Taking Teaching out of the Equation with Technology

将教学从使用科技的思维定式中解放出来

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Abstract: *In this creative non-fiction essay, the authors share experiences from a morning visit with preservice mathematics teachers at a local high school. Through a series of vignettes, the authors share positive and negative encounters with technology as they pose rich questions to students (and their teacher) in a remedial algebra classroom and observe the use of classroom management software (CMS) with the same students.*

摘要: 在这个创新型非小说散文里，作者分享了一段访问当地高中观摩实习数学教师补救代数课堂上的经历。通过展示一系列对学生（和他们的老师）提问并观察学生使用课堂管理软件的场景，作者指出科技（在课堂教学中）起到的积极并且消极的作用。

Keywords: *technology, discourse, problem solving, critical pedagogy*

I

Todd Room 715? Here it is! We've found it.
Zheng Do we just go inside?
Todd Yeah. It's fine. She emailed me yesterday. She's expecting us.

With that, Dr. Zheng Yang—a research mathematician with a specialty in Abstract Algebra—and I—a methods instructor in the Department of Teacher Education—entered Ms. Smith's class-

Todd 715 房间吗？在这儿呢，我们找到了！
Zheng 我们进去吗？
Todd 对呀。她昨天发给我邮件了。她知道我们会来的。

杨铮博士，一位专注抽象代数的数学家，和我，一个来自教师教育系指导教学方法的老师，就这样走进了 Smith 女士的教室。我的那些学生们，即将上岗的中学数学老师，正

room. My students, preservice secondary mathematics teachers, were in the midst of a two-week field experience at Hillsdale High School—a short drive from campus. Zheng was there to gain a better understanding of his students' educational backgrounds—how mathematics is taught in American schools; I was there to check in with my students.

在这所学校开展为期两周的观摩实习，所在学校为 Hillsdale 高中，距迈阿密大学 20 分钟车程。铮是为了更好的，更深入的了解这里学生的学习情况，当然，在他看来，这完全是必要的，即使作为一个在大学教课的老师。铮非常好奇美国数学课堂教学是怎样进行的。对我来说，我必须要看看我的那些学生是怎么教书的了。

Although Zheng and I were initially concerned that our visit might be disruptive—after all, we reached the classroom nearly 15 minutes into first period—our fears were allayed as Ms. Smith greeted us from the back of the classroom. Two teaching candidates—Taylor and Natalie—were standing in a corner on the other side of the room next to a large air conditioning unit. Meanwhile, fifteen teenagers sat motionless—earbuds firmly in place, clicking on boxes to answer multiple-choice mathematics questions on their Chromebooks, faces blue from the faint flicker of lcd screens.

起初我们担心十五分钟迟到会打扰到课堂秩序，但是 Smith 的热情招呼打消了我们的各种焦虑。我的两个学生，Taylor 和 Natalie，站在教室一侧角落的大空调旁边。于此同时，15 个年轻的小朋友安静的坐在那里，戴着耳机，敲击着键盘，盯着笔记本电脑上的选择题，脸上反射出电脑屏幕上的蓝光。

Todd Hello. Thanks so much for taking on our teacher candidates. This is my friend, Zheng. He teaches math at Miami. This is his first visit to an American high school.

Todd 你好。非常感谢你对我们的实习老师进行指导！这位是我的伙伴，铮。他在迈阿密大学教数学。这可是他第一次来访美国的高中。

Zheng Good to meet you, Ms. Smith.

Zheng 很高兴见到你，Smith 女士。

Ms. Smith No problem. The interns have been great. They do a good job helping students with their computer work. It's so good to have some extra hands around here.

Ms. Smith 没问题，这两个实习生表现得都很好。他们帮助孩子们用电脑做功课。他们的工作也带给我们很多方便。

Todd Absolutely. I can only imagine!

Todd 毋庸置疑！

Zheng The computer setup is very interesting, Ms. Smith.

Zheng 这电脑上的东西看上去挺有意思的，Smith 女士。

Ms. Smith Truly, it is. It took some getting used to, but the kids like it. They work at their own pace, and they get instant feedback from the MathXL program. It's really changed how I do business. That was the hardest part. Instead of working through examples and talking with students from the front board, I monitor the students' progress from my computer. When I notice that kids are stuck, I'll go over and help them. I'm more like a coach now. It's more one-on-one. No more planning really, and no more worksheets.

Ms. Smith 确实是的。孩子们很喜欢，尽管他们要花些时间去了解。他们按照自己的节奏学习，而且可以通过 MathXL（一种数学学习软件）得到及时的反馈。这实在改变了我的教学方式，这也是最难的部分。以前都是在白板上做演示和讲解例题，现在取而代之的是我可以在一台电脑上监控学生的学习进展。当我看到某个学生有困惑，我可以及时的发现然后给予援助。我现在更像是他们的教练，对他们更多的是一对一的辅导。再也没有过多的课前准备和练习纸张了。

Todd I see . . . technology really *has* changed how we teach.

Todd 原来这样啊。科技确实改变了我们教学的方式。

Zheng Do you mind if we talk to students?
Ms. Smith Not at all. Go ahead.
Todd Hey, before we do, just out of curiosity ... what do your students know about exponents? Could we ask them about that?
Ms. Smith Go ahead, but don't expect too much. They haven't studied exponents since their Algebra 2 class—last year. We've been focusing on linear relationships — not exponents.

Zheng 您介意我们走走转转和学生聊聊吗？
Ms. Smith 当然不了，请便！
Todd 嘿，在我们去之前，只是出于好奇，我想知道，这些孩子了解指数吗？我们可以问他们有关指数的问题吗？
Ms. Smith 可以，但是不要有太高的期望。他们自从去年修完代数 2 这门课之后再没有学习过指数。我们的重点一直在关注线性关系，没有指数之类的。

II

As we drove to the school, Zheng and I agreed upon two essential questions to frame our visit—namely, *What mathematics content do secondary school students encounter in their classes?* and *How do they encounter this content?* Although both of us are concerned about mathematics content *and* pedagogy, these initial questions reflected our differing professional emphases and roles. As a methods instructor, I'm attuned to the culture of the classroom, student and teacher discourse, and applications of technology in instruction. Zheng, on the other hand, tends to focus on mathematics content that students encounter and how it relates to the university curriculum. Our work and our perspectives necessarily overlap but are not the same. As such, our classroom observations and conversations often lead to new insights that are intellectually nourishing for both of us.

II

在我们开车去学校的路上，铮和我定下了两个关键问题。*我们此行主要调查中学数学课堂内容，以及，这些数学内容是如何被教授的*。尽管我们俩都关心课堂内容和教学方法，但是这些最初的问题体现了我们不同的职业侧重点。作为一个教授教学方法的老师，我更看重的是课堂文化，师生对话，以及科技在教学中的应用。铮，从另一方面，更注重学生学习的数学内容以及这些内容与大学课程的联系。所以我们的工作和观点很可能有交集但却是不一样的。我们的课堂观察以及之后的讨论过程常常会产生有价值的新的见解。

III

After initial introductions and a quick debrief in the back of the room with the teacher candidates, Zheng and I walked around the room and spoke quietly to students. Other than us, the room was silent save the sound of music leaking out of small white earbuds.
Todd Psstt. Hey, can I ask you a question?
Carlos Sure. Whatever.
Todd What do you think about this class?
Carlos It's okay. It's pretty easy.
Todd What about exponents?
Carlos Huh?
Todd I've got a problem for you.

I walked over to the whiteboard near Carlos's desk. In a small corner of the board, just large enough for him to see, I wrote the following problem in

III

在简单地自我介绍和快速的和教室后的实习老师交流过后我和铮就开始在教室里巡视并且开始试着和学生们轻声的交谈。教师里很安静，几乎可以听到那些从学生们小小的白色耳机里渗透出的音乐。

Todd 噗哧，嗨，能问你个问题吗？
Carlos 当然，无所谓了。
Todd 你感觉这堂课上的怎么样？
Carlos 还好吧，挺简单的。
Todd 那指数呢？
Carlos 啊？
Todd 我有个问题问你。

我走到 Carlos 桌旁的白板。那是个小角落，刚好能够他看到，我用一只红色的马克笔写下了：$x^x = 4$。几乎同时，这个学生冒出了答

red, dry erase marker. $x^x = 4$. Almost instantly, Carlos blurted out, "Two!" His voice was loud enough to catch the attention of several classmates.

TODD Why's that?
CARLOS Well, 2 to the second is 4. You know, 2 times 2 is 4.
TODD Okay. Okay. Lucky guess! How about this one? $x^x = 27$.

Intrigued by the discussion in the otherwise silent classroom, Carlos's classmate, Tina, chimed in.

TINA Three!
TODD Aha! Hey! Nice work. Why'd you say three? How do you know your right?
TINA Well, three to the third, 3 times 3 times 3, is 27.

I looked over at Zheng and smiled. *These kids know exponents.*

TODD Okay, we solved the equation for 4 and 27. What about 1? Can the equation equal 1?

TINA That's even easier! One!
ZHENG Is that the only answer?

Silence. At this point, roughly half the class was in on the conversation. We walked over to Ms. Smith's desk. She seemed uneasy.

TODD What do you think about this?
MS. SMITH Well, I just don't know. The kids don't usually see equations like these, and it's not in the Algebra 3 curriculum.

IV

An hour before, Zheng and I sat in the Hillsdale High School parking lot in my silver Toyota Sienna. We had arrived at the school a good half an hour before the first bell—7:35 a.m. to be precise. My eyes were tired from the grading I'd done the night before. Strapped in securely with seatbelts, two men from different worlds—a bald mathematics professor from China and a grey-bearded methods instructor from Southwest Ohio—sat in the parking lot and drank coffee, laughed, and shared recollections of their own high school experiences. I listened carefully as Zheng spoke of China.

案："二！"。他的嗓门之高引起了几个其他学生的注意。

TODD 为什么？
CARLOS 2 的 2 次方是 4，你知道的，2 乘 2 也是 4。
TODD 好。算你猜对了。那这个呢？$x^x = 27$.

这个学生的邻桌，也被我们的讨论吸引了过来，凑了上来。

TINA 3 啊！
TODD 啊哈！漂亮！为啥你说是 3？你怎么知道你是对的呢？
TINA 这个嘛，3 的 3 次方是，3 乘以 3 再乘以 3，是 27。

我看了铮并微笑了一下：*这些孩子是懂得指数的。*

TODD 好的，我们会解这两个方程。那么如果是换成 1 呢？这个等式等于 1 怎么办？
TINA 那更简单啊，1 啊！
ZHENG 只有这一个答案吗？

一片安静。此刻，大约一半的学生已经在聆听我们关于指数方程的对话了。我们走到 Smith 的办公桌旁，她看上去似乎有些不安。

TODD 你对这有什么看法呢？
MS. SMITH 我不知道啊。孩子们一般都没见过这种方程，这也不是代数 3 的教学内容。

IV

一个小时前学校门前的停车场：我和铮坐在银色的丰田西耶那（丰田的一款车）。我们是 7：35 到达，据第一次上课铃打响还有整整半个小时。我的眼睛由于昨晚长时间的批改作业而感到了十分困乏。系着安全带的我们，两个来自不同世界的男人———一个是来自中国的秃顶的数学系教授，一个是来自俄亥俄西南部的满是灰胡子的方法论老师———喝着咖啡，谈笑风生。分享者各自的高中经历。我被铮眼中的中国所深深吸引。

ZHENG The exams were tough. Very tough. We took a test at the end of the senior year. Everything depended on it. The questions were tricky. Very tricky. And, you only got a very limited time to finish them. No one got extra time.

TODD I've heard about those.

ZHENG Yes. Where you go to university depends on those tests. And they list the results on a big wall. Everyone in my school knew how I did. I did not do as well as I wanted. My parents wanted me to study with a tutor to get ready for the test, but I said 'no.' I was ashamed to have a tutor. Now I think I should have had one.

TODD I can't believe that you struggled. You have a Ph.D. in mathematics.

ZHENG I did struggle.

TODD The whole testing thing reminds me of my visit to Asia a few summers ago. My wife and I rode the subway in Seoul and saw kids coming back from night school—test preparation classes—at 10:00, 11:00 o'clock at night. They were still wearing school uniforms. Carrying backpacks. Texting on their giant Samsung phones.

ZHENG Yes. It is similar in China.

TODD At the time, the mathematics educators I met from Korea, Hong Kong, China ... they all wanted to know the same thing. *How can we promote creativity in our classrooms?* And all of the educators visiting from the United States were asking the opposite thing of the Asian teachers—*What's the best way to implement a national curriculum and national assessments?* The two groups—the Asians and the Americans—were like two ships passing in the night.

ZHENG 中国的考试是很难的，非常难。我们在高中的最后一年要参加一个考试（高考）。考试的成绩决定一切。考试的问题都很刁钻，非常难。，你只有固定的时间去完成，没有任何多余的时间。

TODD 我也曾听过些。

ZHENG 是啊。高考决定了你会去什么样的大学。一般来说, 你的成绩也会被公布出来，每个在学校的人都会知道你考得怎么样..... 唉, 我那时学习会遇到很多困难, 我父母也曾劝我找个家教, 但是我觉得很难为情, 所以就破罐子破摔了。现在回想起来, 那时还是应该找个家教的。

TODD 简直难以置信。你居然还读了博士。

ZHENG 是，曾经读书非常吃力。

TODD 这类考试让我想起几个夏天前的亚洲之行。那时我和我的夫人在乘列车去往首尔的路上看到路上从夜校（为准备考试而开展的课程）下课的孩子, 那时都已经夜里十，十一点左右了。他们还穿着校服, 背着书包, 用手里大大的三星手机发着信息。

ZHENG 是啊, 中国也是类似的情景。

TODD 那时我见过的从韩国, 香港和中国大陆来的数学教育学家们都在关注着同样的问题: *到底如何激发学生的创造力?* 与此同时几乎所有的美国教育家们却关注着相反的一面: *什么是最好的方式去实行国际化课程设置和国际化评估方法?* 亚洲的和美国的教育界人士, 这两个群体像是两艘大船从彼此身边悄然驶过, 却不解各自的难处。

V

MS. SMITH I think 0 is another answer, right? Anything raised to the 0 power is 1, right? So, $0^0 = 1$.

ZHENG Interesting ...

MS. SMITH No, no, no. Wait a minute. Zero to the zero is undefined.

TODD Okay ...

MS. SMITH Hang on. I want to type this into my calculator.

V

MS. SMITH 我觉得 0 也可以是答案，对吗？因为任何数的 0 次方都是 1，对吗？所以 0 的 0 次方也是 1。

ZHENG 有意思。

MS. SMITH 不，不，不对，等等，0 的 0 次方是没有定义的。

TODD 好吧。

MS. SMITH 稍等，我想先用计算器试试。

With that, Ms. Smith pulled a graphing calculator out of her desk drawer. Hesitantly, she keyed in 0^0 on the device's liquid crystal screen, holding the calculator up high enough so that we could watch as she typed. As she pressed ENTER, we were greeted with screens similar to those shown in Figure 1.

说着，Smith 从她的书桌抽屉里拿出一个可绘图计算器。她按下了 0 的 0 次方在那黑白的液晶屏上，把计算器举得足够高以至于能让在场所有人看到她所输入的式子。当她按下 ENTER（输入）键的时候，我们看到了类似图 1 所显示的内容：

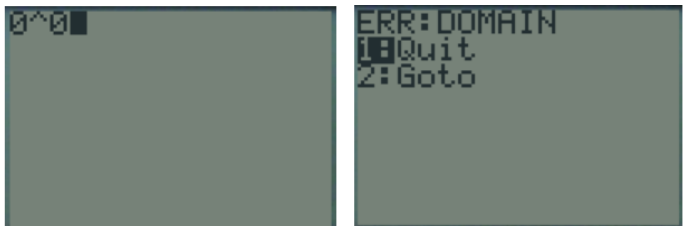

Fig. 1: Calculator suggests that 0 raised to the 0 power is not equal to 1. 图 1：计算器所给出的 0 的 0 次方的结果。

Ms. Smith breathed a sigh of relief, then exclaimed happily "I knew it! I knew it! 1 is the only answer. One to the one equals one, but zero to the zero doesn't work. It's undefined."

Smith 长舒了口气。然后高兴的说"我就知道结果是这样！1 是仅有的答案。1 的 1 次方是 1，但是 0 的 0 次方就不行，因为没有意义。"

Todd That's really weird. That says domain error, doesn't it?

Ms. Smith It does. It does!

Todd But wait a minute. Let's Google 0^0. What does Google say?

Todd 太奇怪了！这上面说定义域错误，真的吗？

Ms. Smith 是呀，是呀！

Todd 等一下，等一下，让我们在 Google(谷歌搜索引擎) 上搜索一下 0 的 0 次方，看看 Google 怎么说。

Abruptly, Ms. Smith stopped smiling. Her eyes seemed to ask, "What are these two professors up to, anyway?" Dutifully, she typed 0^0 into her search bar and hit the ENTER key on her laptop. The results are illustrated in Figure 2.

她突然不笑了，她的眼神似乎告诉我们"这两个教授到底是为了什么？"她按部就班式地在她的搜索窗口输入了 0 的 0 次方，然后敲下了 Enter （回车）键。结果如图 2 所示：

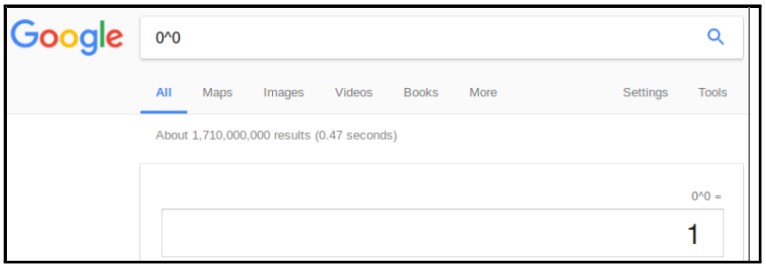

Fig. 2: Google suggests that 0 raised to the 0 power equals 1. 图 2：Google(谷歌) 所给出的 0 的 0 次方的答案是 1。

Other popular search engines—Yahoo, Duck Duck Go, Bing—all returned the same result, namely that 0 raised to the 0 power equals 1.

其他的类似的工具, Yahoo, Duck Duck Go, Bing (其他搜索器的名称), 都统统给出了同样的结果：0 的 0 次方是 1。

VI

Ms. Smith sat in her seat, motionless as the bell rang, dismissing her students for second period. *Who should I believe? Google or Texas Instruments? What does "domain error" mean anyway? That never was a satisfying answer to me. What should I do? The professors don't seem sure about this either. Why aren't they upset?*

Kids shut their laptops, removed earbuds, gathered their book bags, and shuffled off to the next class. Meanwhile, the two preservice teachers, Ms. Smith, Zheng, and I tried to make sense of what had just happened.

Todd What do you two think about all of this?

Taylor Dr. Edwards, I was really surprised that Tina knew the meaning of three raised to the third power.

Natalie Me, too.

Todd But what about that last part? What about 0 to the 0? Do you think it's 1? Or something else?

Zheng Well, in calculus, we looked at this problem using logarithms. Here, I'll write it on the board.

Zheng walks over to the whiteboard.

Zheng Say you start with an equation $x^x = a$. In our earlier examples, using this equation, what had we set a equal to?

Taylor First we let $a = 4$, right?

Natalie Then 27. And finally, in our last example, a equaled 1.

Todd Right! Exactly.

Zheng Note that you can take the natural log of both sides of an equation like $x^x = a$ to solve for x.

Todd Huh?

Zheng Look at $\ln(x^x) = \ln(a)$.

Ms. Smith Yes, yes. I see. We can bring the power of x out front. Kids learn this method in Algebra 2.

Zheng So you get $x \cdot \ln(x) = \ln(a)$. Divide each side by x. We get $\ln(x) = \frac{\ln(a)}{x}$. Are you following me? Is this clear?

Natalie Crystal clear. Really. Kids do this sort

VI

当铃声响起时，Ms. Smith 坐在她的座位上，也没有起身，她与第一节课的学生告别后，准备迎接第二节课的学生。*到底我该相信谁？Google(谷歌) 还是 Texas Instruments(一种美国学生常用的计算器品牌)？"定义域错误"到底什么意思？这绝不是一个我满意的答案。这两个教授看起来也不知道问题的答案，为什么他们不似我这般焦虑。*

孩子们合上电脑，摘下耳机，收起书包，匆匆赶往下一堂课。与此同时，两个实习老师，Ms.Smith，铮，和我还沉浸在刚刚所发生的一切当中。

Todd 你们俩是怎么想的？

Taylor 爱德华博士，我真的觉得不可思议，Tina 居然懂得 3 的 3 次方的含义。

Natalie 我也有同感。

Todd 可是最后那个 0 的 0 次方怎么解释？你认为是 1 还是别的答案？

Zheng 其实我们在微积分里是用对数来解释的。你看，我还是写在黑板上：

铮走向白板前。

Zheng 假定我们给定方程 $x^x = a$。那么之前的例子，a 的取值应该是什么呢？

Taylor 先让 $a = 4$, 对吗？

Natalie 然后第二个例子我们的 a 是 27。最后一个例子取的是 1。

Todd 完全正确。

Zheng 你可以对像 $x^x = a$ 的等式两边取自然对数。

Todd 什么？

Zheng 这样我们得到 $\ln(x^x) = \ln(a)$.

Ms. Smith 对啊。我明白了。我们可以把 x 次方放在前面。孩子们在代数 2 里学过的。

Zheng 所以我们得到 $x\ln(x) = \ln(a)$。等式两边同时除以 x, 就有 $\ln(x) = \frac{\ln(a)}{x}$。你能理解我的意思吗？我表达得还清楚吗？

Natalie 非常清楚。真的。孩子们虽然对处

of equation solving all the time, although they get tripped up by logs sometimes.

TAYLOR I KNOW! It's the name ... logs ... why are we studying trees in math class?

TODD Heh, heh ...

ZHENG Where would you go from here? That doesn't look like an equation kids could solve. There are two x's and no clear way to combine them.

MS. SMITH I know ... let's graph the functions $y = \ln(x)$ and $y = \frac{\ln(a)}{x}$ to see where they meet.

Entering the equations in Desmos, a freely available on-line graphing tool, yields a graph similar to that depicted in Figure 3.

理对数不是那么娴熟，但是他们已经解过很多次这种方程了。

TAYLOR 是啊！这个名字取得... log（在英文中也作木头的意思）... 为什么我们在数学中要学树木呢？（开玩笑）

TODD 呵呵...

ZHENG 接下来该怎么做？这不像是孩子们能解的方程，这其中有两个 x，他们可能会遇到困难把他们结合起来。

MS. SMITH 是呀... 让我们来画函数 $y = \ln(x)$ 和 $y = \frac{\ln(a)}{x}$ 的图像，然后找出它们的交点。

我们可以借助网上免费的画图的工具,Desmos 如图 3 所示。

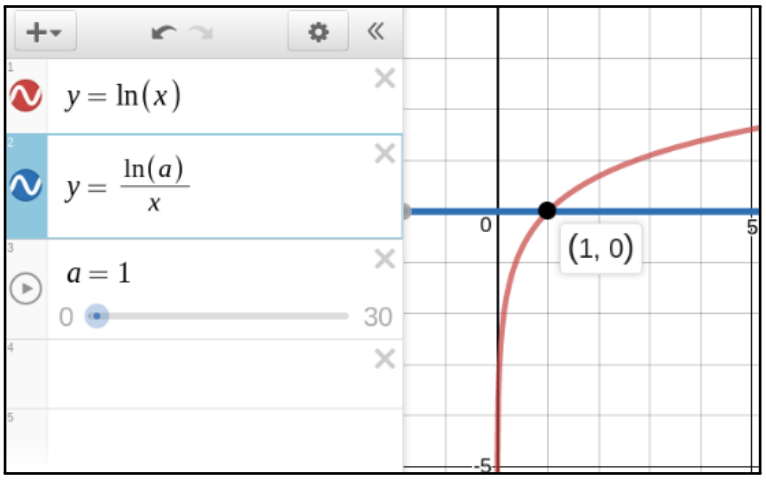

Fig. 3: Solution to the equation $x^x = a$ for $a = 1$ solved graphically with Desmos online grapher.
图 3. 用 Desmos 线上绘图工具得到的当 $a = 1$ 时方程 $x^x = a$ 的解

MS. SMITH So when the slider is set to 1, we're solving the equation $\ln(x) = \frac{\ln(1)}{x}$ which is equivalent to $x^x = 1$?

ZHENG Exactly.

MS. SMITH Look! The two graphs represent the two sides of the equation. They intersect at $x = 1$. That's the answer to the equation.

TODD What do you think will happen if we drag the slider to $a = 27$?

MS. SMITH Since 3 is the solution to $x^x = 27$, the graphs should intersect at $x = 3$, right?

ZHENG Exactly!

TODD Look, you're right!

MS. SMITH 当滑动条的值为 1 时，我们在求解方程 $\ln(x) = \frac{\ln(1)}{x}$ 时，相当于求解方程 $x^x = 1$。

ZHENG 完全正确。

MS. SMITH 你看，这两个图像代表了方程两边，它们在 $x = 1$ 相交。这样就得到了方程的解。

TODD 如果我们挪动滑动条使 $a = 27$ 会发生什么呢？

MS. SMITH 因为 3 是 $x^x = 27$ 的解，所以图像应该在 $x = 3$ 处相交，对吗？

ZHENG 对！

TODD 看，你是对的！

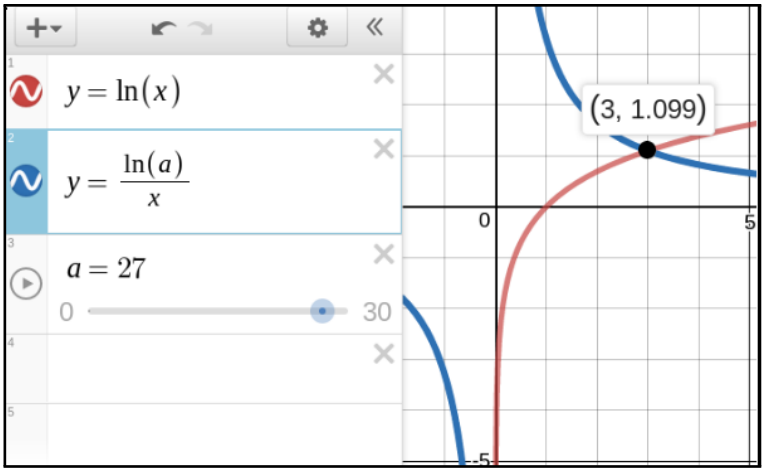

Fig. 4: Solution to the equation $x^x = a$ for $a = 27$ solved graphically with Desmos online grapher.
图 4. 用 Desmos 线上绘图工具得到的当 $a = 27$ 时方程 $x^x = a$ 的解

TODD So, what is my next question? What should I ask next?

TAYLOR Let's follow the same progression as we did with the students. How about what happens when $x = 0$?

Ms. Smith looked worried again. *Will the graph help us see if $0^0 = 1$ or not? Or will the graphs only confuse the issue more?*

TODD Let's move the slider back down to 0 slowly. What happens to the point where the graphs intersect?

Together, we explored a series of graphs similar to those shown in Figure 5.

TODD 那么，我下一个问题是什么呢？我应该问什么呢？

TAYLOR 我们应该沿着我们的思路和之前我们的例子，当 $x = 0$ 是什么情况？

Smith 看上去又有些茫然了。*可是图像会显示 $0^0 = 1$ 吗？还是得出的图像会让整个问题变得更困惑？*

TODD 让我们把滑动条慢慢地挪到 0。与此同时，图像的交点有什么相应的变化呢？

我们一起探索了一连串的图像。如图 5 所示。

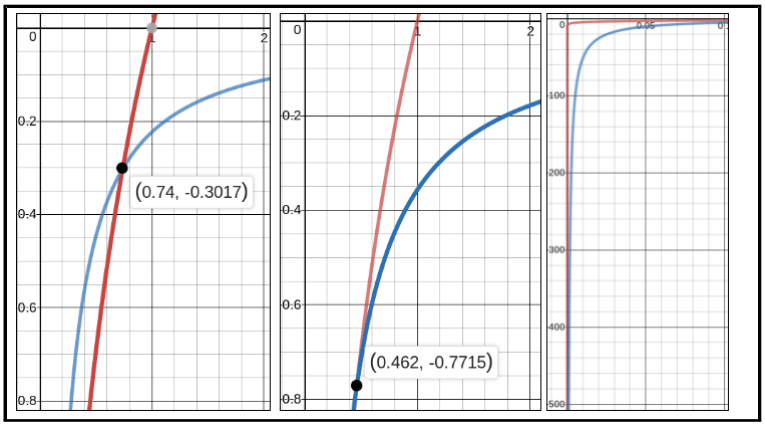

Fig. 5: As the value of a gets closer to 0, the x-coordinate of the intersection gets closer to 0. 图 5. 当 a 趋近于 0 时，交点的 x 取值也趋近于 0

NATALIE The x-value, the solution, is getting closer and closer to 0, and the point is getting more negative. Look at when $a = 0.6$! Geesh! It's difficult to figure out where the intersection even is.

TODD What happens when a equals 0?

MS. SMITH The graph disappears! That's because we're dividing by 0 at that point. That's why the calculator said domain error!

ZHENG Exactly!

MS. SMITH This is the first time I've really understood this. The graph really helped!

TODD Why don't the students look at some of their algebra problems graphically?

NATALIE x 的值，也就是我们的解，不断的趋近于 0。而且图中交点的位置也越来越向负半轴移动。你看，当 $a = 0.6$ ！天呐！已经很难看到交点的准确位置了！

TODD 那 a 等于 0 的时候到底会发生什么呢？

MS. SMITH 图已经消失了！那是因为我们试图除以 0。也就是为什么计算器先前告诉我们定义域出错了！

ZHENG 正是如此！

MS. SMITH 这是我第一次真正的理解了这个题目，图像的作用不可小嘘！

TODD 为什么学生们不借助图像来思考代数的问题呢？

VII

Soon, a second bell sounded and a new group of students shuffled into Ms. Smith's classroom. Although our conversation about 0^0 was certainly interesting, it was fleeting. As a new group of students shuffled into the classroom, Ms. Smith clicked a few buttons within the CMS to load the profiles of her next class. There was no time to pause or reflect on our earlier conversation. A myriad of housekeeping issues required her immediate attention: collecting Chromebooks, taking attendance, loading software, reminding students of the quarterly assessment next Monday. Moreover, Zheng and I had other teacher candidates to meet. We waved goodbye to Ms. Smith and looked on the list that our university's placement office had provided to determine what classroom to visit next.

VII

很快，第二堂课的上课铃响起，下一批学生涌入了 Ms. Smith 的教室。我们关于 0 的 0 次方的讨论虽然很有趣，但我们的讨论已随时间而去。随着学生们的到来，Ms.Smith 在课堂管理软件中点了几个按钮，调出新一批学生的信息，为即将开始的一堂课做着准备。我们没有任何闲暇去反思我们刚才的讨论。无数的闲杂的事物需要现在即刻处理：收集 Chromebooks (一种笔记本电脑)，记录考勤情况，运行软件，提醒学生们下周一要进行阶段性测验。更不用提我和铮还有去拜访其他的实习老师。于是我们告别了 Ms. Smith。根据学校学生实习办公室所提供的安排表格去决定我们下一个要访问哪间教室。

ZHENG What was that last class again? Algebra?

TODD Algebra 3.

ZHENG Three years of algebra in high school? Is that typical?

TODD In some places it is. Back when I was in school, we took two years of algebra—Algebra 1 in 8th grade, Geometry in 9th grade, Algebra 2 in 10th grade. Then onto precalculus and beyond. Trust me, two years was enough!

ZHENG So who takes Algebra 3?

TODD Basically, the kids that struggled in Alge-

ZHENG 刚才那堂课是什么？代数吗？

TODD 代数 3。

ZHENG 高中有 3 年的代数课吗？普遍都是这样吗？

TODD 有些地方是。我以前上学的时候，我们上两年的代数课———8 年级上代数 1 ，9 年级上几何，10 年级上代数 2 。然后开始初级微积分的课程和更深入的学习。相信我，2 年已经足够了！

ZHENG 那么代数 3 是面向哪些学生呢？

TODD 基本上，是面向那些代数 2 有困难的

bra 2.

ZHENG Why do that? Why have 3 years of the same topic—one that students struggle with and seem not to enjoy?

TODD Ha! That's a good question, isn't it? I'm not sure I know the answer to that one! The cynic in me says it's a good way to turn off kids from mathematics.

ZHENG Interesting.

TODD Hey, look. Room 630. Let's sneek in here and say a quick hello to Abby and Kaycie.

ZHENG What class are they teaching?

TODD Precalculus.

As we walked into the room, the difference between this classroom and Ms. Smith's was immediately obvious. I whispered to Zheng.

TODD Do you notice it?

ZHENG What?

TODD The difference between this classroom and the last one.

ZHENG Mmmm ... Yes. No laptops.

TODD True, true. But what else? Do you see any other differences?

ZHENG The students are talking to each other?

TODD But what else? Don't you see it?

ZHENG No, I don't.

TODD There were brown students in the last classroom. The guy in the corner. The one that answered my first question. Remember him?

ZHENG Yes. YES! Very interesting.

Zheng and I walked over to a desk where Abby and Kaycie were sitting, waiting for class to begin.

TODD How is it going? This is my friend, Zheng. He's my friend from the math department. Do you remember me talking about him?

KAYCIE Ha! Yes. You're the x to the x guy, right?

ZHENG Yes. I am that person. *Zheng smiled.*

TODD Is this an honors classroom?

ABBY Why yes, Dr. Edwards, it is. How do you know? You haven't even heard their thinking yet. You've been in here for 10 seconds. How could you know?

学生。

ZHENG 为什么呢？为什么要花 3 年时间去学同样的东西，而且是学生们有很多困难并且看起来也并不感兴趣的课程？

TODD 哈！这是一个很好的问题，不是么？我并不确定我知道这个问题的答案。我心中愤世嫉俗的那个小人告诉我这可是一个让学生不想学习数学的好方法 (反讽)。

ZHENG 很有意思。

TODD 嘿，你看，630 教室。让我们悄悄进去，跟 Abby 和 Kaycie 快速的打个招呼吧。

ZHENG 他们在教什么课程呢？

TODD 初级微积分。

当我们走进教室的时候，马上就能很明显感受到这里和 Ms. Smith 教室的不同。我小声对铮说：

TODD 你注意到了么？

ZHENG 什么？

TODD 这个教室和那个教室不一样的地方。

ZHENG 嗯... 是的，这里没有电脑。

TODD 是啊，是啊。不过别的呢？你还看到有什么不一样的地方吗？

ZHENG 这些学生在互相交流？

TODD 别的呢？你难道没有发现么？

ZHENG 没有了。

TODD 在上一个教室里有棕黄色肤色的学生。就是那个在角落里的男孩。就是那个回答我第一个问题的人，还记得他吗？

ZHENG 对呀！真的是，非常有意思。

铮和我走到 Abby 和 Kaycie 坐的桌子前，等待着这堂课的开始。

TODD 最近怎么样？这是我的朋友，铮。他是我在数学系的好友。你还记得我曾和你说起过他吗？

KAYCIE 哈！当然。你就是那个 x 的 x 次方的伙计，对吗？

ZHENG 对的，我就是那个“伙计”。铮微笑着说道。

TODD 这是那个优秀生班吗？

ABBY 对啊。Edwards 博士，就是。你怎么知道的？你还没听过他们的想法。你刚到这儿 10 秒钟。你怎么会知道？

TODD Look. They're all white. Every last student.

VIII

In our observations in two classrooms at Hillsdale High School, the promise and pitfalls associated with technology use in mathematics classrooms were on full display. As is the case with any tool—from a hammer to a calculator—technology can be used in ways that help or harm. For instance, while a hammer can be used to display a picture, it can also damage one's thumb. Similarly, computers can be used in ways that encourage student problem solving, conversation and collaboration or in ways that isolate students—too often students of color—robbing them of opportunities to share ideas and build mathematical meaning (Hong, 2002).

As teachers become managers—overseeing students as they complete pre-built, multiple choice questions on a computer screen—their pedagogical content growth—their knowledge of mathematics for teaching—is stunted. They ask fewer questions of their own design, have fewer opportunities to reveal their own love and curiosity for mathematics to students, and feel less ownership over their own classrooms. And with little time for professional development or reflection, teachers grow unaccustomed to thinking. For this reason, they may feel paralyzed when confronted with novel questions within the realm of secondary school mathematics yet typically ignored by a state mandated, one-size fits all curriculum. For instance, Ms. Smith was truly taken aback by our questions about 0^0, despite the fact that she had successfully completed multiple calculus courses as part of her preparation to become a teacher and despite the fact that rules of exponents, logarithms, and limits are all part of her curriculum.

TODD What did you think about our visit today?

ZHENG This all was very interesting. Now I have more sympathy for the students in my calculus classes.

TODD Why's that?

ZHENG Well, it is clear to me that solving multiple choice items on a computer has stripped away students' sense of wonder and discovery. This is not mathematics to me.

TODD 看，他们都是白人。在这儿的每一个学生。

VIII

在我们对 Hillsdale 高中两个教室里的观察过程中，科技在数学课中的应用过程中前景与缺陷得到了完美的呈现。科技就像是任何其它工具，例如从锤子到计算器，科技的使用既可以起到帮助也有损害的作用。例如，当锤子可以用来把画钉在墙上，同时也可以砸伤一个人的手指。同样，计算机可以鼓励锻炼学生动手解决问题，促进对话，以及增进合作作用，但同时也可以鼓励学生－更多时候孤立了不同肤色的学生－掠夺了他们分享点子和建立对问题的数学理解的机会 (Hong, 2002)。

当教师们变成了管理者－监督学生在计算机屏幕上完成预先构建的多项选择题时－他们对数学教学方法知识没有得到发展。教师们对自己课程材料设计的质疑越来越少，数学科目的热爱和求知欲的展露越来越少，并且对自己课堂的把控越来越弱。加之有限的职业培训和反思的机会，教师逐渐的失去了对思考的习惯。就是这个原因，当面对高中数学课程标准之外的一些新颖的问题，教师们显得力不从心。例如 Ms.Smith，尽管她成功的完成了作为数学老师所必修的微积分课程（其中涵盖了指数，对数，和极限的内容），但面对我们所提出的问题，她却仍然不知所措。

TODD 你对我们今天的观摩有什么看法？

ZHENG 非常有意思。现在我有点同情我微积分班上的学生。

TODD 为什么？

ZHENG 我现在明白了在计算机上做多项选择题让学生丧失了思考和探索的感觉。这不是我认为的数学应有的精神。

Todd Really?

Zheng Yes, really. To me, the essence of mathematics is uncovering meaning—exploring.

Todd Yes, I agree. We engaged students in this way when we asked them about 0^0, right?

Zheng Yes. *That* was mathematics. We used the teacher's laptop as a tool generate truly thought-provoking questions—what is the value of 0^0?—and to uncover meanings of answers that various tools provided.

Todd Yes, I get it.

Zheng Do American students encounter mathematics in this way?

Todd Too often, I don't think so. Especially, at schools like Hillsdale. The teachers don't have a lot of professional development or time to reflect.

Zheng Why not?

Todd It's complicated. Professional development costs money. Keeping class sizes small costs money. Hillsdale is a relatively poor district.

Zheng But the United States is a rich country.

Todd I know, but not for everybody. It's complicated.

Unfortunately Zheng and I were not asked to make sense of anomalies such as 0^0 when we were in high school, and such investigations are too infrequent in present-day classrooms. It wasn't until we explored mathematics content much later—as graduate students of mathematics and as teachers—that we learned the mathematics behind indeterminate expressions. Too often, we label expressions such as $\frac{0}{0}$ or 1^∞ as "indeterminate" or "undefined" without delving deeper to explain the mathematics behind the results (Watson, 1961).

IX

As a country, we need to take a hard look (a harder look?) at ways we engage our classroom teachers and our students—with (and without) technology. Teachers need the time and space to question more critically ways in which they use technology with students in mathematics class. Placing students in front of a screen does not ensure significant mathematics learning. But neither does lecturing to them about procedures without conceptual understanding.

Todd 真的？

Zheng 真的！数学对于我来讲是揭开问题背后的意义，是探索。

Todd 是的，我同意，当我们问学生 0^0 的问题时，我们正用这种方式激励他们，不是吗？

Zheng 是的。那才是数学。我们用计算机作为一个工具使他们生成真正的想法，去刺激他们问问题，究竟什么是 0^0？我们还可以用我们所拥有的不同工具去揭露背后的意义。

Todd 是的，我明白。

Zheng 美国学生是这么看待数学的么？

Todd 我认为大多时候不是这样的。尤其像 Hillsdale 这样的学校。老师没有太多职业培训和时间去反思其中的问题。

Zheng 为什么不呢？

Todd 这很复杂。对教师的职业培训十分昂贵。保持小班教学也会花费很高昂的费用。Hillsdale 学区相对比较贫穷。

Zheng 但是美国是个富有的国家。

Todd 我知道，但是不是每个人都富有。这是个很复杂的话题。

很遗憾，当我和铮在高中的时候也没有人跟我们提出过 0^0 这么奇怪的问题，这种探讨在如今的课堂上很少出现。这类的内容如果我们不是在之后作为研究生，博士生，或是教师从业者，更深入的学习数学，学习不定式背后的数学理论，大多是接触不到的。太多时候，我们简单地把表达式像 $\frac{0}{0}$ 或是 1^∞ 扣上“不确定”或“无定义”的标签，而不是更深入的探讨结果背后的数学原理 (Watson, 1961)。

IX

最为一个国家，我们需要深刻思考（或说更深刻的思考）我们应该用什么方式去激励我们的任课教师和我们学生，运用（或是不运用）科技在我们的课堂里。教师需要时间和空间去用批判的思维去质问科技是如何被用到数学教育中的。只是简单的将学生置于电脑屏幕前不能保证学生由明显的学习效果。同样，只是单纯向学生演算数学过程而不讲解概念性的理解也是意义不大的。

We must strive to empower teachers to teach mathematics in ways that promote student inquiry. The reason that our $x^x = a$ scenario was successful with students was that it provided students with a novel way—both methodologically (i.e., discussion in small groups) and mathematically (i.e., with an unfamiliar yet accessible task)—to revisit content and build deeper meaning, providing a context for solidifying understanding of content—namely, exponents.

我们必须努力培养教师能够鼓励学生探究的教学方法。之所以我们的 $x^x = a$ 是个成功的范例是因为这道题向学生提供了一种新的数学学习途径。这个范例成功在于展示了在教学方法上启迪（鼓励了学生的小组讨论），并且在数学内容上（提供了一道学生不熟悉但是又可以动手操作的任务）。这道题帮助学生回顾了相关的指数内容同时也强化了对概念的深入理解。

As critical educators, we acknowledge our responsibility as change agents in oppressive school environments (Freire, 1970). During our initial visit, we used several technologies — a search engine, an online graphing tool, and a handheld scientific calculator — to disrupt "teaching as usual" — generating conflicting values for 0^0. Students were genuinely curious about the contradictory findings. *Why do the two tools give different answers? Which one is correct?* Our intent was two-fold: (a) To provide students with novelty to engage them in genuine mathematical thought; and (b) to help the teacher recognize CMS as a tool of distraction that was hindering her ability to engage students mathematically.

最为有批判思维的教育者，我们承认改善存在压迫的学校环境是我们的责任（Freire, 1970）。在我们对（Hillsdale）学校的首次访问中，我们使用了几种科技，包括搜索引擎，在线画图工具和一个手持科学计算器，通过提出富有争议的 00 问题打断了课堂的"常规教学"。学生流露出了对这个富有争议结果的好奇。*为什么用两个计算工具得出不一样的答案？哪一个是正确的？*我们的目的是双重的：（a）通过提出新颖的问题来引导学生数学思维；（b）帮助教师意识到课堂管理软件，最为一种分散学生精力的工具，阻碍了她教授数学思维的能力。

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