Using Monty Python and The Holy Grail to Teach the van Hiele Model for Geometric Thought and Logic

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Abstract: *One of the greatest comedy movies of all time is* Monty Python and The Holy Grail*. The author presents an activity to engage preservice teachers in the van Hiele Model for Geometric Thought and logic using specific clips from the movie. The author provides questions that complement the clips.*

Keywords: *Van Hiele levels, geometry, alternative teaching methods*

1 Introduction

The comedy troupe known as Monty Python's Flying Circus has entertained many people since 1969. Through television sketches and movies such as *The Holy Grail*, their brand of humor has enriched our lives with laughter. Surprisingly, their work also provides educational opportunities for students. For instance, mathematics teachers can use clips from *The Holy Grail* to illustrate van Hiele's Model of Geometric Thought and logic. In the following paper, I present specifics of this approach, sharing background of the van Hiele model and logic and clips from Monty Python sketches that support pre-service teachers' understanding of van Hiele's ideas. All of sketches I present are readily available on YouTube.

1.1 A Caveat

First, a caution. Teachers should preview each and every clip for appropriateness *before* viewing them with students in class. Within scenes, members of the Monty Python troupe sneak in language or innuendo that may be considered inappropriate for certain students. I strongly recommended that you scrutinize each of the scenes I present in this paper to avoid uncomfortable situations.

1.2 My Teaching Experiences with Monty Python Clips

I have been showing clips from *Monty Python and the Holy Grail* in my university geometry classes for years. My students have found this approach helpful as well as entertaining. I teach the van Hiele Model for Geometric Thought, the forms of a conditional statement, and rules of logical reasoning prior to engaging students with scenes from the movie. I use the clips to deepen my students' understanding of the levels of the model and the use of logic in arguments. Before I share specific activities with you, a brief synopsis of the van Hiele model is in order. If you're already familiar with the model, feel free to skip the next section.

2 The van Hiele Model: A Brief Introduction

The van Hiele Model for Geometric Thought was developed by two teachers, Pierre van Hiele and Dina van Hiele-Geldof, and expanded by Pierre in subsequent years. The model based on years of observation of Montessori students—many of whom struggled to learn geometry. The model provides five levels of student understanding. (Note: the original model used levels 0 through 4; later models use 1 through 5, as I do in this paper.) Basically, a student progresses from Level 1 (decision making from appearance) to Level 5 (logical deduction in a different realm than Euclidean geometry). Suppose that a student is asked to identify the shapes depicted in Figure 1. What will they see?

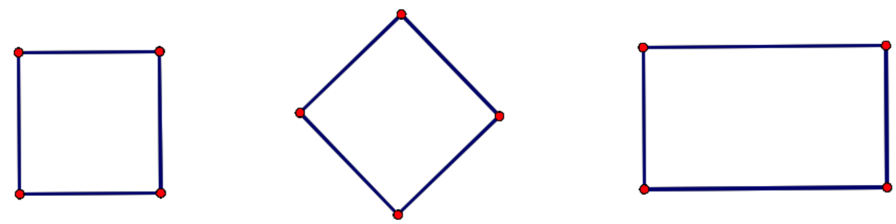

Fig. 1: *Three shapes.*

2.1 Level 1

At Level 1, a student reasons visually, saying "it is true because it looks like it." A student operating at Level 1 would identify the left-most shape in Figure 1 as a square; the middle shape, a diamond; and the right-most shape, a rectangle. The student would see no commonalities among the three shapes.

2.2 Level 2

A student at Level 2 focuses on properties of shapes, typically only one. For example, a square appears to have four sides of equal length. A rectangle has four right angles.

2.3 Level 3

A Level 3 student begins to use logic to deduce properties but cannot fully construct a proof. At this stage, a student is able to understand a proof, but may not fully grasp why a proof is being constructed. For instance, a Level 3 student would understand that a square is a rectangle (since a square has 4 right angles) but would not be able to articulate why this is the case.

2.4 Level 4

A student operating at Level 4 is able to construct a conjecture and will set out to prove it. The main characteristic of a student understanding at this level is the ability to apply the rules of logical deduction in a different environment than Euclidean geometry.

2.5 Progressing Through the Levels

A student progresses through the levels by way of activities of some kind, either physical or mental. Examples of activities include the use of dynamic geometry software, such as The Geometer's Sketchpad or GeoGebra; Miras; geoboards; patty paper; compass and straightedge; or logical deduction. Activities should help students at levels 1 or 2 build an intuitive understanding of the concept to be learned before abstraction and proof.

2.6 Five Properties

There are also five properties of the van Hiele Model: (1) fixed sequence, (2) distinction, (3) adjacency, (4) separation, and (5) attainment. *Fixed sequence* states that a student must pass through the five levels sequentially. *Distinction* refers to the changes in vocabulary that a student uses at each level. For example, shapes that have the same size and shape at an early level become congruent later. The *Adjacency* property refers to relative difficulty of tasks. Specifically, what was difficult for the learner at a particular level is now easy at the next level. The property of *separation* states that the teacher must communicate at the same level as the students for instruction to be successful. Separation is illustrated in the Witch Duck scene, discussed later in this paper. *Attainment* describes the five phases through which a student passes in the process of learning about a particular topic. For more resources concerning the van Hiele Model, please refer to the bibliography.

3 Connecting the van Hiele Model with Monty Python's *The Holy Grail*

And now for something completely different. Let's begin by reviewing the movie *Monty Python and the Holy Grail* looking for applications of geometry and logic. I will discuss the scenes in the order of their appearance in the film.

3.1 King Arthur and the Coconuts

3.1.1 The Scene

The movie begins with an activity in using counter examples to disprove a statement. In the opening scene, King Arthur is seen traveling across England en route to a castle. He is not riding a horse, but galloping as if he were, while his servant knocks two halves of an emptied coconut together to make a galloping sound. A discussion between King Arthur and a person from the castle ensues. Eventually, Arthur is questioned about how he found the coconut, to which he responds "I found it." Reasoning through indirect means, the person from the castle disproves the claim—it's impossible for a coconut to "just be found" in England since England is in a temperate zone while a coconut is a tropical fruit. King Arthur then suggests that the coconut traveled to England in some manner. Arthur uses the example of a swallow that is not native to England but is found throughout the land. The person from the castle asks, "Are you suggesting that coconuts are migratory?" Arthur offers that perhaps a swallow carried the coconut. The response, concerning weight ratios, is that a five ounce bird could not carry a two pound coconut. A second person from the castle now joins in the conversation, commenting that African swallows are notably larger than European swallows. As Arthur leaves, the discussion between the two people from the castle continues with alternative theories of how a sparrow or multiple sparrows could carry a coconut.

3.1.2 Mathematical Connections

This scene illustrates how, in geometry, students make conjectures then set out to test them. For example, a student thinks that the diagonals of a rectangle are the same length. Perhaps the student creates an appropriate correct sketch in GeoGebra, selects a vertex for "click and drag" testing, observes that the measures of the lengths stay the same regardless of the dimensions of the rectangle, and then logically deduces that the conjecture is true. One counterexample is sufficient to show that the conjecture is false. By the end of the coconut scene, many conjectures have been created. We can only hope that experimentation will occur.

This scene can also be used to illustrate proportional thinking, prevalent in similar triangles. If a 5-ounce bird is able to carry a 2-pound coconut, how much could an 80-pound emu carry?

3.2 The Witch Duck

3.2.1 The Scene

The scene with the most applications is the Witch Duck scene. Here, the villagers present Sir Bedemir with a suspected witch. Actually, the scene opens with Sir Bedemir performing an experiment. He has tied a coconut to a swallow to see if the bird could carry it, a Level 1 and 2 activity where Bedemir engages in a physical experiment to gain an intuitive understanding of the problem. When asked by Sir Bedemir how the villagers know that she is a witch, they respond that "She looks like one!", showing that the villagers are operating at van Hiele Level 1 (with respect to witches!). Sir Bedemir then proceeds to lead the villagers in a logical discussion how to determine whether someone is a witch or not. He demonstrates the separation property by communicating at the same level of the villagers. To be more specific, Sir Bedemir has at least a Level 4 understanding since he leads the villagers, at Level 1, in a proof as to why the suspect could be a witch. He leads the villagers through the argument slowly so that understanding is achieved by the end. Eventually, he brings them closer to Level 3 by constructing a logical argument that the villagers can understand but not create.

3.2.2 Mathematical Connections

Sir Bedemir uses the Law of Syllogism (i.e., the Law of Transitivity) repeatedly in his argument. If one relates to a second, which relates to a third, then the first is related to the third. For example, if a pitcher throws a fastball that stays straight, the pitch is hit hard by the batters consistently. If the pitch is hit hard by the batters consistently, the pitcher does not stay in the game very long. One could conclude that, if a pitcher throws a fastball that stays straight, the pitcher does not stay in the game very long.

As an example from the movie, in the Witch Duck scene, Sir Bedemir uses the Law of Syllogism repeatedly to logically prove that:

- If the woman weighs the same as a duck, she floats on water.
- If she floats on water, she is made of wood.
- If she is made of wood, then she would burn.
- If she burns, she is a witch.

His argument also shows that if one either assumes or accepts each of the conditional statements to be true, then the logical conclusion, if she weighs the same as a duck, then she is a witch, must also be accepted as true.

3.3 Knights Who Say "Ni"

3.3.1 The Scene

Our next scene involves the knights who say "ni." In the first part of the scene, as King Arthur and Sir Bedemir are travelling, they encounter the knights who say "ni." In order to pass through these particular woods, Arthur and Bedemir must construct a shrubbery with certain specifications. Otherwise, the knights will continuously say "ni" to Arthur and Bedemir and will not let them pass.

3.3.2 Mathematical Connections

As the scene unfolds, an activity in inductive logic appears. It turns out that there is a word that is not to be spoken in the presence of the knights who say "ni."

Inductive logic is the process of drawing generalizations from a number of specific instances. An example of this concept would be travelling in a foreign country where you do not know the language. Suppose that you are in need of a restroom. You notice only males going in and out of one set of doors, and females using a separate door. Since you do not know the language, you draw a generalization from the specific examples of which gender used which door.

In geometry, activities for students operating at levels 1 and 2 should engage students in inductive logic. Dynamic geometry tools, such as GeoGebra or Geometer's Sketchpad, provide an excellent way to encourage inductive reasoning. As students drag objects on the screen, they observe what properties or relationships change (i.e., are "dynamic") and which ones remain the same (i.e., are "static"). For instance, the previous example concerning the length of the diagonals of a rectangle is a worthy investigation with software. As students drag vertices of the rectangle, they look to see if the lengths of the diagonals remain equal.

In the second part of the scene, when the shrubbery is completed, Arthur and Bedemir are joined by Sir Robin. As the scene continues, the three gentlemen start to use *the word* in their conversation, causing great discomfort among the knights who most recently said "ni." It is interesting to see if the students in the class can identify the word. (Spoiler alert! If you wish to know what the word is, it was now used twice in this paragraph. A thorough examination of the first part of the scene will reveal that the word was used but caused no reaction from the knights who say "ni.")

3.4 The Bridge of Death

3.4.1 Mathematical Connections

The last scene from the movie that I'll explore with you is the Bridge of Death. This scene can be used to illustrate the Law of the Excluded Middle, which states that either a statement or its negation must be true. Suppose that a statement is "Coffee tastes good." Its negation would be "Coffee does not taste good." One of those two statements must be true. For negation, either insert the word "not" into the statement or remove it if it is already there. The negation of "Coffee tastes good" would not be "Coffee tastes horribly." Coffee could taste okay, bad, or great; all that is known is that it is not good.

3.4.2 The Scene

The bridge keeper (i.e., the Old Man from Scene 24) asks each of the Knights of the Round Table three questions. Answering all three questions correctly will allow the knight to cross the bridge to get to the other side. One wrong answer will then plunge the knight into the Gorge of Eternal Peril. The first two questions are the same for each knight: "What is your name?" and "What is your quest?" The third question varies from knight to knight. If the bridge keeper determines that knight answers the third question truthfully, he allows the knight to cross. If the answer is determined to be false, the knight is cast into the gorge. There is no middle ground. For instance, if the knight doesn't know the capital of Assyria or hesitantly answers with a favorite color, the answer is deemed to be wrong.

Interestingly, King Arthur turns the tables on the bridge keeper by making a reference to the opening scene. The bridge keeper asks Arthur about the air-speed velocity of an unladen swallow.

Arthur responds with a question as to which type of swallow, an African or European swallow. The bridge keeper does not know and is consequently thrown into the Gorge of Eternal Peril.

4 In Conclusion

Monty Python and the Holy Grail is an excellent way to teach concepts of the van Hiele Model of Geometric Thought and logic. Not only is the movie entertaining, it's also educational. I am continually amazed how attentive students are while watching scenes from *The Holy Grail* in my geometry classroom. These scenes provide my students with a novel opportunity to see how logic can be used in life outside of mathematics. Sir Bedemirs argument in the Witch Duck scene illustrates that an argument that is logically valid may be devoid of common sense. Inductive logic, as with the scenes with the Knights Who Say "Nee," can be utilized quite often in daily life.

5 Movie Clips

The following resources were available at YouTube at the time of publication. Teachers are encouraged to purchase the entire movie and make it a part of their professional library.

- The whole movie can be found at: `https://www.youtube.com/watch?v=sFbHyju7hN8`
- The opening scene: `https://www.youtube.com/watch?v=6NbCNA0mAtM`
- The witch duck scene: `https://www.youtube.com/watch?v=UTdDN_MRe64`
- The knights who say ni scene: `https://www.youtube.com/watch?v=0e2kaQqxmQ0`
- The Bridge of Death scene: `https://www.youtube.com/watch?v=IMxWLuOFyZM`

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