Examining Textbooks to Support Prospective Teachers' Pedagogical Content Knowledge of Geometry

Ayse Ozturk, Lydia Schramm, McKenzie Milligan, Mickenna Canning, Mya Rapol & Sarah Wilson

The Ohio State University, Newark

Abstract: *This study describes how textbooks can support prospective teachers' pedagogical content knowledge in geometry. Relational content analysis methodology guided the collection of the study's data from five textbooks used in geometry content courses for elementary mathematics teachers to identify pedagogical content knowledge elements and examine the differences and similarities between books. The results suggest that connecting children's ideas and work to the teacher's role in the classroom provides opportunities to develop intertwined knowledge of geometry content, students, and teaching. Using standards and standardized test questions in the books can help develop curricular knowledge of geometry contents in K–5 classrooms.*

Keywords: *Teacher preparation, geometry, textbooks, content analysis*

Introduction

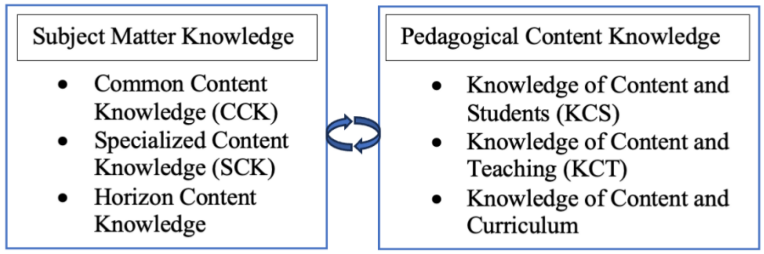
Studies examining teacher candidate preparation in mathematics content courses are limited (Appova & Taylor, 2019). More specifically, how content courses might support teacher candidates' pedagogical content knowledge remains a growing research area (Chapin et al., 2021; Norton, 2019). Recent research aimed at documenting how to strengthen teacher candidates' content knowledge through teacher preparation in mathematics content courses has emphasized developing conceptual understanding (Livy et al., 2019), procedural understanding (Wickstrom et al., 2017), using manipulatives and representations (Bostic, 2016) and textbook content analysis (Jones & Jacobbe, 2014).

The literature on textbook content analysis of teacher candidates' geometric knowledge for teaching primarily focuses on a wide range of geometric concepts, such as providing clear explanations about topics (Max & Welder, 2020), using technology to support problem-solving activities and create multiple representations (Jones et al., 2017), and applying geometry to real-world problems (Litoldo & Amaral-Schio, 2021). Building on the previous research examining textbooks, in this study, we focused on textbooks used in geometry content courses for elementary school teacher candidates (K–5) to discover how textbook contents might support prospective geometry teachers' pedagogical content knowledge.

Conceptual Framework

This study was drawn from Ball et al.'s (2008) practice-based theoretical model that refines pedagogical content knowledge, identifying the two complementary domains of mathematical knowledge for teaching. We adopted this model to examine textbooks' support of prospective teachers' pedagogical content knowledge in geometry teaching (see Figure 1).

Figure 1: *Theoretical model adapted from Ball, Thames, and Phelps (2008).*



The left side of the framework, the subject matter knowledge domain, includes three subdomains. Common content knowledge (CCK) emphasizes the "common" knowledge needed to solve mathematics problems, which is not specific knowledge to teaching mathematics. Specialized content knowledge (SCK) constitutes unique mathematical knowledge and skills for teaching. Horizon content knowledge entails understanding how different mathematical topics are related.

The framework's right side blends content knowledge and pedagogy. For example, knowledge of content and students (KCS) facilitates predicting common student mistakes when teaching geometrical shapes. Knowledge of content and teaching (KCT) guides teachers in sequencing examples when introducing geometrical shapes and their attributes. Knowledge of content and curriculum highlights teachers' knowledge of standards, state assessments, grade levels when particular topics are typically taught, and more.

The framework guided the current study's data creation and analysis process to learn how textbooks support prospective teachers' pedagogical content knowledge in geometry teaching. We chose this framework (a) as a well-known, highly detailed framework that provides a comprehensive understanding of two areas of mathematics knowledge for teaching along with six subcategories and (b) as useful for creating and associating codes in performing relational content analysis of textbooks.

Methods

To answer the research question, the research team collected the three most commonly used textbooks reported by U.S. mathematics course instructors for elementary teachers (Max & Newton, 2017) and two widely used textbooks in Ohio (Gosh Hajra, & Ozturk, 2020). Our examination of textbook contents followed Elo et al.'s (2014) relational content analysis methodology in order to (a) systematically analyze the content of geometry textbooks and identify the presence of pedagogical content knowledge elements while considering the conceptual framework and (b) examine the differences and similarities across the textbooks.

Data Collection and Creation

The selection criteria for the textbooks examined in this study included three essential components. Each book had to include 2-dimensional geometric concepts (e.g., shapes) and measurement concepts (e.g., area). Textbooks had to reflect research-based studies (e.g., cognitively guided teaching and mathematical problem-solving approach). Lastly, each textbook had to offer examples of children's reasoning, discuss the teacher's role, and include instructional activities that could be used in elementary mathematics classrooms.

Using our conceptual framework (Ball et al., 2008), we first brainstormed how to find instances of pedagogical content knowledge of geometry teaching in the five books. We engaged in a selective reduction process that entailed reading the book and reducing the text to categories, from which we could focus on developing codes for informing our research question. For example, after reading the first book twice together as a research team, we found that the data codes should include a reference to the Common Core State Standards of Mathematics and a student's work sample. The inclusion of other connections (e.g., learning standards or classroom activities) other than pure mathematical content was critical for the code development because we aimed to interpret in a later data analysis step how our codes could be linked to the components of the right side of the conceptual framework, thus revealing patterns to describe how these textbooks supported prospective teachers in developing geometrical teaching-related knowledge.

After creating the initial codes, we examined the books individually and then came together to compare and contrast the patterns found in each book. Thus, we built a shared understanding of the codes and avoided missing any codes. Research team meetings also helped us revise our descriptions of each code and share examples for the same code in different books to check the consistency and coherency of the codes' descriptions. In the sixth meeting during the code-creation phase (about 14 hours in total), we finalized the codes (see Table 1) for relational content analysis.

Table 1: *Codes, Descriptions, and Examples Across Five Textbooks*

Codes	Descriptions	Examples
Instancing children's written work	Providing examples of children's work samples	A child's drawing to show the comparison of different length of the lines (Battista, 2012, p. 54)
Instancing children's ideas	Providing examples of children's reasoning within the classrooom discussions or assignments	Student Julie is confused about unit conversions, specifically why we multiply by 3 to convert 6 yards to feet (Beckman, 2018, p. CA-234)
Instancing teachers' role and their instructional activities in K-5 classrooms	Providing examples of classroom dialogues or/and mathematical activities to discuss how a teacher would respond to children's reasoning	A vignette shows how a teacher facilitates her classroom discussion oabout finding the retangular area with her 4th graders (Sowder et al., 2017, p. 519)
Use of learning standards/mathematical practices and connection to K-5 curriculum	Referring content standards or standards for mathematical practices or curricular materials	Describing area and perimeter of polygons in different grade levels (Bennett et al., 2016, p. 736)
Use of assessment questions	Providing example questions from standardized tests	Questions about 3-D shapes' attributes for 4th graders in NAEP 2013 (Billstein, 2020, p. 728)

Table 1 contains the code names, descriptions, and examples of each code in the different books. We used this information to identify the patterns related to the commonalities and differences described in the five books. We conducted separate book reviews for each book, followed by research team code-creation meetings, to increase the reliability and validity of the study findings.

Data Analysis

Relational content analysis (Elo et al., 2014) guided our data analysis process (totaling about 6 hours), reflecting our research focus on defining the patterns related to the pedagogical content knowledge of geometry teaching that emerged from five textbooks and investigating the relationships between the patterns.

Our relational content analysis comprised four steps. We began by using our code descriptions (see Table 1) to capture the evidence of code instances in each book and report their frequencies. Next, we identified co-occurring codes and included their frequencies in our final report. We then created the final table containing the code frequencies within each textbook and across all codes (see Table 2) to draw conclusions. Lastly, we made inferences about the differences or similarities of data and focused on how codes are associated with the conceptual framework.

When the research team met to organize the frequencies of codes, we eliminated instances that included children's names but lacked examples of these children's reasoning. Although they were initially included in the codes, we decided to exclude them from the revised code descriptions since our codes reflected children's mathematical ideas, not merely names. For example, Beckmann (2018) included the following problem:

> Carmina and Antone measured the distance between spots where they are standing as 10 feet 7 inches . . . If Carmina and Antone had measured the distance between them along the floor, would the distance be different than what they measured? (p. 576).

This instance was similar to the second code in Table 1 but did not explain the children's reasoning; thus, we eliminated such examples.

Including the co-occurrence of codes in the final report (see Table 2) was critical, as they changed the codes' total number and percentages. A common example of co-occurring codes is a code for an example of a child's mathematical ideas while inviting prospective teachers to discuss how to respond to these ideas or an example of an instructional activity to facilitate classroom discussion around the child's ideas. For example, Beckmann (2018) included the problem,

> Tiffany says that angle at A in figure 10.29 is bigger than the angle at B. Why might she think this? How might you discuss angles with Tiffany? (p. 463).

This instance was a co-occurring code because it exemplified both a child's ideas and the teacher's role.

Findings and Discussions

Table 2 displays the code frequencies within each textbook across all codes. Observable patterns are evident regarding the frequency of each code aligned with the total number of codes counted across all books. Out of 780 total codes, the highest percentages of codes focused on children's ideas (34.35%), mostly drawn from Battista's (2012) book which provides many examples of how children reason through geometric concepts by exemplifying typical student errors and the development of reasoning levels over time.

The second-highest code percentages focused on the teacher's role and classroom connections, such as discussing teachers' responses to children's ideas to advance the latter's geometric reasoning. Billstein et al. (2020) demonstrated the most examples related to classroom connections and the teacher's role. The other books had similar percentages of instances of the code for the teacher's role, except for Beckmann (2018), which had the lowest percentage. Instead, Beckmann's book focused on pure content and learning standards, which reportedly led to this book's wide application in elementary mathematics content courses for U.S. teachers (Max & Newton, 2017).

Table 2: *Codes, Frequencies, and Percentages Across Five Textbooks*

	Books					
Codes	Battista (2012)	Beckmann (2018)	Bennett et al. (2016)	Billstein et al. (2020)	Sowder et al. (2017)	**Total Codes: (780)**
Instancing children's written work	43 (75.43%)	1 (1.75%)	0	3 (5.26%)	9 (15.78%)	57 (7.30%)
Instancing children's ideas	177 (66.04%)	21 (7.83%)	21 (7.83%)	13 (4.85%)	36 (13.43%)	268 (34.35%)
Instancing teachers' role and classroom connections	49 (22.27%)	15 (6.81%)	45 (20.45%)	90 (40.9%)	21 (9.54%)	220 (28.2%)
Use of learning standards	0	60 (31.57%)	70 (36.84%)	19 (10%)	41 (21.57%)	190 (24.35%)
Use of assessment questions	0	0	0	45 (100%)	0	45 (5.75%)
Number of co-occurred codes	5 (9.09%)	8 (14.54%)	17 (30.09%)	13 (23.63%)	12 (21.81%)	55** (7.05%)

The codes with the two highest percentages (exemplifying children's ideas and the teacher's role) often co-occurred. Specifically, 55 out of 780 codes (7.05%) occurred together, almost the same degree as another identified code that focused on children's written work (7.30%). This observation suggests that authors tend to emphasize the importance of the teacher's role when responding to children's ideas, introducing a new concept, or connecting different geometric concepts. Aligning with Max and Weldner's study (2020), this result empirically demonstrate that the books exemplify classroom situations designed to lead prospective teachers to consider what they would do in particular situations rather than representing a direct teaching method.

The third-highest percentage of codes focused on the use of learning standards. With the exception of Battista (2012), all other books revealed links between learning standards and geometry content. While Beckmann (2018) included a list of learning standards next to each relevant session, Bennett et al. (2016) and Sowder et al. (2017) provided examples of how learning standards could be linked with instructional activities in progressing through grades K–5.

The fourth-highest percentages were related to the code for children's written work. This code was mostly found in Battista (2012), which focused more on how children think and how teachers could communicate with them to sharpen their reasoning and learning of geometric concepts. For this reason, the book was reportedly used as a complementary resource instead of as a major content book in elementary mathematics content courses for teachers (Authors, 2020). Children's written work samples would allow prospective teachers to see concrete examples of children's reasoning through geometric concepts. All of the other books demonstrated similar patterns in exemplifying actual children's written work. Bennett et al. (2016) included no examples of children's work but, like Beckmann (2018), primarily focused on pure geometry.

The code with the least frequency related to the use of assessment questions, providing sample questions from actual standardized tests for K–5 students. Only Billstein et al. (2020) included this code, but retaining it in our final codes seemed necessary because such sample questions from standardized tests can familiarize prospective teachers with the content for different grade levels, which deepens their curricular knowledge of geometry (Livy et al., 2019).

The findings also revealed three correlations between the codes and our conceptual framework. For example, the first two codes in Table 1 were related to KCS because those instances invite prospective teachers to understand and predict typical student responses, student mistakes in different grades, and reasoning levels for different geometric concepts. The third code, the teacher's role and classroom connections, was related to KCT. The fourth and fifth codes were associated with knowledge of content and curriculum; learning how geometry contents, learning standards, and standardized test questions are related helps prospective teachers develop a contextual understanding of teaching certain geometric concepts at different grade levels. Although previous studies (Jones & Jacobbe, 2014; Max & Weldner, 2020) emphasized relating textbook contents with subject matter knowledge, our results primarily focus on identifying pedagogical content knowledge elements in the books.

Although we specifically sought correlations between the codes and the pedagogical content knowledge side of the framework, we also found a correlation between our codes and two subdomains of subject matter knowledge. Exemplifying the teacher's role and classroom connections requires access to specific knowledge for teaching geometry (SCK). In addition, the codes related to understanding children's ideas and children's written work require access to knowledge of the content (CCK) in order to make sense of them. This result reflects Norton's (2019) description of the relationship between mathematical content knowledge and pedagogical content knowledge. Lastly, examining Table 2 by columns enables summarizing the similarities and differences across the books in four ways:

- Beckmann (2018) and Bennett et al. (2016) focus on content and curriculum, especially how Common Core Standards apply to each chapter. These books offer practice questions, hands-on activities for prospective teachers to apply content knowledge, and a summary section in each chapter.

- Battista (2012) is unique in emphasizing understanding children's strategies, analyzing their actual written work, and studying teachers' probing questions. This book has no connections to learning standards or standardized assessments.

- Billstein et al.'s (2020) work is unique in that it is the only book that provides sample questions from actual standardized assessments. Like Beckmann (2018) and Bennett et al. (2016), Billstein et al. (2020) emphasize pure content knowledge with detailed explanations. Each chapter concludes with real-life practice problems, questions from state-standardized tests, and discussion questions about teachers' roles and students' ideas.

- Sowder et al. (2017) include examples of children's written work and classroom activities for prospective teachers to utilize, similar to Battista (2012). Each section features "Issues in Learning," categorizing children's responses to geometry problems by grade level. Like Beckmann (2018) and Bennett et al. (2016), Sowder et al. (2017) emphasize the use of learning standards though differing in their presentation of the standards.

As suggested in other studies (e.g., Chapin et al., 2021), depending on the instructor's purposes and practices, textbooks could be used to support prospective teachers' knowledge in geometry content courses.

Conclusion

This study examined five geometry textbooks to explore how textbook contents might support pedagogical content knowledge of prospective teachers in geometry. The findings suggest that using connections to children's written work, ideas and the teacher's role in the classroom provide opportunities to develop intertwined knowledge of geometry content, students, and teaching. Furthermore, the books' presentation of learning standards and standardized test questions actively connect geometry activities in teacher candidates' content classrooms with their future work in K–5 classrooms. Mathematics teacher educators can apply the coded instances of pedagogical content knowledge in the textbooks in their content courses to engage and motivate prospective teachers to learn concepts needed to teach in their geometry classes.

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Ayse Ozturk (ozturk.25@osu.edu), is a lecturer in the Department of Teaching and Learning and the Department of Mathematics at the Ohio State University, Newark. Her research interests include equitable teaching practices and mathematics teacher education.

Lydia Schramm (schramm.75@buckeyemail.osu.edu) is a primary education major in the Department of Teaching and Learning at the Ohio State University, Newark. She plans to specialize in special education and use her degree to help children on the autism spectrum.

McKenzie Milligan (milligan.207@buckeyemail.osu.edu) is a primary education major in the Department of Teaching and Learning at the Ohio State University, Newark. Her research interests include play and mathematical problem solving with young children.

Mickenna Canning (canning.26@buckeyemail.osu.edu) is a primary education major in the Department of Teaching and Learning at the Ohio State University, Newark. Her research interests include the use of board games to teach young students basic mathematical concepts.

Mya Rapol (rapol.3@buckeyemail.osu.edu) is pursuing degrees in primary education in the Department of Teaching and Learning at the Ohio State University, Newark. Her research interests include exploring connections between algebra and geometry.

Sarah Wilson (wilson.4446@osu.edu) is is a primary education major in the Department of Teaching and Learning at the Ohio State University, Newark .Her research interests include connections between sports and mathematics to further student understanding and engagement.