracting one impacts progress towards the game goal.

6 Extensions and Additional Activity Ideas

A key element in the design of *Mondrian Math* was to consider how teachers could connect the work of Mondrian across disciplines. We have identified extensions of the game for mathematics, art, and English language arts.

6.1 Extending the Mathematics

One extension activity asks children to calculate the area and perimeter of the portions of the game board that are not covered. This activity provides children with additional opportunities to determine area and perimeter while exploring the concept of area decomposition—children divide composite figures into rectangles in order to calculate area (NGA Center & CCSO, 2010). Another extension activity uses the book *Mathterpieces* by Greg Tang. In this book, readers explore artists such as Monet, Seurat, and Mondrian. Tang includes works from each artist and selects an element for a math challenge. For example, Tang challenges readers to use squares from Mondrian's work for the following number puzzle: "...7's made from SQUARES are great, the different groupings number EIGHT!" (Tang, 2003, p. 20). Children could be challenged to create similar puzzles. While the puzzles in *Matherpieces* focus on combinations of squares as addends for a target sum, an alternative connection between the game and the extension would be to stress the connections between multiplication and area. With this emphasis, the children could create puzzles based on combinations of various factors to create a target product.

6.2 Reinforcing Connections between Mathematics and Art

Collaborating with art teachers provide ways for children to use their game boards to extend their knowledge of Mondrian's work. The interested reader is encouraged to refer to the *National Coalition for Core Arts Standards* (2014) for specific standards related to these ideas. For example, children could take a completed game board and use it as the basis for creating a finished piece of art in the style of Mondrian. Using examples of Mondrian's work as a guide, children could add the black lines that Mondrian used to connect the shapes in art pieces.

6.3 Language Arts Connections

In addition to mathematical extensions, *Mondrian Math* provides children with opportunities to connect their mathematical thinking with key ideas in English Language Arts (NGA Center & CCSSO, 2010). After children finish playing the game and completing their art piece, the class could create an art gallery of their work. As part of this activity, children could write an informational text describing their artwork focusing on the area and perimeter of shapes as they use academic language in a mathematical context (NGA Center & CCSSO, 2010). Children could be challenged to write descriptions with enough mathematical detail that others could identify their work from the text alone. For instance, children might include perimeters of the rectangles or total area of the rectangles in their written descriptions.

7 Conclusion

Introducing *Mondrian Math* to children provides them with a context for learning about perimeter and area while giving them time to consider the relationships between these mathematical concepts. Van De Walle, Karp, and Bay-Williams (2016) write, "Learning mathematics involves developing rich connections between mathematical ideas." While playing this game, children will have many opportunities to discuss their emerging understanding of these concepts as they strategize how to make the best use of the roll to achieve the goal of the game. This type of activity helps them to see these connections in action and provides a fun context to learn these complex ideas.

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