Towards More Inclusive Mathematics: Opening up the Standards for Mathematical Practice

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Abstract: The authors use the concept of expanding the curriculum to demonstrate how the Common Core Standards for Mathematical Practice can include a space for kindness within the mathematics classroom. The authors examine current mathematics practice standards and reflect on how personal tenets create a space for teaching mathematical practices framed in kindness.

Keywords: Standards of Mathematical Practice, curriculum, kindness

1 Introduction

1.1 An Initial Question

We asked a group of preservice education students at the College of Charleston to discuss their experiences as students of mathematics. Candidates considered the question: *What are your thoughts about math and math class in general?* As they discussed responses in small groups, a surprising array of topics rose to the surface—timed tests, SATs, memorization, pressure to show work.

I felt confident in math, but the timed thing threw me off.

As the discussions continued, the comments became more passionate.

I remember the timed tests; they were very stressful. I would think, what matters is that I know how to do it, not how fast I can show that I know it.

Why do I have to show my work to prove what I got is right? If what I got is right, then it is \$#?&%! right, you don't have to see my methodology!*

Nothing made me feel stupider than math, all the rules and theorems to memorize. Why? In real life couldn't you just look them up?

Perhaps none stung more than the following: *Beyond basic math, when will I ever use math in real life?* Such comments made us wonder why students felt so much negativity about mathematics.

1.2 What Research Says

1.2.1 A Dislike of School Mathematics

Many students in the United States see mathematics as difficult and express a dislike for the subject in school. Studies researching attitudes and emotions towards mathematics suggest that students frequently establish negative emotions towards mathematics, which have been shown to be associated with feelings of anxiety, inadequacy and hopelessness (Larkin & Jorgenson, 2016; Brady & Bowd, 2005; Zan & Di Martino, 2007). For instance, a study of elementary students showed that they felt that tasks such as creating dots to represent numbers became a meaningless exercise. The students didn't understand why they were required to complete such work when they could figure out numerical problems using other strategies (Green, 2014). Since students weren't aware of the reason for drawing dots, they came to see this activity as a strange ritual with no connection to their own thinking about mathematics.

When mathematics is reduced to a set of rules and procedures—when it is seen as uncreative, dull, and decontextualized—school mathematics creates a scenario of oppression and exclusion (Wagner, 2011). Wagner (2011) asserts that most mainstream mathematics teaching represents mathematics as a set of closed procedures that are devoid of creativity, rather than as a space where ideas can be freely explored. *If we present mathematics as an authoritarian and totalitarian discipline, aren't we also contributing to the promotion of the authoritarian and totalitarian ideas outside of the classroom?* Considering this question, we work within the context of mathematics education to describe the act of expanding the Common Core Standards for Mathematical Practice in order to imagine a different kind of mathematics (Jagger, 2014; 2018).

1.2.2 Planned and Lived Curriculum

We applied Aoki's (1993) notion of planned and lived curriculum to guide our curriculum expansion work. Aoki described planned curriculum as a curriculum by designers and subject matter experts influenced by current educational paradigms and political currents. For example, the curriculum standards in our home state of South Carolina are named the "College and Career Ready Standards" (SCDOE, 2015). These standards show that curriculum writers' goals and visions focus on career and college preparedness. In reality, a curriculum is more than a document; it is a lived experience of students, teachers, and parents. Aoki's term "lived curriculum" is the multiplicity of lived curricula: what is actually happening in the classroom. The lived curriculum encompasses all nuances of implementation and all responses to the planned curriculum. The lived curriculum has the potential to create positive and organic change, but it also can produce mathematics that is rigid, rule following, decontextualized, and oppressive. Through the act of opening up the mathematics curriculum for our students, we explore possibilities within the lived and planned curricula—a process that requires flexibility to allow for multiple interpretations (Jagger, 2014; Hunter-Doniger, 2018).

1.2.3 Opening Up the Mathematics Curriculum

Opening up the curriculum requires curriculum writers to move away from the transactional discourse of the Common Core standards that are rooted in creating individuals who can compete in the global economy. Instead, curriculum writers need to shift to a transformative discourse that includes listening to everyone's ideas while using mathematics to speak the truth (Miller, 2010). In doing so, educators interpret mathematics in ways that empower students and community (Martin, 2009). The act of opening up the standards to be more inclusive has to be done through continuous and collective discussions involving educators, students, and parents. In 2017, Francis Su, the outgoing president of the American Math Society (AMS), addressed the AMS and the Mathematical

Association of America in a joint conference. He discussed how mathematics should be presented in a way that promotes human flourishing, suggesting that approaching mathematics through virtues such as play, beauty, truth, justice, and love could bring joy to learning (Su, 2017). By embracing such virtues, educators can promote a path to inclusivity in mathematics. These virtues welcome all students, embrace them and say, "I see you, and I share the same transcendent passion for math that you do, and you belong here" (Su, 2017, p. 16). While math is a human construct, it is indisputably found in nature—both throughout the world and in the cosmos. Yet, in mathematics education, too often certain people are singled out as math-worthy. This selection process disregards students who may be interested in math but struggle with it or see it in alternative ways.

2 Mathematics process and practice standards

2.1 Process Standards

The process of standardization in mathematics education has produced two categories of standards: content standards (which include the subject matter of mathematics such as algebra) and process standards. The process standards are divided into problem solving, reasoning and proof, communication, connections, and representations and are the result of educators looking for more authentic ways to describe mathematical proficiency beyond mastery. Process standards recognize that mathematics should be relevant and engaging, providing students with "ways of acquiring and using content knowledge" (NCTM, 2000, p. 29 as cited by Kosko & Norton, 2012).

In recent years, the Common Core State Standards for Mathematics (CCSS-M) have overshadowed the National Council of Teachers of Mathematics (NCTM) standards. With the mathematical practices, the Common Core expands on the idea of process standards. The mathematical practices "describe ways in which developing student practitioners of the discipline of mathematics increasingly ought to engage with the subject matter as they grow in mathematical maturity and expertise" (National Governors Association, 2010). The Standards for Mathematical Practice (SMP) list eight expectations for students:

- 1. Make sense of problems and persevere in solving them;
- 2. Reason abstractly and quantitatively;
- 3. Construct viable arguments and critique the reasoning of others;
- 4. Model with mathematics;
- 5. Use appropriate tools strategically;
- 6. Attend to precision;
- 7. Look for and make use of structure;
- 8. Look for and express regularity in repeated reasoning.

The planned purpose of the practice standards is to create an understanding of mathematics that is better aligned with authentic mathematical practice. There is strong evidence that teachers who implement the Standards for Mathematical Practice in their lessons create tasks that are more cognitively challenging. Unfortunately, "low achieving" students are much less likely to engage their students in the mathematical practices (Schweig, Kaufman, & Opfer, 2020). We argue that *all* students need to understand why and how mathematics works, because without this understanding, students become disengaged and are less likely to "do mathematics." Since the concepts of why mathematics works and how it is found in the world are fundamental in the Standards for Mathematical Practice, focusing on them makes mathematics more inviting and less restrictive to students, which in turn may bring more students to mathematics.

2.2 Opening Up the Standards for Mathematical Practice

We suggest opening up mathematics standards to create a lived curriculum that respects teachers' and students' voices while also being compatible with the tenets of creative and innovative mathematics. According to bell hooks (1994), engaging in dialogue is one of the simplest ways that teachers, scholars, and critical thinkers can begin to cross boundaries between the curriculum writers and curriculum implementers. In other words, opening up is a collaborative process. Opening up curriculum standards does not mean that the practice standards should be abolished or even replaced. Rather, it means that we must take a deeper look at the mathematical process standards and extract the opportunities to make mathematics more inclusive.

On paper, the Standards for Mathematical Practice offer opportunities for students to "do mathematics" rather than to simply memorize the facts and procedures. As part of the process of opening the standards, we first acknowledge embedded barriers to teaching and learning. The act of opening up speaks to the truth of the discipline while advocating for learners. Su (2017) conceives mathematics in a way that embraces attributes of the discipline emphasizing being friendly, generous, considerate and in essence, kind. He focuses on the play, beauty, truth, justice, and love found in mathematics. As such, the concept of inclusivity becomes not just a pedagogical practice but rather a quality embedded within the content. We explore inclusivity while maintaining the rigor and structure of mathematics in our classrooms. In the paragraphs that follow, we discuss a reframing of mathematics as a connected, collaborative, creative, and authentic area of study. We invite educators to think about how existing standards can be re-envisioned using Su's (2017) virtues described in Table 1.

Virtue	Classroom Implementation
Play	Bringing a sense of joy to learning by dissolving the traditional curricular boundaries
	and incorporating math into other disciplines and other disciplines into math
Beauty	Seeing aesthetics in mathematics
Truth	Listening to everyone's ideas and using math to speak the truth
Justice	Bringing in social justice to mathematics
Love	Embracing innovative ways to interpret mathematics so that the classroom becomes a
	place of joy and empowerment

Table 1: Su's virtues and their connection to mathematics classrooms.

Using Su's (2017) virtues as a framework, we use past practices and educational experiences to incorporate play, beauty, trust, justice, and love in a more inclusive mathematics curriculum.

2.3 Examples of Play, Beauty, Truth, Justice, and Love in Mathematics

2.3.1 Play

Approaching mathematics through play brings joy to learning, especially when other disciplines are incorporated into the study of mathematics. In such contexts, students and their teachers have opportunities to explore possibilities and interrelationships. Building on Koch's earlier work (1970), Triandifillis (2006) asked elementary school students to create poems based on mathematical lies (e.g., "addition makes numbers always bigger"). This exercise gave students the opportunity to be playful and humorous about mathematics, and to explore mathematical concepts through contradictions. In practice and in our research, we have asked preservice teachers to create stop motion animations to open possibilities of what a given concept entails both within and outside the discipline of mathematics (O'Byrne et al., 2018). In the study, teacher candidates' notions of randomness evolved as they combined the arts, technology, and mathematics in their creations.

They moved from seeing randomness as a surprise action or lack of pattern to something deeper (e.g., that the random processes create a pattern, such as cutting a folded piece of paper in random places, which turns into a snowflake when unfolded).

Examples such as these illustrate the creation of new curricula through the combination of traditional content areas. This is not a new concept; for instance, romantic philosopher, poet, and scientist Friedrich von Hardenberg (1771–1801), better known as Novalis, had a vision of human knowledge with blurred borders between disciplines and new disciplines created by combining the old. Novalis (2007) expressed his ideas in *The Romantic Encyclopedia*—a document that crystalizes his vision of unified science through cross-curricular entries covering musical mathematics, poetical economy, spiritual physics, and the combination of mathematics and grammar (among other topics). Novalis' died at the age of 29 without finishing his encyclopedia. Nevertheless, many of his fragmented entries have been quite popular throughout the last two centuries. The idea of combining seemingly incompatible disciplines has been slowly incorporated into educational practice and curricula. For instance, in our courses students create mathematical literature as they incorporating mathematics concepts into original poetry to better understand their world and their place in the Universe (see for example, Hunter-Doniger (2018)).

2.3.2 Beauty

In her book *Mathematical Mindsets*, Jo Boaler laments that most students (and adults) in the United States see mathematics as a "performance subject" in which the goal is to "get the questions right" (Boaler, 2016, p. 21), so students rarely see beauty in mathematics and its objects. Yet, Sinclair (2006) noted that students enthusiastically explored beauty by painting, drawing, and playing musical instruments. *If children find beauty and pleasure in the arts, then we can help them see and use beauty in their mathematical explorations*. One way to achieve this is to use the element of surprise. For example, when odd numbers in a Pascal triangle are shaded, the resulting picture is a fractal (Figure 1).



Fig. 1: Shading odd numbers in the Pascal triangle reveals an otherwise hidden structure.

Seeing the world aesthetically involves seeing an underlying structure of an object. We explore mathematical and algebraic structures while working with bilingual and emerging bilingual students in an exercise that includes creating poems in which the syllabic length of the line corresponds to the term in a sequence. For example, Chloe, one of the inservice teachers in our graduate course and a native Mandarin speaker, created a "linear poem" in which each line corresponds to an arithmetic sequence 1, 2, 3, 4, etc. As shown in Figure 2, the poem has aesthetic beauty that represents the mathematical properties of the characters in each line.

Chloe shared her original poem with non-Mandarin speakers in her class, and they reflected on the poem's structural geometric beauty. Chloe also presented the English translation of the poem:



Fig. 2: Chinese linear poem by Chloe.

Spring, The grass turns green, The earth smells fresh, All creatures begin to revive. The queue of wild geese is under the shadow of the cloud, The eagle hovers in the sky high above, It is the starting time of the year. Don't waste then precious time. Go out and enjoy nature!

The students who were not Mandarin speakers could still interact with the poem by reflecting on its geometric structure and the aesthetic differences between the structure and the English translation.

2.3.3 Truth

The act of opening up the curriculum will alter the transactional discourse of the Common Core standards, which focus on creating individuals who can compete in the global economy. Shifting to an expanded curriculum involves transformative discourse that includes listening to everyone's ideas and using math to speak the truth (Miller, 2010). There are two ways that we can see the truth in mathematics. First, we can recognize that mathematics contains universal truths such as 1 + 1 = 2. These universal truths contribute to the power of mathematics. For example, Wagner (2011) states,

Mathematics is powerful. It enables us to model and thus visualize phenomena that the physical tools at our disposal, cannot access. It enables our imagination to explore spaces that conventional wisdom scorns as unreal, impossible or insignificant. It facilitates the management and arrangement of data that exceeds human ability to sense. The list of its powerful qualities goes on. (p. 110).

There is another side of the truth in mathematics. This side comes from challenging "universal truths" through individual resistance to widely accepted ideas. For example, mathematician Tartaglia accepted the roots of negative numbers as mathematical objects in order to explore the structure of the solutions of cubic equations at a time when such views were not popular.

The mathematical truth becomes truths (plural) if we also recognize that mathematics is a cultural activity and that different cultures view mathematical concepts differently (D'Ambrosio, 1985). As Wagner (2011) noted, it is important to recognize that all mathematics are cultural products and not just the mathematics that stem from informal settings. Academic mathematics is structured in a way that may appear culturally neutral and objective, however, the representations as well as the problems come from a specific cultural context. For example, Gerofsky (2011) pointed out that the way that the coordinate system is depicted (with positive numbers going up and negative numbers down) may be linked to Western (and other) religious traditions that view "up" with positive heavenly connotations and "down" with negative connotations and hell.

To help students explore truth and truths in mathematics, educators need to encourage students to communicate their ideas and thoughts on mathematics. In our work with eighth graders, we positioned students as mathematicians and then asked them to define group norms. In doing so, we gave our students the opportunity to define their own practices. One student proposed that listening to other's ideas is something that mathematicians do. We also went beyond the Common Core definition of justifying arguments and critiquing the others' reasoning by reverting to a more organic form of communication: explaining and listening to each other's ideas. We positioned all learners as mathematicians with valuable and important input (Yeh, Ellis, & Koehn Hurtado, 2017). Situating truth this way, we show how justice can be obtained in the mathematics curriculum.

2.3.4 Justice

Mathematical knowledge is powerful, but this is not the case for all mathematics learned in school. Yeh, Ellis, and Koehn Hurtado (2017) used the term "powerful mathematics" to describe the kind of mathematics that we should teach students. Knowledge of powerful mathematics focuses on conceptual understanding, logical thinking, and problem solving. It also involves "reading and writing the world with mathematics" (Gutstein, p. 4). In other words, students should learn how to use mathematics to understand the world and to identify social injustices. Andersson and Valero (2016) point out that although "mathematics education counts in society … society does not necessarily count in mathematics education" (p. 199). The decontextualization of classroom mathematics is not only unfortunate, it is also unethical—mathematics teachers have a moral and ethical imperative to not "close the door (i.e., close off the world) in their mathematics classroom" (Stinson, 2014, p. 2). In other words, there is an ethical imperative to teach social justice issues in mathematics at all levels.

Teachers can "open the door to the world" for their students in a variety of ways. For example, students could analyze the characteristics of Anthropocene, the current geological era characterized by anthropogenic climate change. Analyzing historical and current data including information on sea levels and average temperatures gives students an opportunity to explore the mathematics of climate change (Hunter-Doniger, 2018). In addition, students could look at the mathematics of extreme weather to investigate the risk of tropical storms and hurricanes and their possible link to climate change. An important part of this project is investigating how climate change and extreme weather affect economically-challenged communities. Furthermore, a socially just mathematics curriculum provides opportunities for all students without discouraging children because of race or gender. Incorporating justice in the curriculum illustrates the rightful place of love in the classroom.

2.3.5 Love

Chilean mathematician and educator Jorge Soto Andrade addressed the International Congress of Mathematics Education and proclaimed that "millions of children are exposed to mathematics in a way that turns out to be an inescapable torture for most of them" (Andrade, 2016). In the same lecture, Andrade cited the work of Johnston-Wilder and Lee (2010), stating that regular mathematical "drill and kill" procedures are a form of cognitive abuse. From a strictly reactive point of view, if we want to claim that education is about inclusion we have to eliminate cruelty from mathematics instruction.

One way to go beyond the "drill and kill" exercises is to empower students through schoolwork critiques. For example, we presented seventh and eighth grade students with unrealistic word problems (e.g., "Ali gets \$120.50 a week allowance. A dog costs \$1,578.99. How long will it take Ali to save up for the dog?"). We asked eighth graders to identify what is unrealistic about the problems. They construct more realistic versions and presented them to preservice teachers. The preservice teachers solved the problems and returned the answers to the students. Subsequently, students evaluated and critiqued the preservice teachers' solutions.

Another way to view love is in the context of learners' love and enjoyment of mathematics. If we believe that all students should love and enjoy doing mathematics, then the question of love becomes a question of equity. Therefore, making sure that all students enjoy mathematics should be an important step in lesson planning.

3 Conclusion

In order to show the full creative power of mathematics, we need to expand learning opportunities for students who might otherwise be disengaged or resistant to school mathematics. The act of opening up challenges the finality of definitions. In the case of the mathematics curriculum, we question the static nature of current mathematical practices and invite educators to deconstruct, rearrange, change, and create new practices. Opening and expanding will require teachers to go beyond the planned curriculum to ensure that the lived curriculum is accessible to all students.

Opening up is not easy to achieve, but it is essential. Opening up requires teachers who see mathematics as a "context dependent enterprise in which the focus is on knowing (something dynamic) rather than knowledge (something static)" (Hunter-Doniger, 2018). The curriculum openings we outlined stem from our own experience with our own lived curriculum. Using Su's (2017) virtues that help people flourish as a framework with play, beauty, trust, justice and love within mathematics can be a springboard to create more inclusive mathematical spaces. We invite teachers, students, and parents to create and share their personal openings to contribute to a vision of open, engaging, and creative mathematics curriculum for all.

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