Using Visual Modeling Tools to Reach Students with Learning Disabilities

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Abstract: *Teachers can use electronic visual modeling tools to help students with learning disabilities visualize and understand mathematical concepts such as proportions, dilations, and scale factors. In this article, the authors describe strategies for using static and dynamic visuals for supporting the memory and processing of students with learning disabilities as they engage in challenging mathematics.*

Keywords: *visualization, differentiation, special needs instruction*

1 Introduction

When your students are struggling, sometimes a picture is worth a thousand words. Sometimes, a moving picture can be even more valuable—especially with respect to important topics like ratio and proportion, scale drawing, dilation, and similarity. Strategic use of dynamic geometry tools can help students with learning disabilities (LD) visualize relationships between mathematical concepts while providing a key scaffold for these students' memory and processing.

1.1 Geometry and Students with LD

Students in the United States often struggle with geometry concepts such as the connection between transformations and similarity (Seago et al., 2013). Yet, both general and special education teachers are under increased pressure to teach such complex mathematical concepts not only to general education students, but also to students with LD. These students are expected to progress through a learning trajectory where they develop conceptual connections between ratio and proportion, scale drawing, dilations, and similarity (Confrey et al., 2012) and they will be expected to succeed with related problems on high-stakes assessments (e.g., Partnership for Assessment of Readiness for College and Careers [PARCC], 2014).

Students with LD are certainly capable of high levels of success with complex mathematical concepts, especially when they are supported with visual representations (Marita & Hord, 2017). However, students with LD often struggle with working memory (i.e., processing, storing, and integrating multiple sets of information) especially when they encounter multi-step problems (Swanson & Siegel, 2001). Teachers can give these students better opportunities for success in multi-step situations if they help students with LD utilize visual representations to organize their thinking and minimize the demands on working memory (e.g., van Garderen, 2007). For instance, when students with LD are solving a two-step problem, if they can store information from the first step of a problem on paper, process the second step, store information from the second step on paper, and then connect information from the two steps, they have a better chance of processing, storing, and integrating all parts of the problem. Therefore, helping students with LD store and process information as they work through transformation and similarity tasks may be essential for student success with these problems.

1.2 Access to Geometry through Technology

While visual representations on paper are often effective tools for teaching students with LD (Marita & Hord, 2017), teachers should also consider the benefits of moving representations. Dynamic geometry software has long been a productive tool for supporting geometric thinking (see Olive, 1998). This software can be used to construct and explore geometric objects and their properties, propose and test mathematical conjectures, and develop informal concepts of proof (Battista, 2009). The ability to precisely construct and move shapes on a computer screen mitigates the limitations of static textbook images and can help to make patterns more visible. By moving a point or changing the length of a segment as they explore a mathematical concept, students have rich opportunities to see relationships between geometric figures.

Given these benefits, teachers can use dynamic geometry software both to establish high expectations and provide key supports for students with LD (National Council of Teachers of Mathematics [NCTM], 2000). When teaching students with LD and using dynamic technology, teachers must consider which technologies and tasks are the best fit for students, including how visuals align with students' prior experiences and whether the models prepare students for their future experiences. For example, in the context of ratio, proportion, and similarity, a secondary teacher might consider how students with LD have previously experienced similarity in elementary classes as well as how the visuals introduced for solving scale-drawing problems in the eighth grade might support or limit a transformational conception of similarity in high school.

1.3 Case Study: Building a Dynamic Conception of Similarity

As a special education student since elementary school, Jasmine has worked diligently and has made significant progress. However, mathematics is becoming more complicated for Jasmine in the eighth grade. For high school, she must develop a more sophisticated and abstract understanding of mathematics. During this transition, she is likely to benefit from scaffolded conversations with teachers using visuals of many types (static and dynamic) to support her memory and processing as well as her learning of challenging mathematics.

We based our work with Jasmine on the Triangles and Transformations Learning Trajectory (Confrey et al. 2012), teacher research about transformational approaches to similarity (Seago et al., 2013), and related Common Core Standards (2010) (e.g., 7.G.1—Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale. and 8.G—Understand congruence and similarity using physical models, transparencies, or geometry software). We began with the following sample PARCC problem:

Sample PARCC Problem

A right triangle has legs measuring 4.5 meters and 1.5 meters. The lengths of the legs of a second triangle are proportional to the lengths of the legs of the first triangle.

Which could be the lengths of the legs of the second triangle? Select each correct pair of lengths.

a) 6 m and 2 m
b) 8 m and 5 m
c) 7 m and 3.5 m
d) 10 m and 2.5 m
e) 11.25 m and 3.75 m

Jasmine began reading the word problem, whispering to herself, and underlining key words such as measuring, meters, lengths, proportional, and legs of the second triangle. Over the course of several minutes, she typed numbers into her calculator and recorded the series of calculations (see Figure 1). The teacher, Mrs. Karr, asked for clarification: "You are doing some division?" Jasmine

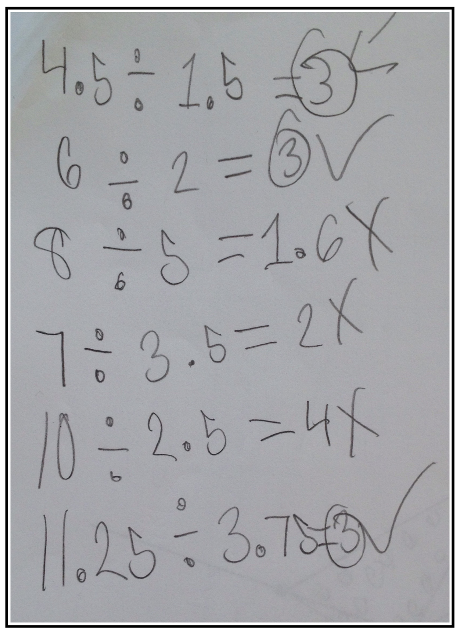

Fig. 1: *Record of student findings from calculator.*

replied, "Yes . . . To see if they have the same answers . . . The reason I am writing this down is so I can get the answers that go in this spot" (pointing to the multiple choice responses). "I like to write for some reason. I always like to check my work."

Jasmine used productive strategies to manage demands on her working memory (e.g., offloading information from short-term memory onto her scratch paper to allow her to not have to hold so much information in her head as she thought about upcoming steps in the problem; see Risko & Dunn, 2015) as she worked through this problem including underlining, using a calculator, recording her progress carefully, and checking her work that she strategically organized on paper. Moreover, she determined the correct answer. Yet, Mrs. Karr needed to know more about Jasmine's problem solving process to plan subsequent instruction.

MRS. KARR Your work is very organized. It's excellent. Let's talk more about this problem. What is the problem about?

JASMINE It's about measurement in meters. Who's got the same ratio.

MRS. KARR I don't see the word ratio at all. Why do you know it has to do with a ratio?

JASMINE (*She paused for a moment and reread the problem out loud. When she got to the word proportional, she exclaimed*) That's it! Proportion means ratio. I had it in my mind, but as soon as I started like doing the work, it just went out of here.

MRS. KARR So, you thought of proportion and you decided to divide all of the pairs, and see if any of the pairs . . .

JASMINE (*jumping in*) The only way to figure out if they have the same ratio is to divide these two (*pointing to her the side lengths listed in her work*).

MRS. KARR So, the sides had the same ratio. So, you are not picturing the triangle when you think about this problem?

JASMINE No.

MRS. KARR Okay, that's fine. Your work is awesome. Now, we are going to go back and talk about proportional triangles in a whole different way.

Building on her existing knowledge, Mrs. Karr wanted to support Jasmine's thinking about the proportional figures using a transformational approach. She had Jasmine use a ruler to draw a generic triangle and they worked together to make a larger, similar version of that triangle using the process of dilation (Figure 2). During this process, Mrs. Karr made the points for the larger triangle and Jasmine connected the dots.

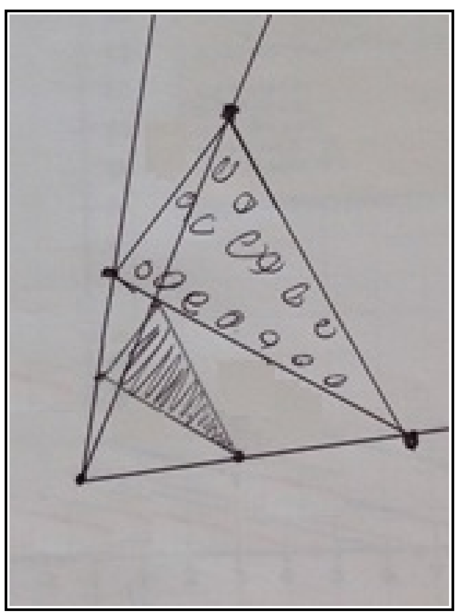

Fig. 2: *Sketch of dilation transformation.*

JASMINE (*As she made the triangle*) It's a bigger version of the other one! And, if we want to make it three times as big we go on farther.

MRS. KARR Yes! (*Then, she shaded the smaller rectangle and drew polka-dots inside the larger triangle.*) The one with polka-dots in it is twice as big, but it's still the same shape.

JASMINE Yeah, but it's going to have different . . . The meters . . . Because, it is twice more than the other number.

Then, Mrs. Karr gestured to one side of the triangle moving the tip of her pencil back and forth along that side (i.e., a dynamic representation in the form of a moving gesture) to demonstrate the length of that side.

MRS. KARR Let's look at these sides. If this is three (*pointing to the side on the smaller triangle*), what is this? (*pointing to corresponding side on the larger triangle while gesturing back and forth with her pencil tip along that side*)

JASMINE It's six!

MRS. KARR You are getting the big idea. This is about two (*pointing and gesturing along another side of the smaller triangle and then to a corresponding side on the larger triangle*) so what is that?

JASMINE Four.

At this point, Mrs. Karr decided Jasmine was ready to transition to dynamic representations and a discussion of scale factor. They both made different versions of similar triangles by dilating a triangle using the dynamic geometry software, GeoGebra (see Figure 3). After Jasmine was able to see a moving representation of dilation to create similar triangles (i.e., the stretching and shrinking process), Mrs. Karr discussed with Jasmine how dividing corresponding sides of similar triangles will always lead to the same answer (i.e., the scale factor). Then, Mrs. Karr transitioned to a dynamic

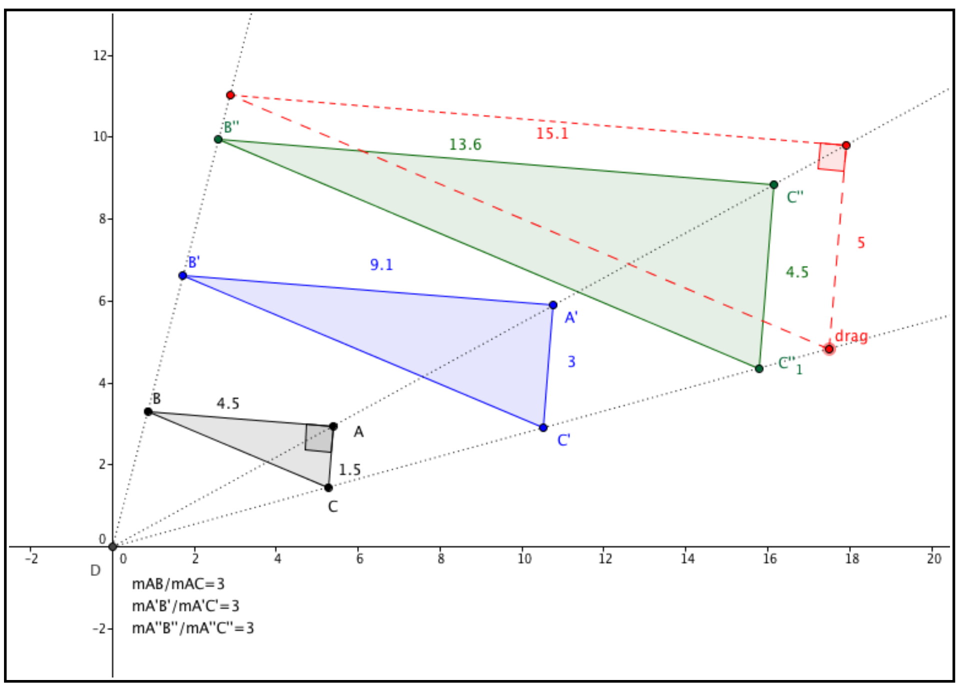

Fig. 3: *Dilations in GeoGebra.*

visual created using Robo-Compass software (which she has also printed on paper so she could later draw on the screenshot with a pencil; see Figure 4). Mrs. Karr constructed a right triangle of the same dimensions as the triangle in Problem 1 and subsequently dilated it by scale factors of two and three. She asked Jasmine questions such as, "Which triangle is dilated (from the red triangle) by a scale factor of two? Three?" and Jasmine was able to answer with the correct triangles with blue and green, respectively. Mrs. Karr guided Jasmine to check her work on the calculator for a scale factor of three, "Check 4.5×3 and see if you get 13.5. And then, 1.5×3 should be 4.5."

They experimented with using numbers with decimals as scale factors (e.g., 1.5 as a scale factor; see the pencil-drawn triangles). Jasmine said, “You can still make a triangle out of that.” Mrs. Karr observed, “It is still the same shape as the other one.”

At that point, Mrs. Karr moved Jasmine back to answer choices in the original problem. They began using the edge of a piece of small piece of paper on the Robo-Compass printout to connect points along the $x-$ and y-axes (i.e., the hypotenuses of the triangles). Then Mrs. Karr asked, “Is the triangle with sides of six and two going to work?” Jasmine replied, “Yeah.” Mrs. Karr said, “What

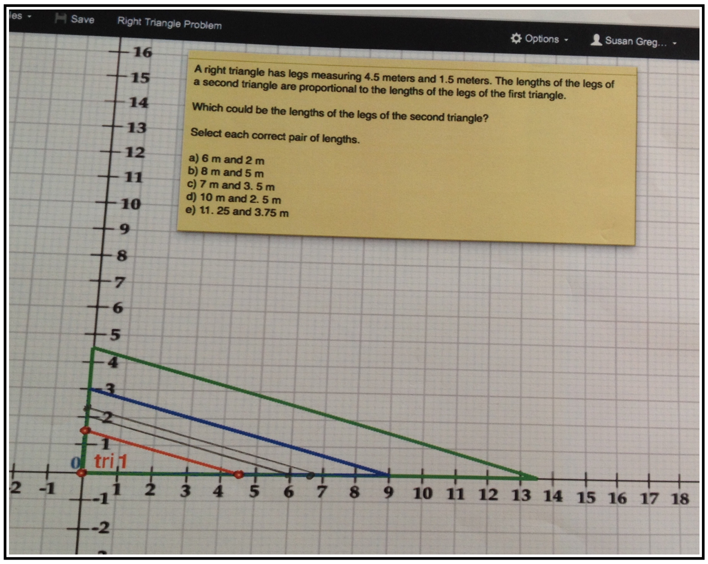

Fig. 4: *Robo-Compass.*

about $8-5$?” Jasmine used the paper to connect the points and immediately realized that a triangle with sides eight and five would not be similar to the red triangle (see Figure 5) which seemed to be obvious to Jasmine when she saw the visual.

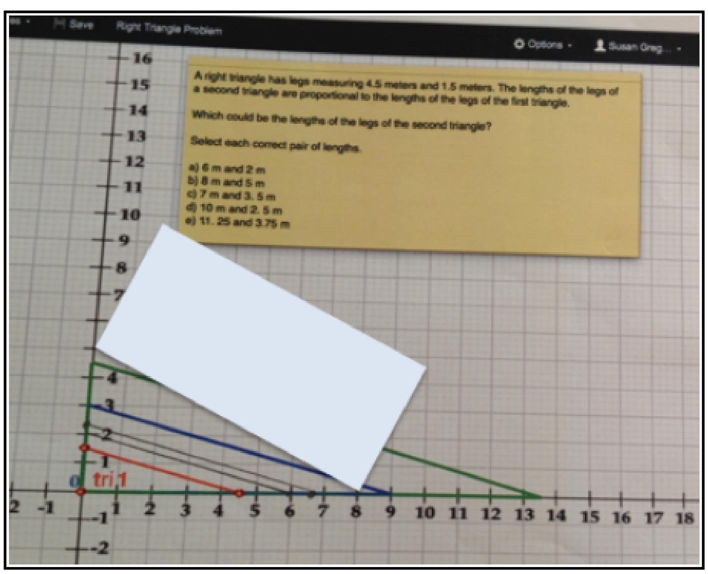

Fig. 5: *Informal test of similarity.*

Then Jasmine easily finished checking her calculations against the visual. Both of these tasks allowed Jasmine to connect a proportional situation with which she had some understanding, to a geometric model chosen to support a transformational view of similarity.

2 Summary

To find ways to support students with LD as they engage with geometric concepts, we recommend that teachers experiment with different combinations of visual representations from a multiple sources. We recommend trying a variety of software products in combination with drawing on paper (and gesturing at times to support the other visuals) to facilitate productive mathematical conversations. High quality representations support better mathematical thinking and discussions for all students (Sfard, 2002); we stress that these representations are possibly even more crucial for students with LD due to the working memory deficits these children need to overcome to succeed with challenging mathematics.

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