Fostering Mathematical Inquiry with Language Independent Board Games

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Abstract: *The authors discuss an approach for engaging students in mathematical problem solving and argumentation in a manner that supports* all *learners, including those for whom English is a non-native language (i.e., ELLs). Building on the work of Norval and Castaneda (2020), Wanko (2017), and others, the authors engage students in a language-independent game,* Where are the Cherries?*. English-speaking students learn the rules of the game as they analyze an instructional video in Chinese. Students strengthen their problem solving skills as they develop a greater appreciation of the language challenges that ELL classmates face on a daily basis.*

Keywords: *English language learners, ELLs, language-independent math games*

1 Introduction

Given the central role that language and communication play in the teaching and learning of school mathematics, it's not surprising that English Language Learners (ELLs)—those for whom English is not a first language—often struggle in English-only classrooms. For instance, Norval (2019a, 2019b) and Castaneda (2020) have written about ways in which test items—including those from highly regarded sources such as the National Assessment for Educational Progress (NAEP)—privilege native speakers. As Norval (2019a) notes, "Although some may view mathematics as a universal language, this is not the case" (p. 30). Citing Wolf and Leon, Norval notes that mathematical tasks are often linguistically demanding for ELLs.

> "They (ELLs) must be taken into account in our education system as students with unique challenges and considerations. Treating students as a monoculture without accommodating for individual needs impedes efforts of ELLs to thrive academically" (2019a, p. 29).

Wanko (2017) and others have advocated using language-independent puzzles to engage *all* students in constructing mathematical arguments. Wanko notes that language-independent puzzles "(don't) require any reliance on knowledge of any specific language . . . once the solver knows the goal and the rules that govern a puzzle, it does not matter if the puzzle appears in a Japanese, a Polish, or an American publication—the solver still knows what to do. This is very different for crossword puzzles and other language- (and culture-) dependent word-based puzzles" (p. 515).

Rather than directly providing game play rules to his students, Wanko presents sample pencil-and-paper puzzles along with their solutions and encourages students to determine rules inductively.

In the following sections, we discuss another approach for engaging students in language-independent problem-solving. Rather than posing paper-and-pencil tasks to students, we share videos of game-play in an unfamiliar language (e.g., Chinese). As native English-speaking students watch math videos in another language, they must do so carefully, relying on non-verbal cues rather than spoken explanations to determine the rules of play. Invariably, students generate questions about gameplay as they watch—for instance, *What is the objective of the game? Does the game have a winner and a loser? What mathematical aspects of the game can be modified? How do these modifications impact gameplay?* In this regard, videos and paper-and-pencil puzzles serve a similar role—namely, engaging students in inductive reasoning. In addition, non-native videos provide an additional benefit—through first-hand experience with another language, the videos encourage native speakers (and teachers) to more fully appreciate the challenges that ELLs face in mathematics classrooms.

2 *Where Are the Cherries?* Basics of Game Play

2.1 Game Board

The variation of the game we play uses a board similar to Monopoly. Figure 1 illustrates that the rectangular board has four spaces along each side, with a large center region.

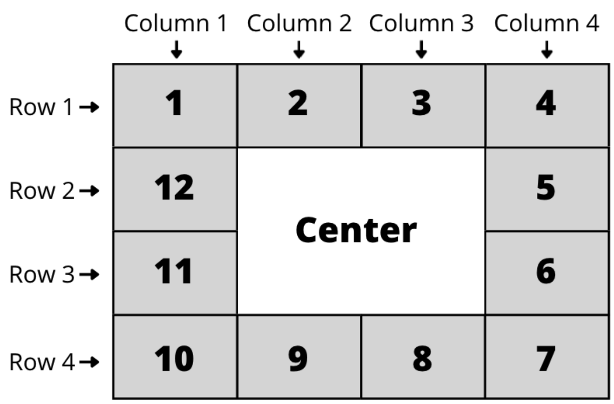

Figure 1: Where are the Cherries? *game board.*

In Figure 1, we've numbered spaces in a clockwise fashion, starting with the upper left space as "1." Note that we can also refer to spaces by their row and column. For instance, "Space 3" and "Row 1, Column 3" are equivalent locations. In our gameplay discussions, we'll refer to spaces on the board by number or position, whichever makes the most sense in context.

2.2 Game Pieces

Traditionally, students play the game with cherries as play pieces; however, any small object will suffice. Pennies, chips, tiles, and small candies all work well.

2.3 Setting up the Game Board

Initially, we place pieces so that each row and column have the same number and the center isn't empty. Figure 2 illustrates a game board with six pieces in each row (i.e., Rows 1 and 4) and six pieces in each column (i.e., Columns 1 and 4). We've placed three pieces in the center. Figure 3 emphasizes that each row and column of the board have the same number of pieces.

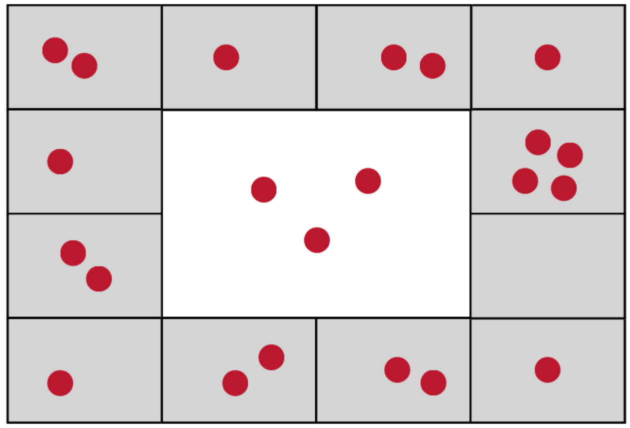

Figure 2: *Where are the Cherries? game with six pieces in each row and column of the board.*

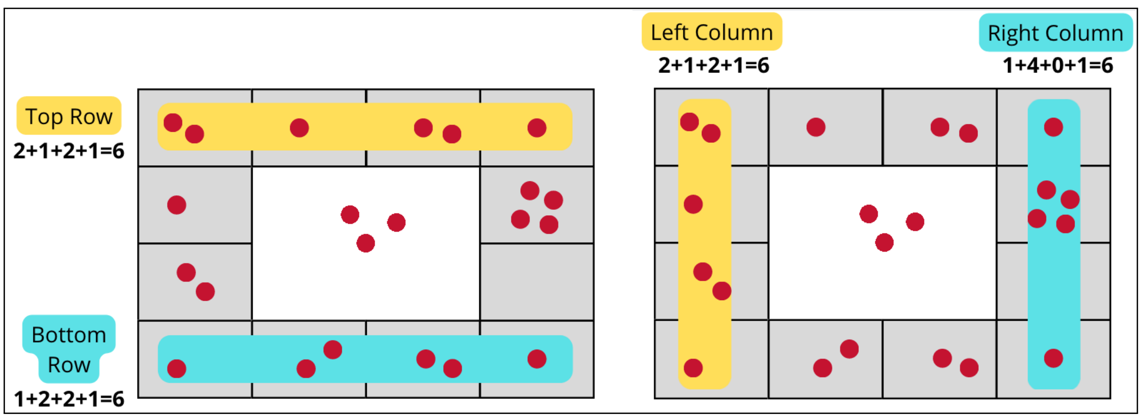

Figure 3: *At the start, each row and column of the board should contain the same number of pieces.*

(*Editors' Note: Students can explore some genuinely exciting mathematics when they set up the board. For instance, try constructing a setup with seven boundary pieces with three in each row and column. Doing so helps acquaint students with the game and its rules*).

2.4 Legal Moves

Students take turns moving pieces out of the center while keeping the number of pieces along each row and column the same (i.e., constant). At each turn, two pieces are moved: (1) a player moves a non-center piece to an adjacent space, and (2) a player moves one piece from the center to any of the board's 12 spaces. Once a player moves a piece from the center, it must remain in the boundary. In Figure 4, we provide a link to a video example that highlights various aspects of gameplay.

2.5 Object of the Game

The game's object is to move all pieces out of the center while keeping the total number of pieces in all columns and rows constant. The game can be played competitively (with the first player clearing off the board deemed the winner) or collaboratively (with players moving pieces together until the center is cleared or no more valid moves are possible).

3 Introducing *Where are the Cherries?* to Students

Rather than initially revealing rules of play to our students (like we did with you, our readers!), we share a video of *Where are the Cherries?* being played. Although the game is explained wholly in

Chinese, our English-only students learn how to play the game by watching (and listening) carefully. Before reading further, we encourage *you* to watch the video in its entirety using the link in Figure 4. *Do the rules of the game seem clearer to you now that you've watched a round of the game in action?*

Figure 4: *Chinese teacher introduces the* Where are the Cherries? *activity in a short YouTube video (Source:* `https://tinyurl.com/where-are-the-cherries`*).*

The following dialogue illustrates how we introduce the game to typical secondary-level mathematics students.

CANDACE : What is *this*? Mr. Edwards, you've outdone yourself this time! *The guy in the video. He's speaking in Chinese. [Rolling eyes sarcastically]* We don't know Chinese, *Mr. Edwards.*
LUIS : Yeah! How are we supposed to do this?
TEACHER : Calm down! Just watch! Give it a chance!

When the video reaches the 0:19 mark, the Chinese teacher shows the game board for the first time. A screenshot of the initial setup of the board along with game pieces (i.e., cherries) is provided in Figure 5. Note that this board has 8 cherries in each row and column, with 6 cherries in the center.

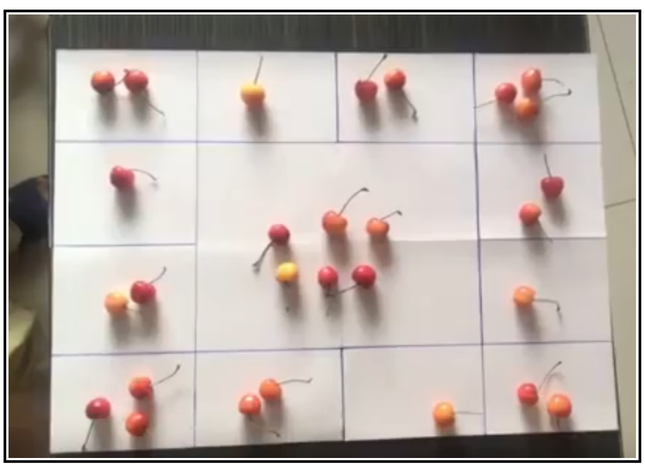

Figure 5: *Initial setup of* Where are the Cherries? *game board from the YouTube video.*

TEACHER : [*Pausing the video at 0:19*] Class! Check this out. [*Walking to a student in the back of the classroom*] Lisa, what do you notice about the game board?
LISA : Uggh . . . I don't know! [*Several classmates laugh*]
TEACHER : [*Reassuringly*] That's okay. Let's look at the board together. What is the teacher using as game pieces?
LISA : Cherries?
TEACHER : Yes! Of course! Good, you *can* see! [*Turning to another student*] John?

JOHN : Yes?
TEACHER : How many cherries do you see?
LISA : I see 28. There's 6 in the center and 22 along the edge.
JOHN : [*Pointing his finger in the direction of the screen*] Oh! I see two here, one there, two there, and three there.
TEACHER : Where are you talking about? Along this top row? [*Targeting his laser pointer along the top row of the board from left to right*] *Two-one-two-three*.
JOHN : No! That's not where I was looking. Look along the left column. [*Pointing to the top left corner*] *Two*. [*Then moving down one space*] *One*. [*Moving down one more*] *Two*. [*Moving to the bottom left corner*] *Three*.
LISA : Hey! They *all* have 2-1-2-3.
TEACHER : What do you mean, Lisa?
LISA : Well. The game board has two rows and two columns. In the top row, there's 2-1-2-3. In the bottom row, there's 3-2-1-2. That's the same numbers, but in reverse. The two columns do the exact same thing.
TEACHER : What do you think about this, class?
SEVERAL VOICES : She's right!
TEACHER : Okay. These are *terrific* observations. Let's watch a bit more of the video and see if these patterns continues once the game begins. [*Unpausing the video at 0:19*]

At approximately 0:51, the teacher moves a cherry out of the center of the board to Column 1, Row 3. This is quickly followed by a move of another cherry out of the bottom left corner (i.e., Space 10) to an adjacent space (i.e., Space 9). These steps are represented graphically in Figure 6.

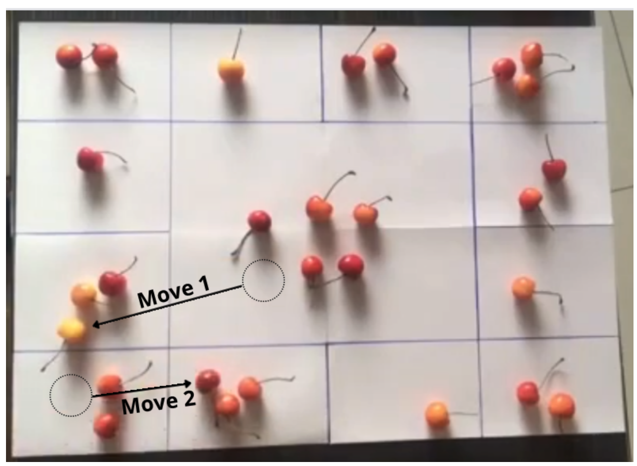

Figure 6: *Initial teacher moves for* Where are the Cherries? *game.*

LUIS : Hey! He moved one out of the center and another one out of the corner!
LISA : [*At the 0:55 mark*] The same numbers are still the same, but in a different order. [*Pointing to the first column*] See? 2-1-3-2. [*Pointing to the bottom row*] See? 2-3-1-2.
LUIS : Since they're the same numbers in each row and column, the sums must be equal.
TEACHER : What do you mean, Luis?
LUIS : The number of cherries in each row and column never changes. There were eight cherries in each row and column BEFORE the center piece was moved AND after.
LISA : He took one out of the center without changing the number in each row and column.
TEACHER : Oh my gosh! Do you see what she's talking about? [*pointing to rows and columns of the game board*]

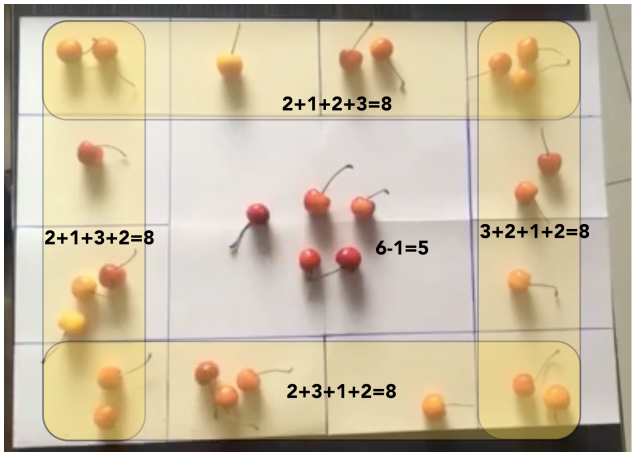

Figure 7: *Each row and column has eight pieces. One piece has been removed from the center (`0:55`).*

TEACHER : See?? [*Pointing to the top row.*] 2-1-2-3. That's 8!

STUDENTS : Uh huh.

TEACHER : The bottom row has 8, too. [*Pointing to bottom.*] 2-3-1-2. Again, 8!

LISA : But there's one less in the corners. There were ten corner pieces. Now there's only nine!

TEACHER : That's a terrific point, Lisa! We've all made some excellent observations. Let's watch the rest of the video and see if we can determine the game's rules.

We continue similarly, watching the video and pausing to discuss gameplay details. As we watch, the goal of the game becomes increasingly evident. By the end of the video, students understand that they need to move all pieces out of the center while holding row and column sums constant. The screenshot in Figure 8 illustrates a completed game board.

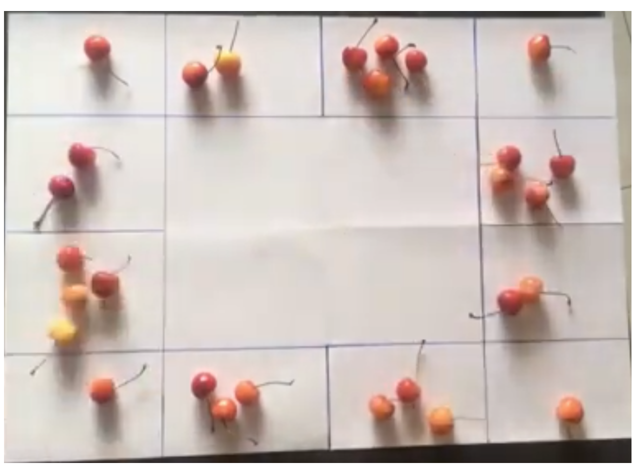

Figure 8: *A completed game board with no center cherries and eight game pieces in each row and column (`2:37`).*

Our scaffolded-viewing approach is helpful for students in at least two ways: First, it reduces possible frustration with language and content demands associated with the Chinese instruction. Secondly, it models problem-solving strategies, particularly those involving the decoding of symbols and language.

4 The Mathematics of Game Play

Recall two key features of the example game from the video (see Figure 5): (1) Initially, there were ten cherries in corner spaces 1, 4, 7, and 10; (2) The center contained six cherries. Based on these observations, we say that a $(10, 6)$ board is "solvable" since we were able to complete the game for a board with six initial center (c) pieces and ten corner (k) pieces. This raises a series of interesting questions—namely,

- Is every (k,c) board solvable?
- If so, how can we verify this?
- If not, which (k,c) configurations yield unsolvable boards?

We explore these questions in the remainder of this paper.

4.1 Are All Boards Solvable?

The figures below show the process of moving cherries using a simplified board with one cherry in each corner and one in the center. We use a board with fewer pieces to emphasize the importance of corners in the game.

4.1.1 Helpful Observations

Note that in this initial configuration, $k = 4$, $c = 1$, and each row and column sum equals 2. Note, too, that cherries in the corner are counted twice: once vertically and once horizontally. Thus, moving a game piece out of a corner and into a row reduces the sum of a column by 1 (e.g., a move from Space 1 to Space 2 reduces the sum of Column 1 to 1). Similarly, moving a game piece out of a corner and into a column reduces the sum of a row by 1 (e.g., a move from Space 1 to Space 12 reduces the sum of Row 1 to 1). We highlight this latter move in Figure 9.

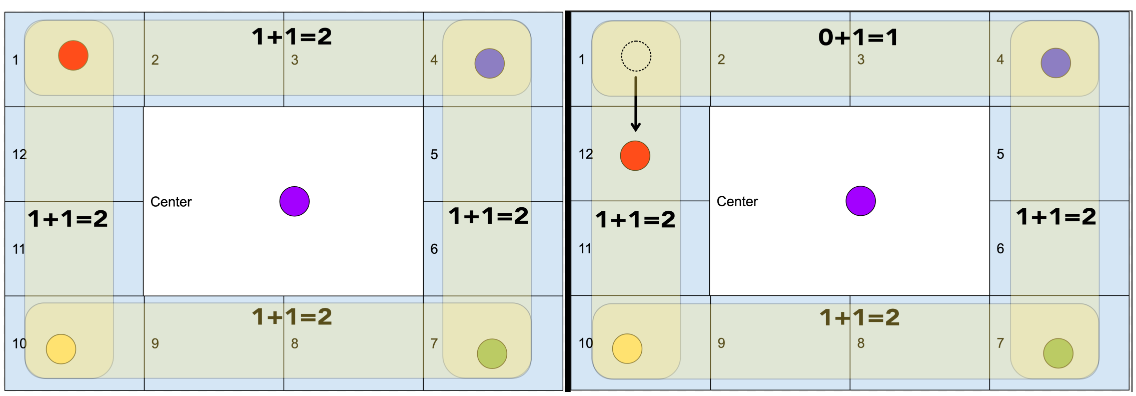

Figure 9: *(Left) Initial game board configuration; (Right) Moving a piece out of the top, left corner reduces the sum of row 1 by 1.*

To keep the sum of pieces fixed in each row and column for the board in Figure 9, we need to move the center piece to Row 1 to compensate for the missing piece. We highlight this approach in Figure 10.

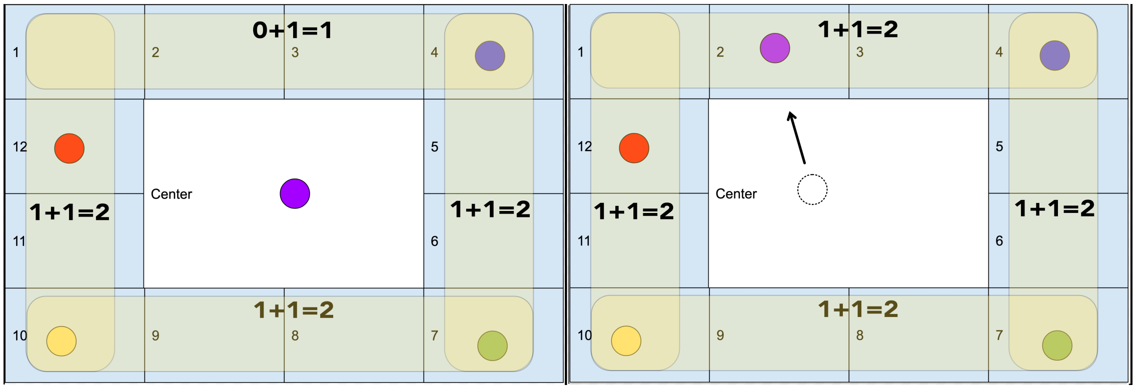

Figure 10: *(Left) Initial game board from Figure 9 right; (Right) The board is solved by moving the center piece to Row 1, Column 2 (i.e., Space 2).*

The preceding example shows that a $(4, 1)$ board is solvable. Note that the two boards we've explored thus far were both solvable. Note, too, that in both the $(4, 1)$ case and the $(10, 6)$ case, the number of corner pieces, k, was strictly greater than the number of pieces in the center (i.e., $4 > 1$ and $10 > 6$, respectively). What if we made the number of center pieces greater than the number of corner pieces? Will the board remain solvable?

4.2 A Counterexample

Consider the $(4, 5)$ board in Figure 11, left.

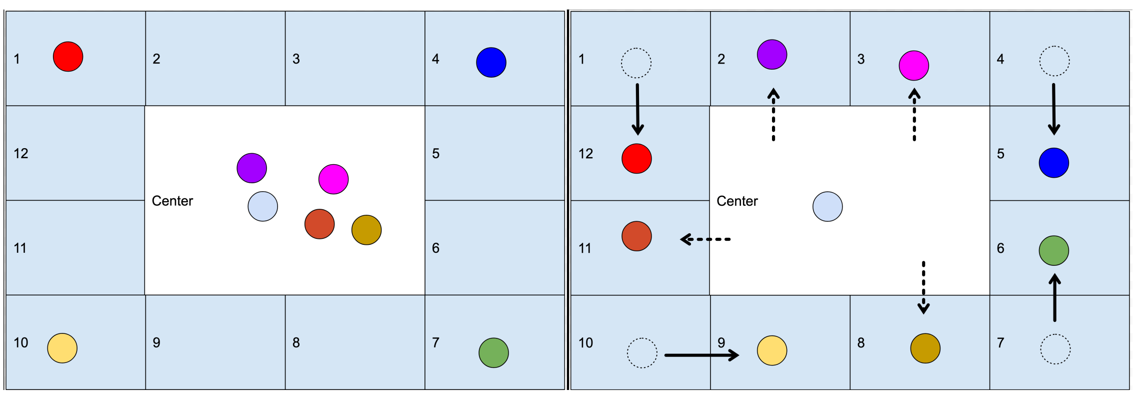

Figure 11: *(Left) Initial board configuration; (Right) Our earlier strategy leaves one piece in the center.*

In Figure 11 (right), four pieces were moved out of corners. Moving a piece out of a corner reduces a row or column sum by 1, so moving four corner pieces provides "room" for four pieces to come out of the center. Since we started with more than four pieces (namely five), we won't be able to complete the game. Moving the remaining piece out of the center will result in one row or column having a sum of 3, with the others summing to 2. Thus, the $(4, 5)$ board is not solvable.

4.3 Solvable Pairs

4.3.1 Looking for Patterns

Based on the discussions in the previous sections, we invite readers to play several rounds of *Where are the Cherries?* with different values of k and c. Before reading further, look for an easy method to determine if a (k, c) board is solvable or not. Try your approach with several initial configurations. Record your findings for each ordered pair.

4.3.2 A Conjecture

Theorem 4.1. *If k is the initial number of corner pieces, then (k, c) is solvable if and only if $k \geq c$.*

Below, we provide two possible arguments in support of Theorem 4.1. The first is our "official" proof. The second is a less formal alternative, written with minimal notation for students who are less familiar with mathematical argumentation. We encourage you to play *Where are the Cherries?* with your students. While you're welcome to use the theorem and proofs provided in this paper with your students, it's absolutely essential that your students move beyond reading others' work. Encourage them to generate their own conjectures and write their own arguments so they may come to see mathematics as a vehicle for creative thought and expression.

Proof (Official version). Let m denote the fixed sum of each row and column. Let t represent the total number of pieces on the board and let a_i denote the number of pieces placed on space i. Let $b = \sum_1^{12} a_i$ denote the total number of pieces along the board's boundary. Since the sum of every row and column is held constant, we know that

$$\begin{aligned} a_1 + a_2 + a_3 + a_4 &= m \\ a_4 + a_5 + a_6 + a_7 &= m \\ a_7 + a_8 + a_9 + a_{10} &= m \\ a_{10} + a_{11} + a_{12} + a_1 &= m \end{aligned}$$

Adding all four equations above, we obtain $b + (a_1 + a_4 + a_7 + a_{10}) = 4m \implies b + k = 4m \implies k = 4m - b$. Next, note that pieces lie within the boundary or center at all stages of gameplay, so $t = b + c$. Combining these results,

$$\begin{cases} k = 4m - b \\ t = b + c \end{cases} \implies$$

$$\begin{cases} k = 4m - b \\ c = t - b \end{cases}$$

To confirm desired inequality, we argue that $4m \geq t$. Note that when there are no corner pieces (i.e, $k = 0$), every boundary piece is counted exactly once (i.e., $4m = t$). When a piece lies in a corner, the piece is counted as both a row and column piece, and thus the total number of boundary pieces is strictly less than the number obtained by adding each row and column sum. In other words, double-counting makes $4m$ larger than t. Thus, we have shown $4m \geq t$.

$$k = 4m - b \geq t - b = c \implies k \geq c$$

□

Proof (Alternative version). Let k refer to the total number of corner pieces and c refer to the total number of pieces in the center. We perform the following two-step process k times until no pieces remain in the corners of the board.

1. Move a piece out of a corner to reduce by 1 the total number of pieces in one row or column.
2. Move one center piece to a non-corner piece in the row or column with the reduced number of pieces (to restore the fixed sum).

Moving a game piece out of a corner is the *only* way to "make room" for an additional center piece. Once all pieces have been moved out of corner positions, no additional center pieces can be moved without exceeding the fixed sum for one or more rows and columns.

If $b < c$, then $k \leq b < c$ (i.e., we have fewer pieces in the corners than in the center) and we will end up with center pieces that can't be moved. □

5 Implications for Students and Teachers

5.1 Problem Solving and Problem Posing in Language-independent Games

Presenting language-independent mathematics games such as *Where are the Cherries?* benefits students (and teachers) in numerous ways. For instance, the games:

- **Focus students on mathematics.** When we provide students with language-independent mathematics videos in an unfamiliar language, they must concentrate on relevant details of the game (usually the math itself) and make meaningful observations.
- **Encourage inquiry.** In the case of *Where are the Cherries?*, students begin by making a series of guesses about the goal and rules for the game. They modify their conjectures as they continue to watch the video as well as later when they play the game themselves. This way of learning through inquiry more closely approximates authentic research conducted by professional mathematicians.
- **Foster independent problem-solving.** Rather than passively learning from the teacher or the text, students explore language-independent board games without any clear step-by-step instructions or known answers. They formulate rules and strategies in small groups and on their own. In this way, language-independent games help to stimulate students' thinking while providing a novel approach for promoting problem posing and problem-solving.
- **Transform the role of teacher.** As the conversations in section 3 illustrate, the teacher plays a different role when presenting language-independent puzzles. Rather than telling students exactly what to do, teachers set the stage for learning by asking questions, recording students' observations, and discussing alternatives.

5.2 ELL connections

A significant portion of students in U.S. public schools (10.4%) are English language learners. According to the National Center for Education Statistics (NCES), the number of ELLs has increased rapidly over the last ten years. Gong and Gao (2018) note that English proficiency is instrumental to ELLs' academic success in the mathematics classroom. Since mathematics may be taught in a way that requires less language demand, some may assume that math class is less difficult for ELLs. However, such assumptions overlook challenges such as specialized mathematical terms and expressions (e.g., radius, circumference, radical) and cultural differences. Wells (2003) points out that mathematics has a language system that conveys special-purpose meaning. ELLs may feel overwhelmed when their teachers deliver specialized content knowledge in English.

From our research and classroom experiences, we have found language-independent games to be a powerful teaching tool. Games reduce the language burden for ELLs. On the other hand, the games benefit English-only students by encouraging them to focus on mathematics. We believe that *all* students can play games successfully in other languages with minimal teacher intervention through context clues and collaboration. ELLs can visualize mathematical procedures in the videos—connecting spoken conversation and academic language to the images on their screens. As one of our colleagues noted, "When we (use language-independent puzzles), we remove the teacher telling students what to do. We allow students to make conjectures and explore."

Some students have a better understanding or sense of mathematics than others. This is undoubtedly true of ELLs. Too often, teachers misinterpret challenges with English as challenges with mathematics content. It's important to remember that students using their second or foreign language to learn mathematics will likely find it more challenging to understand than native speakers. Watch the video again and imagine how you would feel learning mathematics in a Chinese classroom. Using language-independent video can help you and your students become more understanding and patient with ELLs in your classroom.

6 Future Work & Research Ideas

Our initial work with language-independent video has raised many questions that require future exploration. Below, we list several areas for future work.

- As we've explored *Where Are the Cherries?* over the past year, we have realized that **the mathematics behind the game is not obvious**. Unlike typical textbook exercises, it's not always clear how students will use mathematics and logic to solve or explain the game. Concerning games in math class, McFeetors and Palfy (2017) ask, "How can we be sure that mathematical reasoning is going on" (while students play games)? More research is needed to understand the various ways that different groups of students employ mathematics as they explore *Where Are the Cherries?*
- To implement language-independent games effectively and to better understand their rightful role in the mathematics classroom, more research is required to **explore connections between mathematics games and maths learned in standard textbooks**. For example, are there any differences in terms of reasoning patterns? If so, what are they? Is it sometimes more effective to use a language-independent approach? How do we incorporate games into our classroom teaching? In the paper, "We're in Math Class Playing Games, Not Playing Games in Math Class," McFeetors and Palfy (2017) include ideas for eliciting reasoning as students engage in gameplay. Which of these prompts are most useful for struggling students? How might they be modified for ELLs?
- Future research is needed to **compare and analyze responses from students at different levels**. Is it the case that students at all levels gain mathematical understanding from playing language-independent games? What adaptations can be made to the game to better support *all* learners? Noddings (1984) and Yoon (2008) remind us that teaching ELLs involves more than language issues. "Teachers roles should extend to include their cultural and social needs" (Yoon, 2008, p. 496). What cultural and social needs arise when playing language-independent games with students?
- Future research is needed to **compare and analyze the differences between responses from the Chinese students and American students about the video and their solution strategies**. We have informally interviewed Chinese and American students and teachers about the game; however, more research is needed to understand how mathematics, language, and culture impact learning and gameplay.
- Teachers and educational researchers need to **explore how language aids learning the game** (such as the rules, the goal, and how to explain it). Language independence may be an illusion. Although we call the game "language-independent," students don't stop communicating as they play—for instance, they speak to each other, they write, and they share ideas non-verbally. Perhaps language plays a more important role in language-independent games than we think.

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