A Modeling Activity: Analyzing Snowflakes Using Geometry

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Abstract: *The* Snowflakes Task *is a mathematical modeling task that explores the geometry of snowflakes. During this task, students categorize and analyze five purposefully selected snowflakes using geometric concepts such as symmetry and angles and use their knowledge of geometric attributes to create a mathematical model that could be used to analyze other snowflakes. By the end of the task, students will become familiar with different snowflake shapes, further their understanding of geometric concepts, and describe the relationship between snowflakes and geometric attributes. The authors implemented the* Snowflakes Task *in two different settings: once with prospective teachers in a virtual setting and once with middle school students in person. In this paper, the authors share the task sequence, a summary of implementations, including adaptations made to support our implementation settings (virtual or in person), and student thinking revealed during data analysis.*

Keywords: *Mathematical modeling, geometry*

Introduction

"Mathematical modeling is a process that uses mathematics to represent, analyze, make predictions or otherwise provide insight into real-world phenomena" (COMAP & SIAM, 2016, p. 8). Modeling tasks elicit student thinking and enable students to interpret situations and find solutions (e.g., Aguilar & Battista, 2017; Mousoulides, Pittalis, Christou, & Sriraman, 2010). Using this rationale, we developed a modeling task for which the real-world phenomenon is snowflakes. In this task, students (1) categorize and analyze snowflakes and (2) generalize their findings to develop a model that could be used for future snowflake analyses. The task aims to foster student experiences where geometry is used to describe and explain real-life situations.

The Common Core Standards for Mathematics describe modeling as "the process of choosing and using appropriate mathematics and statistics to analyze empirical situations, to understand them better, and to improve decisions" (NGA Center & CCSSO, 2010). While solving modeling tasks, students have the opportunity to create their own models (COMAP & SIAM, 2016). Using multiple approaches, they generate different answers based on their unique models (Cirillo et al., 2016). The focus is to encourage students to develop and apply mathematical methods and help them reason to make assumptions and decisions that would improve their models (COMAP & SIAM, 2016). Therefore, modeling tasks lend themselves to multiple entry and exit points and opportunities for differentiation.

Learning Goals of the *Snowflakes Task*

We (one pre-service teacher and one mathematics teacher educator) developed the *Snowflakes Task* where students explore the geometric structures of snowflakes. Students analyze a series of snowflakes using geometric concepts and attributes to create a mathematical model to be used when analyzing other snowflakes. By the end of the activity, students will further their understanding of geometry concepts and describe the relationships between defining characteristics of snowflakes and geometric attributes. Also, by noticing the connections of a real-world phenomenon to mathematics, students will experience the necessity of mathematics in explaining real-life situations.

The *Snowflakes Task* aligns with multiple Mathematics Content Standards and Standards for Mathematical Practice (NGA Center & CCSSO, 2010). Depending on students' current knowledge of geometry, the specific standard(s) addressed by the task can vary. A sample of aligned standards are as follows: 3.G.A.1., 4.G.A.1., 4.G.A.2., 4.G.A.3., SMP-3, and SMP-4.

Description of the Snowflakes Task

The *Snowflakes Task* consists of four parts: (1) recording notices and wonders of a snowflake video, (2) categorizing a series of snowflakes, (3) analyzing the snowflakes, and (4) developing an analysis guide (i.e., model). The original task was intended for virtual instruction but later adapted for in-person instruction. In this article, we share two implementations. The first implementation was with university students who were prospective teachers (PTs). The instruction took place on Zoom in individual, small group (in breakout rooms), and whole class formats. To allow students to work collaboratively and share work with the entire class, Google Jamboard was used. We collected data from 20 PTs. The second implementation was with eighth-grade middle school students. Students worked individually and in groups to complete their handouts (see Snyder and Kara [2021] for an example handout of the task). We collected data from 28 middle school students. The first author was the primary teacher during both implementations.

Focusing on geometric concepts, we analyzed students' work to identify common themes within each part of the task sequence and determined frequency counts of each theme. Next, we share the task sequence, a summary of implementations including adaptations made to support our implementation settings (virtual or in-person), and evidence of student learning revealed during data analysis.

Part 1: Notice and Wonder

In the first part of this modeling task, students are shown a video of a slow-motion microscopic view of a growing snowflake and asked to record what they notice and wonder (see Figure 1 for the final image of the snowflake on the video). Once students record their responses, the teacher orchestrates a discussion during which students share their ideas. The teacher highlights notices and wonders that appear most frequently and statements that focus on geometric attributes. These discussions function as a great transition into the next part of the task.

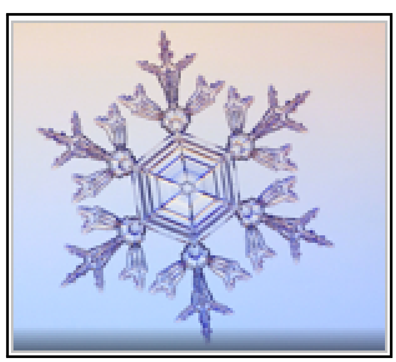

Figure 1: *The end image of snowflake video (image taken from Libbrecht, n.d.).*

In our implementations, notices and wonders from both groups revealed several common themes. The most common themes in participants' responses were (1) identifying different geometric shapes in the center of each of the snowflakes(s), where both groups stated that they noticed various patterns (e.g., hexagon) within the center of the snowflake(s), and (2) identifying (line) symmetry in the snowflake(s).

Part 2: Categorizing Snowflakes

The second part of this modeling task is motivated by the scenario: "Olaf was outside playing in the snow with Elsa, Ana, and Sven when he noticed different snowflakes falling from the sky. Olaf wants to sort the snowflakes he created." Students are then placed into groups and prompted to sort each of the five snowflakes (see Figure 2) into categories they determine. Anticipated responses include reasoning through various angles, line segments, shapes, edges, and symmetry. During this time, the teacher asks assessing and advancing questions driven by the learning goals of the task. Some examples of assessing questions include "How did you determine that the snowflake has [or does not have] that particular attribute [e.g., line symmetry]?", "How did you come up with your categories?", "How did you decide that snowflake goes into that particular category?" Some examples of advancing questions include, "What could be another set of categories that can be used?" and "How would you modify Snowflake [X] to fit into that category?"

Figure 2: *Pictures of slowflakes (images taken from Libbrecht, n.d.).*

In our implementations, students worked in small groups, recorded their responses on their handouts, and later shared their work during the whole-class discussion. Additionally, PTs recorded their work on Google Jamboard (Google Workspace, n.d.). While monitoring students' work on categorizing snowflakes, one particularly helpful prompt was asking for consideration of geometric attributes; especially those related to angles, shapes, and angle measurements. After groups completed their categorization, they participated in a gallery walk, during which students observed other groups' written work and asked questions to each other for further explanation and/or clarification. The gallery walk provided an opportunity for students to understand each other's reasoning, compare their own work with others' work, and share feedback. Discussion of similarities and differences of snowflakes through categorization served as a transition into Part 3 of the task sequence.

Both groups of PTs and middle school students used categories related to snowflakes' geometric attributes (e.g., symmetry, shape, and angles). The most common attribute in both groups' categorizations was related to (line) symmetry. Out of 20 responses from PTs, 11 of them stated the snowflakes shared the similarity of symmetry. Other categories identified by the PTs included "edges (rounded vs. pointed)," "concaveness versus convexness," and "open versus closed" qualities within the snowflakes. Another common category in the responses of middle school students was related to the number of sides/segments of snowflakes, such as sharing the similarity of six sides.

Part 3: Analyzing Snowflakes

For the third part, students analyze the snowflakes following the prompt, "How should Olaf analyze the group of snowflakes? Make a mathematical argument about how geometric shapes and attributes could be used to describe Olaf's snowflakes." The teacher's role is again to assess and advance student thinking towards the learning goals of the task. Some examples of assessing questions include, "What specific characteristics are noticeable in Snowflake [X]?", "What types of angles do you notice in Snowflake [Y]?", and "What [geometric] shapes do you observe on each snowflake?" Some examples of advancing questions include, "What other symmetry types can be observed in these snowflakes?", "How do concaveness and convexness differ in our snowflakes?", and "How could we use angles and their measurements to extend our thinking in snowflakes?"

In our implementations, students' analyses included more in-depth mathematical reasoning than their work in the previous parts. Identifying the symmetrical structure of snowflakes was the most common theme in both groups' responses. Another common theme in the PTs' responses was identifying concaveness and/or convexness within different parts of snowflakes. The second most common theme in the middle school students' responses was the six-sided structure formed in each figure. The most common themes and frequency counts of these observations for each snowflake are provided in Table 1.

Table 1: *Common themes and frequency counts identified in participants' analyses*

Implementation 1: PTs' responses during virtual instruction					
Description of Theme	**Frequency for Snowflake #1**	**Frequency for Snowflake #2**	**Frequency for Snowflake #3**	**Frequency for Snowflake #4**	**Frequency for Snowflake #5**
Symmetry	9	9	11	9	15
Concaveness and/or convexness	8	10	12	10	13
Angles included in the snowflake	8	14	11	10	3
Polygons inside of the snowflake (e.g., hexagon)	3	5	7	9	5
Made of line segments	11	3	2	3	3
Implementation 2: Middle school students' responses during in-person instruction					
Description of Theme	**Frequency for Snowflake #1**	**Frequency for Snowflake #2**	**Frequency for Snowflake #3**	**Frequency for Snowflake #4**	**Frequency for Snowflake #5**
Symmetry	15	14	11	12	12
Six-sided figure	10	12	8	10	11
Edges (pointed vs. round)	19	6	6	5	6
Polygons inside of the snowflake (e.g., hexagon)	2	9	4	9	8
Rotational symmetry	6	6	6	7	6

Part 4: Analysis Guide for Snowflakes

In the final part, students are asked to create a model/analysis guide to analyze future snowflakes and explain how their guide would help Olaf better understand snowflakes in an organized way. They are asked to work in groups and use their analyses from earlier parts.

The term “analysis guide” is new to most students. During our implementations, some groups created a guide to relate all snowflakes, whereas others made rubrics, bullet points, and full explanations. One PT group made a yes/no checklist for the geometric attributes they identified. Some groups considered what a snowflake “could have” based on its geometric attributes. It was beneficial to ask students to consider other snowflakes and prompt them to think about what similar/different characteristics snowflakes could have. Another helpful prompt was asking students to consider what questions they would use in their analysis guide.

In our implementations, students considered similarities and differences in snowflakes’ geometric attributes while creating their analysis guides. The most common theme in the PTs’ guides was again line symmetry. Eight out of 20 PTs and 24 out of 28 middle schoolers stated all snowflakes have line symmetry. One PT noted “all snowflakes have lines of symmetry” in their analysis guide whereas a middle school student stated this attribute in their guide with “whether it is symmetrical.” Themes and their frequency counts of these themes are summarized in Table 2.

Table 2: *Common themes and frequency count identified in participants’ analysis guides*

Implementation 1: PTs’ responses during virtual instruction	
Themes	**Frequency Count**
Line symmetry	8
Concave vs. convex	7
Polygons inside of the snowflake	6
Exterior structure of a snowflakes (end shape)	5
Implementation 2: Middle school students’ responses during in-person instruction	
Themes	**Frequency Count**
Line symmetry	24
Number of sides	16
Point or rounded edges	14
Made of different shapes	13
Exterior or end shape	7

Extensions and Modifications

In modeling tasks, it is fundamental to consider students’ background knowledge. Introduction and closure activities serve as valuable informal assessment tools. We used a notice and wonder activity to introduce the task. In terms of geometric attributes, other tasks such as “Which snowflake does not belong?” could also be helpful to assess students’ initial understanding of snowflakes.

A potential closure activity is having students create their own snowflakes and analyze these snowflakes using the guide they created. They can use or notice new geometric attributes during the activity and make revisions to their guides if needed. We included this extension in both implementations (see Figure 3 for examples of snowflakes created by students). PTs used online resources (e.g., Snowflake Maker, n.d.; The Math Learning Center, 2022a, 2022b) to create their snowflakes and later shared them with Jamboard. During the in-person implementation, middle-school students created snowflakes using construction paper and scissors. Many of the snowflakes created by the students included line symmetry, which was an attribute they noticed during their earlier analyses. For this closure activity, some examples of potential assessing questions include, “Why did you decide to make your cut on this side of the snowflake?", “How does cutting straight lines differ from curved/ segments?”, “What made you come up with a hexagonal shape in the center instead of a rectangle [or other shapes]?”, “What are the lines of symmetry on your snowflake?”, “What polygons can be observed in your snowflake? Why do you think these particular shapes occurred in your snowflake?", and "Describe the concave and convex shapes that you observe in your snowflake." Some examples of advancing questions include, "How does folding the paper help us make a pattern?" and "If you were to make a second snowflake, what would you change or do differently? Why?"

We also recommend a closure activity during which students are presented additional resources on snowflakes' characteristics and pictures (e.g., resources on the design of snowflakes and why two snowflakes will not be the same). Next, students can write a reflection paragraph on their takeaways from the lesson and their thoughts on snowflake designs. Having students describe their own takeaways will help them (1) organize their thoughts in terms of the key ideas of the task and (2) take the ownership of their own mathematical reasoning and learning.

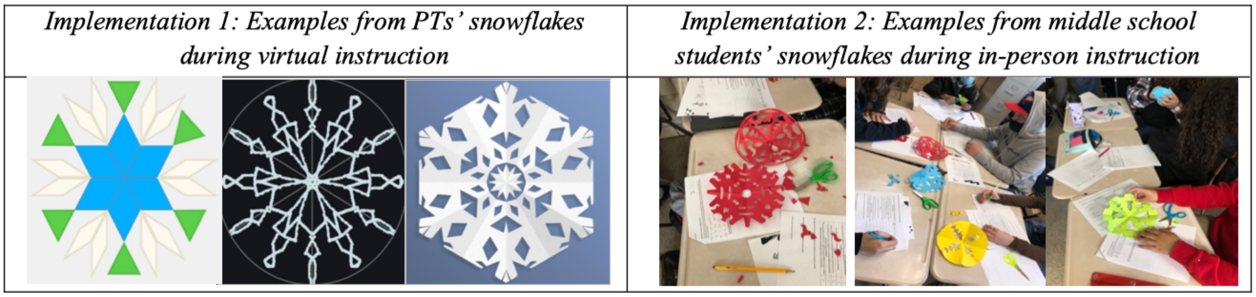

Figure 3: *Examples of students' work from implementations.*

Final Thoughts

In the *Snowflakes Task*, the sophistication of students' analyses was related to their current understanding of geometric concepts. Working in groups, students communicated their mathematical ideas and built on each other's ideas, which led to revisions and refinement of their understanding and analysis of snowflakes. Finally, they did gallery walks to share their feedback and thoughts on their peers' work.

The *Snowflakes Task* provided nontraditional learning opportunities. Students formed new ideas by applying their prior geometry knowledge in each scenario. By participating in modeling activities, students are given opportunities to build meaningful experiences, while connecting mathematics to real life. This activity serves as an informative example when creating other modeling activities that focus on using mathematics to understand real-world situations. The format of the task can be adapted to other relatable contexts such as cultural artifacts, structure of traditional ornaments, or sports. Teachers can choose the context based on their students' interests. Applying students' interests in modeling tasks makes the mathematics of these tasks more relevant and engaging.

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