Exploring Circle Area with Radius Squares

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Abstract: *The author describes an activity designed to foster student understanding of the formula for the area of a circle using radius squares. In the lesson, sixth graders discover that π number of radius squares are needed to cover a circle.*

Keywords: *geometry, algebra, measurement, hands-on activity*

1 Introduction

Finding the area of the circle is a concept that middle school and high school students frequently refer to when solving problems. Although students may conclude that $A = \pi r^2$ using a variety of strategies, many struggle to understand the formula conceptually—namely, that there are π number of radius squares r^2 that cover the area of the circle. The activity I describe below has been designed to help these students better understand the *meaning* of the formula using a visual, hands-on approach.

I observed the implementation of the activity in a classroom with 18 sixth-graders in Spring 2019. During the following summer, as part of my work in an advanced methods course, I revised the accompanying lesson plan with a small group of practicing mathematics teachers.

2 Before the Lesson

Prior to this lesson, students spent two days exploring properties of circles (e.g., parts and definition of a circle). On Day 1, students discovered circumference using a variety of different circular objects (e.g., aluminum cans). Students measured the diameter of each object and then, using string, found the length around. Using these measurements, students found and compared ratios of circumference to diameter. The students recognized that each ratio was close to 3. The teacher explained that the ratio of a circle's circumference to its diameter is constant—and that it's commonly referred to as pi (π). From here, students used this observation to derive the formula for circumference as $C = \pi d$.

On Day 2, the day before the Radius Squares lesson (i.e, the focus of this paper), students began an exploration of circle area. Students were given circles printed on grid paper. They estimated the length of each circle's radius and covered each with unit squares to estimate each area. After recording the approximate area and radius of their circles, students looked for a relationship between radius and area. Some students were able to see this relationship on their own while others struggled to find patterns.

3 Lesson Details

3.1 Alignment to Common Core State Standards for Mathematics (CCSSM)

The lesson I observed was adapted from the Connected Mathematics Project (CMP) *Filling and Wrapping* 3.3 activity (Lappan, Phillips, Fey & Friel, 2014). The lesson aligned to Common Core standard 7.G.B.4 "Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle." Although this is a seventh grade math standard, it was routine for the teachers at the school to present the activity at the end of 6th grade, revisiting circle content in greater depth in 7th grade.

3.2 Main Objectives

The teacher identified the following learning objectives for the activity:

- Students will understand the meaning of a radius square and use them to cover a given circle.
- Students will develop a formula for the area of circle and be able to interpret what the meaning of each part of the formula is in relationship to circle drawings.

The intent of the activity was to prepare students for upcoming real-world problems with circles, such as determining how much grass can be watered from a sprinkler that rotates along a circular path.

3.3 Student Activity

3.3.1 Launch

Students were placed in mixed ability groups of 3 or 4 then discussed how to determine the circumference of a given circle. They also summarized findings from the previous day's estimating area activity.

3.3.2 Main Investigation

Two different worksheets were distributed to students—(a) a white paper with circles of various sizes and accompanying questions; (b) a yellow labsheet covered with radius squares. Original versions of these worksheets are available for download at `https://tinyurl.com/circles-quest` and `https://tinyurl.com/yellow-ws`. Figure 1 illustrates several important features of these handouts.

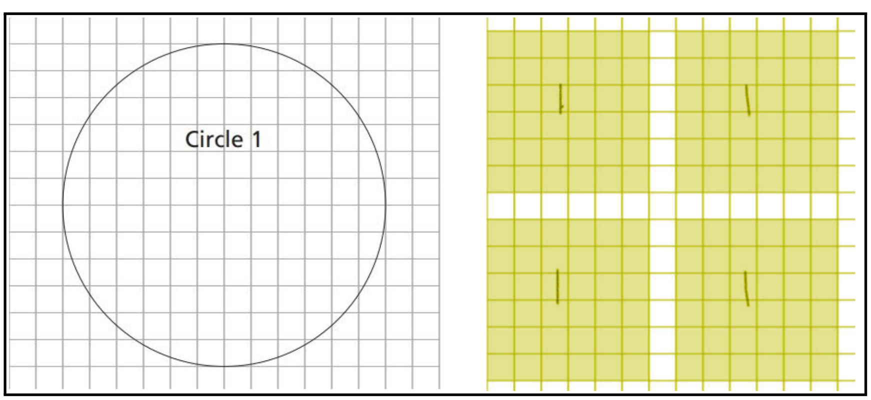

Fig. 1: *(Left) Circle for students to measure; (Right) Four "radius squares."*

The 6th graders estimated the area of the circle shown in Figure 1 (Left) using the "radius squares" shown in Figure 1 (Right). As the name suggests, "radius squares" have side length equal to the radius of the given circle. As students worked to describe the area of the circle, the teacher posed the following questions:

- "How can the area of the radius square help us to figure out how much of a radius square I used?"
- "Do you think I will use a smaller number of radius squares on a smaller circle?"
- "Is there a pattern that you notice with how many radius squares I need to cover a circle?"

4 Student Data

4.1 Sample Student Work

Figure 2 provides several examples that illustrate the variety of methods students chose to cover their circles with unit squares.

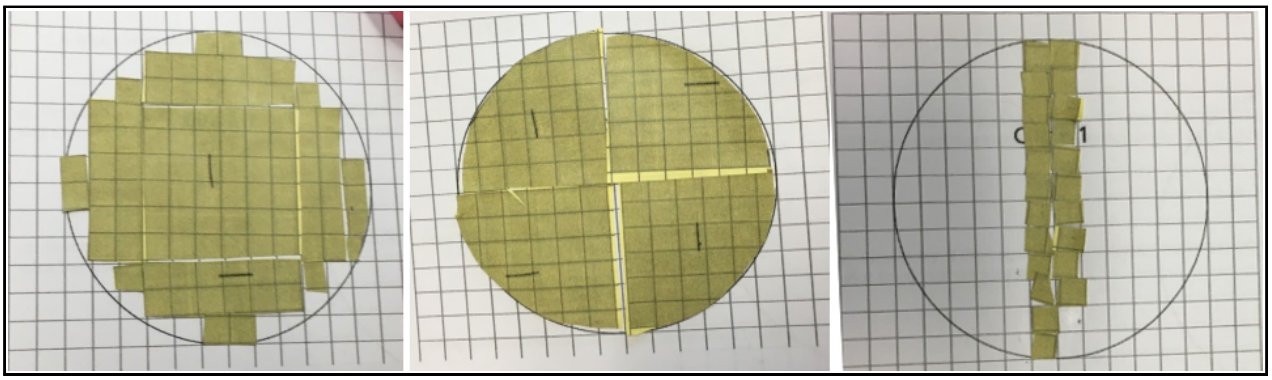

Fig. 2: *(Left) Cover the circle using 1 radius square at a time (cutting and pieces and placing symmetrically around the circle's center); (Middle) Cover the circle with 4 whole radius squares and cut off excess; (Right) Cut out small squares, each ($\frac{1}{36}$) of a radius square, and cover the circle with them one-by-one.*

Figure 3 illustrates student written explanations of their area estimation strategies using fractions and percents. Figure 4 highlights a strategy using decimals.

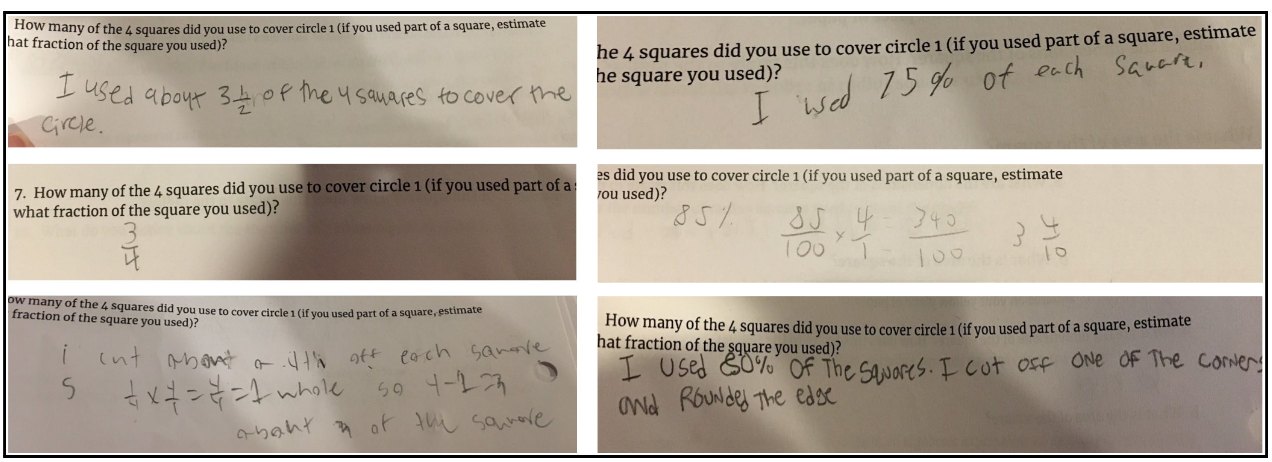

Fig. 3: *(Left) Student work highlighting the use of fractions (Left) and percents (Right).*

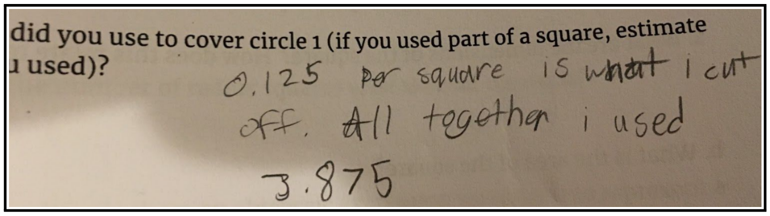

Fig. 4: *(Left) Student work highlighting the use of decimals.*

Overall, 11 of the 18 students estimated the area of the circle to be between 3 and 4 radius squares. Seven 6th graders provided incomplete work. The students' preferred solution methods and performance are summarized in Figure 5.

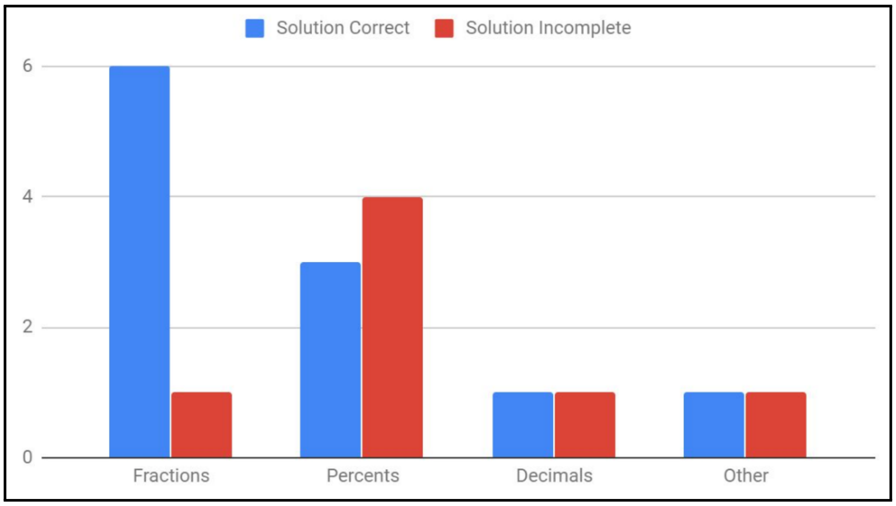

Fig. 5: *Summary of student work.*

4.2 Analysis of Student Work

When students covered their circle using the radius squares, many struggled with the concept of a radius square and believed that when a circle was smaller, they would need less squares to cover the circle. It wasn't obvious to students that the dimensions of the radius square were based on the size of the circle itself. After the students covered their circles, they were asked to determine how many of the 4 radius squares were used. While students were successful in covering their circles, the transition from the visualization portion of the activity to the computational phase (i.e., calculating how many squares were actually used) was a cognitive hurdle for many. Several students considered this problem in terms of percent, noting that they used a certain percentage of each square. However, students found it difficult to translate their percentage into a number of squares used. The same held true for some students who expressed their findings in terms of fractions of radius squares.

5 Revision Analysis

Part of my work in the Advanced Methods summer course (EDT 566) involved the crafting research-based revisions of the *Radius Squares* lesson. My revisions were informed by conversations with colleagues and the following readings: "How Many Times does a Radius Square fit into the

Circle?” (Flores and Regis, 2003) and “Developing Mathematics Identity” (Allen and Schnell, 2016). Specifically, I chose to revise four aspects of the lesson—namely, (1) materials used; (2) activity structure / sequencing; (3) providing students with more voice; and (4) enhanced modifications and accommodations. I discuss revisions in each of these areas in more depth in the paragraphs that follow. A copy of the revised lesson is available for download at `https://tinyurl.com/revised-circle-lesson`.

5.1 Use of Materials

In the article, ”How Many Times does a Radius Square fit into the Circle?”, Flores and Regis (2003) note that there are benefits in providing students choices of what type of materials they choose to use for the activity. I modified the original lesson to provide students with more options. For instance, by printing radius squares on patty paper (or 8.5×11 tracing paper), students can cut around the edges of the circle more accurately, because they are able to see through the paper. Likewise, modifying the activity to use different colors for each radius square helps students see how many radius squares they used to cover the circle more clearly, particularly for students who choose to use one radius square at a time (as in Figure 2 (Left)). This revision will help students determine how many radius squares were used.

5.2 Structure / Sequencing of Activity

Begin the lesson by having all students estimate the area of the same-sized circle, then follow this up with pairs of students exploring circles of different measurements. This idea was inspired by the readings and reinforced by my teacher colleagues. Flores and Regis (2003) note that “by using circles of other sizes, students can also be convinced that the ratio between the area of the circle and the area of the radius square is the same for circles of any size” (p. 365).

- As soon as the first circle is distributed, the teacher circulates around the room, recording the number of radius squares needed for various groups on the class whiteboard. Students find the average of these estimates and record this value.
- The teacher follows a similar procedure as pairs work on unique circles—emphasizing that estimates are relatively close once again, even with different sized circles.
- With these examples, students are encouraged to see that, regardless of a circle’s size, it is covered completely with π radius squares, leading students to the area formula, $A = \pi$ radius squares $= \pi r^2$.
- After this discussion, students are provided with an exit ticket (available for download at `https://tinyurl.com/area-exit-ticket`) to see where their level of understanding is on the area of a circle formula.

5.3 Giving Students a Voice

In “Developing Mathematics Identity,” Allen and Schnell (2016) focus on the importance of giving students a voice in the mathematics classroom. In my revision of the lesson, I emphasize the importance of the teacher looking for opportunities for students to share their strategies with their classmates in small groups and whole-class discussions. This provides students with multiple perspectives on problem solving and supports growth of all students. Those who struggle with the activity are provided with new approaches that encourage continued exploration. Students who understand the problem are provided with opportunities to hone their presentation skills—as we all know, there’s no better way to learn mathematics than to teach it to someone else.

5.4 Modifications and Accommodations

I've created a half-sheet handout to support students who struggle with covering the circle or finding the number of radius squares needed (available for download at `https://tinyurl.com/half-sheet-handout`). The sheets give students more direct guidance, without revealing answers.

For students who finish the activity early, I have incorporated two different extension ideas into the lesson. First, students can be provided with problems that require students to work backwards—that is, they are provided with circle areas and work to determine radii. Secondly, students connect their work with circle area to parallelograms. Specifically, students are provided with a circle that is cut into a specific number of equal pieces (i.e., "sectors" or "pizza slices"). Students are prompted to rearrange these pieces to build a parallelogram (see Figure 6). From here, they're asked to determine the area of the parallelogram—recognizing the height of the parallelogram (i.e., the radius) and the base of the parallelogram (i.e., half of the circumference).

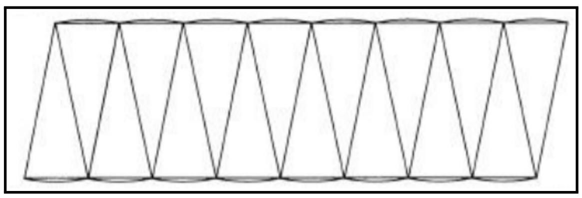

Fig. 6: *Cutting up a circle into slices to create a parallelogram (Flores & Regis, 2003)*

6 Conclusion

The revised lesson provides students with the opportunity to not only find the formula for the area of a circle, but also to visualize the meaning behind the area of the circle. If we are able to help our students make the connection between the formula and the actual meaning of each part of the formula, then the area of a circle formula is one that will stay with them throughout their entire mathematical careers.

References

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