Taco Cart Lesson: A Three Act Task

Hannah Dietz, *Loveland High School*

Abstract: *The author offers a revision of a lesson plan which uses Dan Meyer's Three Act Task format to lead students through a problem involving the Pythagorean Theorem. The Taco Cart problem uses a real life situation to engage students. The research-based revisions that the author offers helps increase clarity for students and teachers.*

Keywords: *geometry, algebra, trigonometry, Pythagorean Theorem*

1 The Assignment

In the Summer of 2019, I enrolled in *Mathematics Misconception Diagnosis & Remediation* (EDT 566), a graduate level curriculum course for practicing secondary-level mathematics teachers. In preparation for the course, I implemented a rich task in my classroom in Spring 2019. I collected my students' written work and video recorded their conversations as they worked on the task. During our three week summer course, teachers shared these collected artifacts in small groups. We read articles related to our rich tasks and suggested research-based revisions to each other's lessons as part of our EDT 566 course work. I decided to engage my students in a variation of *Taco Cart* (`https://threeacts.mrmeyer.com/tacocart/`). As a "Three Act Task" (Meyer, 2011), *Taco Cart* is presented to students in three separate parts: (1) In Act 1, the problem is posed and students make initial conjectures regarding possible answers; (2) During Act 2, students identify information and work towards a solution; (3) In Act 3, students compare their predictions and solutions with a teacher-provided answer.

2 Context

Students enrolled in my Math II course—a blocked, 90 minute class—spent 45 minutes working on *Taco Cart*. On the day of the lesson, 15 students were present—12 males and 3 females, including 4 students with teaching and learning accommodations. In my original plan, I included the two sequels to *Taco Cart* (discussed in more detail below).

Students completed *Taco Cart* in the Spring of 2019, after end-of-course state testing, so they had already engaged in the entire Math II curriculum prior to the lesson. My students had spent significant time throughout the year studying the Pythagorean Theorem and basic right triangle trigonometry.

3 *Taco Cart* Lesson

3.1 Act 1: Introducing the Task

Taco Cart opens with a video describing the problem situation (readers are encouraged to watch the video at `https://threeacts.mrmeyer.com/tacocart/` before reading further.

Two friends, Ben and Dan, are on a beach when they notice a taco cart nearby. Ben believes that the quickest route to the cart is to walk in a straight line from their current location to the cart through sand. Dan thinks that walking along the street at a right angle will be faster since it's easier to walk on pavement (see Figure 1).

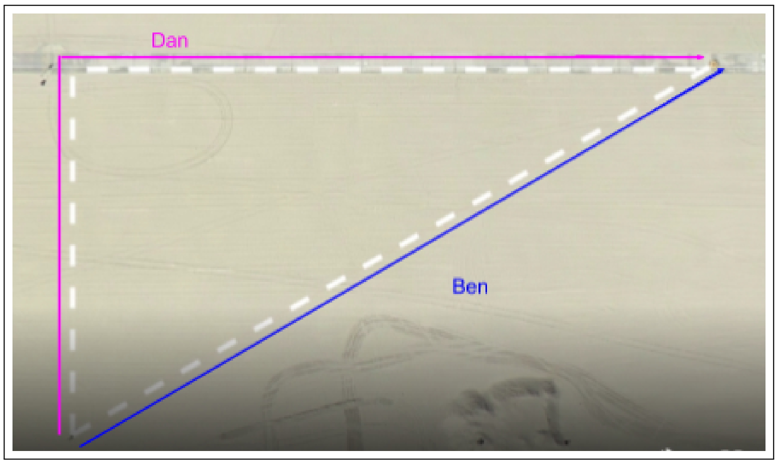


Fig. 1: *Paths chosen by Dan and Ben to the taco cart.*

After watching the video, students considered the following questions (See Items 1 and 2 in Appendix B for more details).

1. *Who will reach the taco cart first? Explain your thinking.*
2. *At what time will this person reach the taco cart? Explain your thinking.*

I used a Think-Pair-Share format to engage students in these questions. Students first answered the questions independently, then shared their initial responses with a partner. Following small group conversations, pairs shared their observations in a whole group setting.

3.2 Act 2: Constructing Student Solutions

I began Act 2 by asking students what information they needed to solve the problem (see Item 3 in Appendix B). After providing students with time to consider my question, I displayed the two distances that Dan took (i.e., lengths of the two legs of the right triangle depicted in Figure 1) along with walking speeds in sand and on the street (see Figure 2). With this information, students constructed initial solutions. Afterwards, I asked the students the two follow-up questions listed below (see Items 4 and 5 in Appendix B for more details).

4. *Do you want to change your answer based on the given information? Why or why not?*
5. *What is your new guess of who and when the first person will reach the taco cart?*

As students considered these questions in a Think-Pair-Share format, they revised their answers from Act 1.

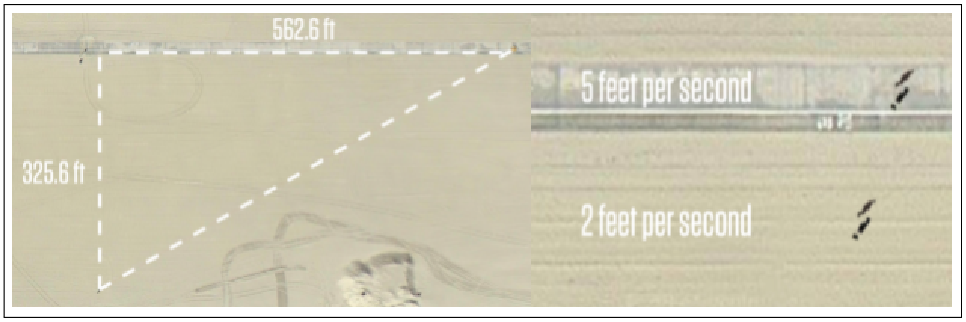


Fig. 2: *Information provided during Act 2 to solve the problem.*

3.3 Task 3: Comparing Student Solutions to Teacher-Provided Answer

During Act 3, students watched a video showing Ben and Dan walking to the Taco Cart along their proposed paths. After the video was presented, students answered the following wrap-up question (see Item 6 in Appendix B for more details).

6. *What are your final thoughts? Were you always right? Were you surprised? Was one piece of information more vital than another?*

3.4 Alignment to Standards

Taco Cart provides students with a real life scenario for exploring the Pythagorean Theorem. The task encompasses many Standards for Mathematical Practice—requiring students to model mathematics, determine appropriate problem-solving tools, and construct viable arguments. The lesson addresses standard `G.SRT.8: Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems`.

4 Student Data

Students progressed through the worksheet of questions (see Appendix B for more details) using a Think-Pair-Share format—initially working independently on each question, then in pairs with an assigned partner, and finally as a whole class. Since multiple revision opportunities existed throughout the lesson, analysis of student written work wasn't as useful as video since the recordings captured student thinking "in the moment" better than the worksheet. I will restrict my analysis of student work to examples to those that reveal student missteps and early thinking (i.e., examples with previous work that wasn't erased).

4.1 Summary

As we analyzed my students' data, it was clear that several struggled using the Pythagorean Theorem, with a number placing known values incorrectly. All but one student used the Pythagorean Theorem to find the missing length. One student used trigonometry to find a missing angle and then used trigonometry again to find the missing side length. A student work samples is provided in Figure 3.

Fig. 3: *One student's method of finding missing side length.*

Examining the video, my colleagues and I noticed that almost all students struggled with speed. Some students omitted this information. Others forgot to compute times required to tranverse the two legs of the right triangle separately. Figure 4 shows erase marks fixing this mistake.

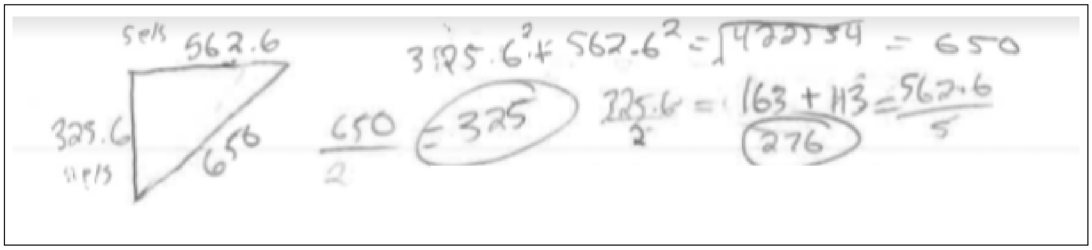


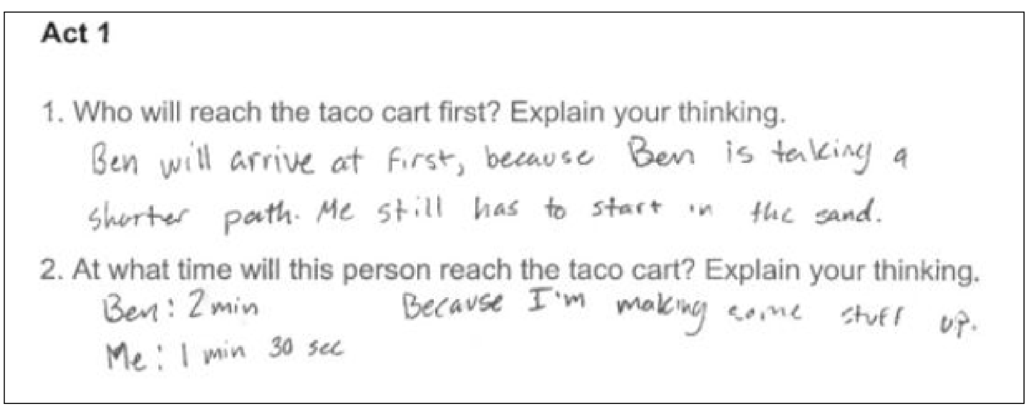
Fig. 4: *Eraser marks indicate revisions.*

Table 1 summarizes student answers after each Act. Data represented in the first two columns was gathered from the class video. Data for the third was gleaned from student handouts. During Act 1, most students said that Ben would reach the taco cart first since Ben has the shortest path. After Act 2, the majority switched to Dan. After Act 3, we discussed lingering misconceptions with speed. Many students "fixed" their answers during this time, with all ultimately indicating that Dan would reach the cart first.

	After Act 1	After Act 2	After Act 3
Dan (me)	0	11	15
Ben	10	4	0
Tie	5	0	0

Table 1: *Student solutions to the task at different stages of the lesson.*

During class discussion and the throughout the handout, I asked the students to explain their thinking. Comparing the video to student samples, my classmates and I agreed that the in-class conversations provided better evidence of mathematical thinking than than written explanations for the majority of my students. As Figures 5 and 6 suggest, depth of student explanations varied. Most of the students did attempt justify their statements; however, without access to speed or distance information, students were unable to provide mathematical justifications (e.g., "I'm making some stuff up," "The track isn't that far away").


Act 1

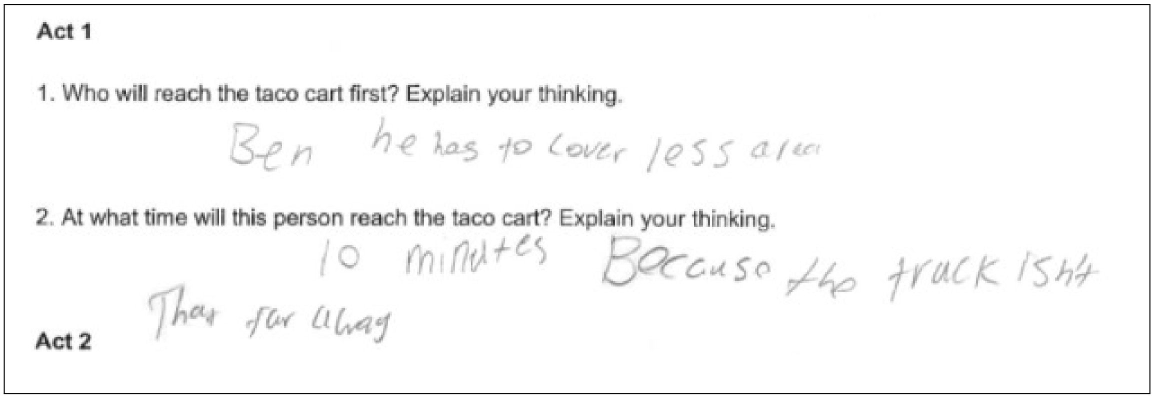
1. Who will reach the taco cart first? Explain your thinking.

Ben will arrive at first, because Ben is taking a shorter path. Me still has to start in the sand.

2. At what time will this person reach the taco cart? Explain your thinking.

Ben: 2 min Because I'm making some stuff up.
Me: 1 min 30 sec

Fig. 5: *Student answers show a wide variety of depth of explanation.*


Act 1

1. Who will reach the taco cart first? Explain your thinking.

Ben he has to cover less area

2. At what time will this person reach the taco cart? Explain your thinking.

10 minutes Baccuse the truck isnt Thar far away

Act 2

Fig. 6: *Student answers show a wide variety of depth of explanation.*

Table 2 shows typical explanations for the first two questions in Act 1.

Explained thinking on both problems	Explained thinking on #1 only	Explained thinking on #2 only	Did not explain thinking on #1 or #2
8	5	1	1

Table 2: *Act 1 explanations on questions 1 and 2.*

5 Literature Review

Teachers shared their students' work, video recordings of student conversations, and research articles as they revised initial lessons in small groups. Below, I discuss two readings that informed my colleagues' understanding of Three Act Tasks and our revisions of the *Taco Cart* activity.

5.1 Promoting Growth Mindset with 3 Act Math (Ehlert, 2015)

In the article "Promoting Growth Mindset with 3 Act Math," Ehlert discusses three instructional advantages provided by Three Act Tasks—namely, (1) accessibility; (2) multiple solution paths; and (3) revision. I discuss each briefly, along with anecdotes from my own classroom.

Ehlert notes that Three Act Math tasks are accessible to students regardless of level (i.e, they have a "low entry" point). All students can participate meaningfully in Act 1 by making a guess at the beginning of the task. Since the Taco Cart activity doesn't provide numerical information at the beginning, students aren't initially burdened by mathematical computation.

Students perceive problems differently and bring different experiences and mathematical understandings to tasks. The Taco Cart activity provides students with "multiple routes to a reasonable solution" while challenging the popular student belief that "they are no good at math and never will be" (Ehlert, 2015).

One of my core teaching goals is to actively engage students in their mistakes. Ehlert notes that Three Act Math tasks encourage "learning from mistakes" (2015). After Act 2, a few of my students didn't have the correct answer. This provided an ideal context for group discussion. As my students analyzed their peers' strategies collaboratively, they considered the task from perspectives different from their own.

5.2 Mr. Meyer's Taco Cart (Nguyen, 2013)

In her blog entry, Nguyen (2013) discusses the implementation of *Taco Cart* in her own classroom. Nguyen starts off by introducing the problem, but then asks her students to brainstorm other questions that they could investigate. One student suggests finding the fastest route, a sequel question in Meyer's original Three Act task. Nguyen's inquiry-based approach allows students to take ownership of their work.

In addition, Nguyen gives her students a working place-mat based on the image provided in Act 2. Along each side of the triangle, there is a white box in place of the dimensions, including the hypotenuse (which was not given in the original 3 Act Task). After a discussion of what is needed to solve the problem, Nguyen tells the students that she will only tell them one side length. The class chooses the hypotenuse. She also gives the students the two different speeds. This created a whole different way to find the missing side lengths because the Pythagorean Theorem was out of the question, although many of Nguyen's students started off with the theorem.

After some time, Nguyen's students realized that they could measure the sides with a ruler and set up proportions. I thought that this was a fantastic idea. Many times when students see a right triangle, they think Pythagorean Theorem. That was what almost all of my students thought when I showed them the first video. I think it is important for students to realize that there are restrictions to the Pythagorean Theorem and the theorem can not be used all the time. This then requires them to think of other mathematical tools they have that will allow them to solve the problem.

The way Nguyen designed this lesson, the final answer was not as important as the process of finding a solution. Students would not be able to find a solution without checking and guessing a million times, as this lesson was not designed for a calculus class. This is valuable because of the increased emphasis on how to get to the final solution and why are we taking these steps to get there.

6 Lesson Revisions

6.1 Strengthen Differentiation with Student-Generated Questions

I will still pair my students so they may work together. Once the Act I video ends, I will ask students two questions: *What do you notice? What do you wonder?*. I'll have the pairs of students write down at least two different questions that they came up with on a blue post-it notes and place these on the front board.

I took the idea of coming up with different questions from Nguyen's article. Once all post-its are on the board, I will arrange them so that like questions are placed together. We'll discuss the different questions as a whole class then pairs will choose the question they wish to pursue. By having students create their own questions, they have more ownership over their work. Although all students answer a question for the task, the level of difficulty varies based on student choice. This allows all students to find success which helps promote growth mindset (Ehlert, 2015).

6.2 Scaffold Information with Working Mats

To start off Act 2, I will provide students with Working Mat to organize their work (see Figure 7). Students will write their question on the top of the mat. Students will record data on their mats after they discuss required information as a whole group and after I provide two distances and the two speeds. The Working Mat will help students keep their work organized, will ease the process of grading their work, and will help keep all students engaged in the task.

6.3 Promoting Student Explanations

After information is provided, students have time to work on their question. Once students have solved their problem, I'll provide them with a revised handout that will include the following questions:

1. *What question did you try to solve?*
2. *How did you solve this problem? Include a step by step detail of what you did* (please attach your work to this handout), and
3. *Pick another question that was discussed at the beginning of class. What is that question? How would you START solving this problem?*

After looking at student samples, I was not completely happy with the written responses from the students, which is why I changed the questions. By having the students explain their mathematical process in solving the problem, I will be able to determine more readily their conceptual understanding of the task and how they computed their answer. If students finish early, I will ask them answer their second question. Once everyone is finished, students will share their handout responses in a whole group setting.

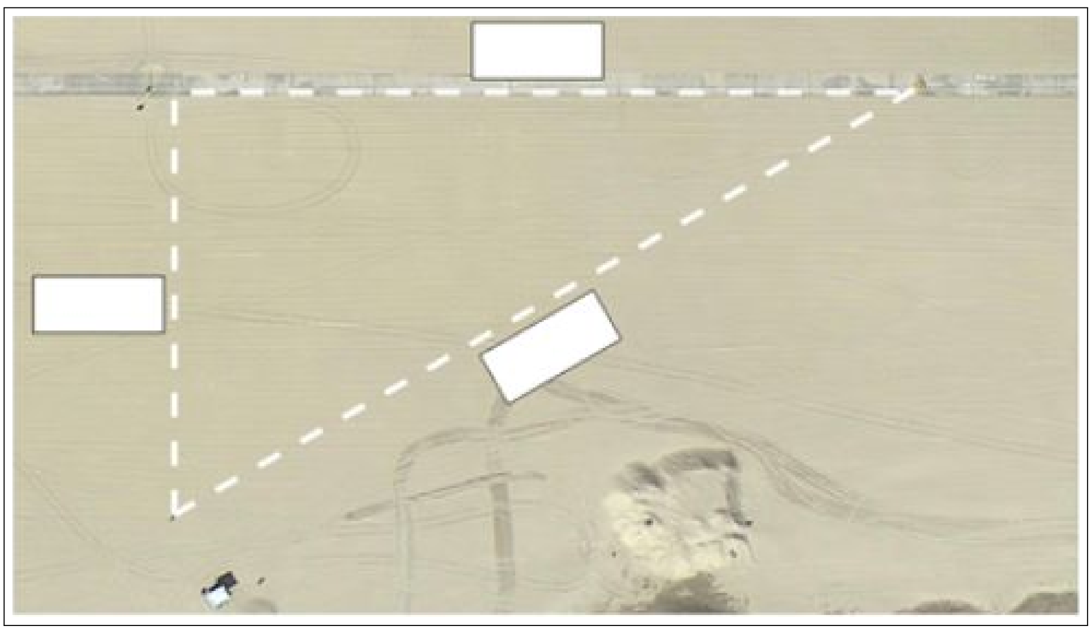


Fig. 7: *Working Place Mat (Nyguen, 2013).*

7 Conclusion

In the preceding article, I've discussed Three Act tasks as a vehicle for promoting meaningful problem solving, collaboration, and revision in the secondary mathematics classroom. Moreover, and arguably more importantly, I've illustrated the power of research and collaborative planning as an approach to build better lessons for students. Although *Taco Cart* (Meyer, 2011) was already an engaging lesson, my classmates and I were able to tailor the activity to my students' needs through careful analysis of student video and written work.

References

Ehlert, D. (2015). *Promoting Growth Mindset with 3 Act Math*. Retrieved May, 2019, from https://www.nctm.org/Publications/Mathematics-Teaching-in-Middle-School/Blog/Promoting-Growth-Mindset-with-3-Act-Math/

Meyer, D. (2011). *The Three Acts of a Mathematical Story.* Retrieved May, 2019, from https://blog.mrmeyer.com/2011/the-three-acts-of-a-mathematical-story/

Nguyen, F. (2014). Mr. Meyer's Taco Cart. Retrieved May, 2019, from http://fawnnguyen.com/mr-meyers-taco-cart/

Hannah Dietz, (DietzHa@lovelandschools.org), teaches mathematics at Loveland High School in Loveland, Ohio. Hannah is currently a graduate student in the Masters of Arts in Teaching (MAT) Program in the Department of Mathematics at Miami University. Her professional interests include mathematical discourse and the use of technology in the teaching and learning of school mathematics.

APPENDIX A: Revised Lesson Plan

Hannah Dietz
EDT 566 Lesson Plan (2019)

Lesson Title
3 Act Task - Taco Cart

Grade Level
10th Grade (Math II)

Lesson Objectives
- Students use prior content knowledge to tackle a problem
- Increase problem solving skills
- Increase mathematical reasoning skills

Ohio Standards, Benchmarks, and Grade Level Indicators
- G.SRT.8
  - Solve problems involving right triangles.
    - Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems
- F.IF.4
  - For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities

Materials, Technology, Resources
Student Resources:
- Handout
- Working mat of the beach
- Pink and Blue Post-Its

Teacher Resources:
- 3 Act Task Videos

Lesson Procedures (Act 1, Act 2, Act 3)
Act 1 (10minutes)
- Pair students, Pass out blue post-its
- Use the link provided in Teacher Resources to access the videos
- Play the video for act 1
- ask two questions: What do you notice? What do you wonder?
- pairs of students will write down at least two different questions on post-its
  - Students place post-its on board
- arrange post-its them so that like questions are placed together.
- Class discussion of the different questions.
  - What was the most popular questions
  - What would be fun to solve
- Student pairs will decide what question they will try to solve

Act 2 (25 minutes)
- Pace out working mat
    - Have students write down the question they are working on at the top of page
- ask students what information they would like to solve their problem.
    - Have them write done different pieces of information on separate pink post-its.
    - Post-its will go to the board and be organized according to like responses.
- Class discussion on the responses
    - "Why would each piece of information be useful?"
- Students work in partnership to solve their question
    - Float around
    - Refer students back to the given information

Act 3 (10minutes)
- Pass out handout
- Students will complete the handout
- If students finish early, have them solve a second question completely.
- Class discussion- wrapping up lesson
    - Have students share information written on the handout
        - "What was most challenging?"

Assessment
Students will be assessed through group discussion and the handout. The group discussion and the handout encourages students to explain their mathematical thinking. The students will be assessed based on the thoroughness of their explanation and mathematical language.

Accommodations
(There is a wide variety of students in my Math II classes. I have a handful of students that have an IEP or 504. I also have students who pick up content very quickly.)

- Scaffold when necessary
    - For example: Is there a shape that is formed by the two men walking?
- Pairing students by ability level and have students pick their own question to work on.
    - Different questions have different levels of difficulty, allowing lower level students to work on less difficult questions and higher level students work on a more challenging question.

APPENDIX B: Original Student Handout

3 Act Task: Taco Cart **Name:** ______________________

Act 1

1. Who will reach the taco cart first? Explain your thinking.

2. At what time will this person reach the taco cart? Explain your thinking.

Act 2

3. What information do you need?

4. Do you want to change your answer based on the given information? Why or why not?

5. What is your new guess of who and when the first person will reach the taco cart?

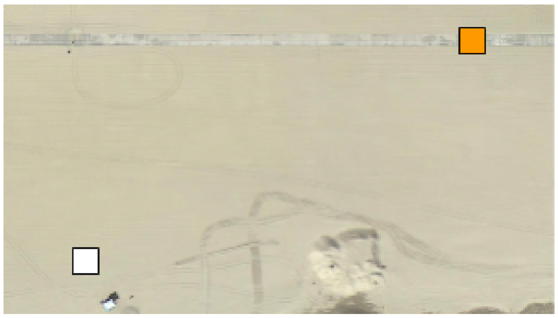
Act 3

6. What are your final thoughts? Were you always right? Were you surprised? Was one piece of information more vital than another?

The Sequel Part 1

Act 1

1. Where would the taco cart have to be so that both people would reach it at the same time? Draw the point where you think the taco cart should be.



2. What time will they get to the taco cart?

Act 2

3. What information do you need?

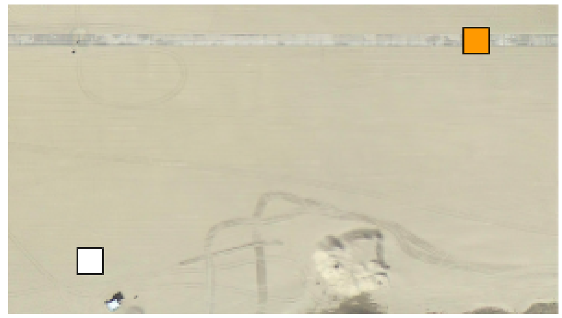
Act 3

4. What do you think about the answer in relation to your guess? Where you close? If so, how did you come up with your answer? Where you far off? If so, what do you think you did wrong or did not consider?

The Sequel Part 2

Act 1

1.What path to the taco cart would take the least amount of time? Draw the path that you think would take the least amount of time.



2. What time will they get to the taco cart?

Act 2

3. What information do you need?

Act 3

4. What do you think about the answer in relation to your guess? Where you close? If so, how did you come up with your answer? Where you far off? If so, what do you think you did wrong or did not consider?

APPENDIX C: Revised Student Handout

3 Act Task: Taco Cart **Name: ______________________**

1. What question did you try to solve?

2. How did you solve this problem? Include a step by step detail of what you did (please attach your work to this handout), and

3. Pick another question that was discussed at the beginning of class. What is that question? How would you START solving this problem.