GPS & SONAR: Connecting Military Applications to Classroom Mathematics

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Abstract: *The author explores the integration of GPS and SONAR applications in middle school mathematics classrooms. Highlighting the relevance of STEM in the military, the author presents practical problems and activities that connect math concepts to real-world scenarios.*

Keywords: *Global positioning, SONAR, GPS, middle grades mathematics, applications*

1 Introduction

Science, **T**echnology, **E**ngineering, and **M**athematics, also known as STEM, is utilized throughout all six branches of the military which involves the Army, Marine Corp, Navy, Air Force, Space Force, & Coast Guard. As a participant in the immersive Joint Civilian Orientation Conference (JCOC), the oldest public liaison program hosted by the Department of Defense (Department of Defense, n.d.), I was one of three educators that had the opportunity to experience STEM in action where I started at the Pentagon and then traveled to bases from each military branch throughout the Western part of the United States.

While there are numerous connections of STEM being applied throughout the military, this article will focus on math connections as related to global positioning, GPS, and SONAR. As classrooms are full of students with different backgrounds, interests, levels of abilities, and future pathways, a variety of contexts are presented within the article that incorporate problems focused on math concepts to build or reinforce skills as well as activities that can provide opportunities for higher order thinking as well as discussion of math concepts, cross-curricular/personal connections, and historical/current events. The problems and activities presented within this article can be integrated into lessons focused on middle school math concepts or teachers can choose to use all of the problems and activities as part of an exploratory unit on "STEM in the Military" to build & reinforce skills and concepts within the middle school mathematics curriculum.

2 Global Positioning System (GPS) & Intersecting Circles

The mobilization of military vehicles, aircraft, and surface level vessels are assisted with Global Positioning System (GPS) satellites which provide position, navigation, and timing data that ensure that equipment, supplies, or troops reach a prescribed location at a specified time. Under the command of Space Force Guardians located at Schriever Space Force Base in Colorado, GPS Satellites are constantly monitored and sent updates to keep data accurate for both military and civilian users (Wise, 2021).

GPS satellites broadcast radio signals providing their location, status, and precise time with on-board atomic clocks. This signal can be picked up by a GPS receiver such as those found on vehicles, aircraft, and surface vessels as well as smartphones which can further calculate how far the device is from a satellite. To determine a precise location on Earth, a GPS receiver must use at least three satellites in a process called trilateration which ultimately involves the intersection of three circles to identify a precise location on the Earth (*How Your Smartphone Knows Where You Are*, 2015). While GPS works with 3D trilateration with satellites in space, GPS can be illustrated with 2D trilateration by drawing at least three circles on a computer screen such as with Google Earth Pro or by graphing circles on a coordinate plane and finding the point of intersection.

2.1 Problem 1

Download the Google Earth Pro Desktop version if not already installed to your computer, then complete the following:

A. Watch `https://youtu.be/NQTE9KSTVyo` to learn how to draw a circle to determine the set of points representing your possible location if you are 337 miles from Albuquerque, New Mexico. Then, draw a circle to determine in which states you could be located.

   Answer: If I am located 337 miles from Albuquerque, NM, then I could potentially be in Arizona, Colorado, Kansas, Oklahoma, or Texas within the United States or I could ultimately be in Mexico.

B. Using the concept of intersection of circles and understanding that you are also 487 miles from Omaha, Nebraska, draw a circle and determine the two states in which you could potentially by located.

   Answer: Video of result found at `https://youtu.be/e_XDtBBrWpk` indicates that I could be in Colorado or Texas.

C. Understanding that you are also 373 miles from Salt Lake City, UT, and using the concept of 2D trilateration and the intersection of circles, draw another circle, and identify the city and state in which you are located.

   Answer: Denver, CO as shown in Figure 1 and in the video found at `https://youtu.be/5uOWtTNT1jw`

Figure 1: *Screenshot from video showing the intersection of the three circles to be Denver, CO.*

2.2 Problem 2

Identify a mystery city within the United States that is known only to you. Provide three clues on how far the city is from three known US cities using Google Earth Pro and have a partner follow your clues to determine your location.

Answer: The given mystery city and state can vary, but an example would be St. Louis, MO. The various clues can also vary for a given mystery city. As such, here are clues using St. Louis, MO as an example:

- I am 309 miles from Cincinnati, OH.
- I am 472 miles from Atlanta, GA.
- I am 457 miles from Oklahoma City, OK.

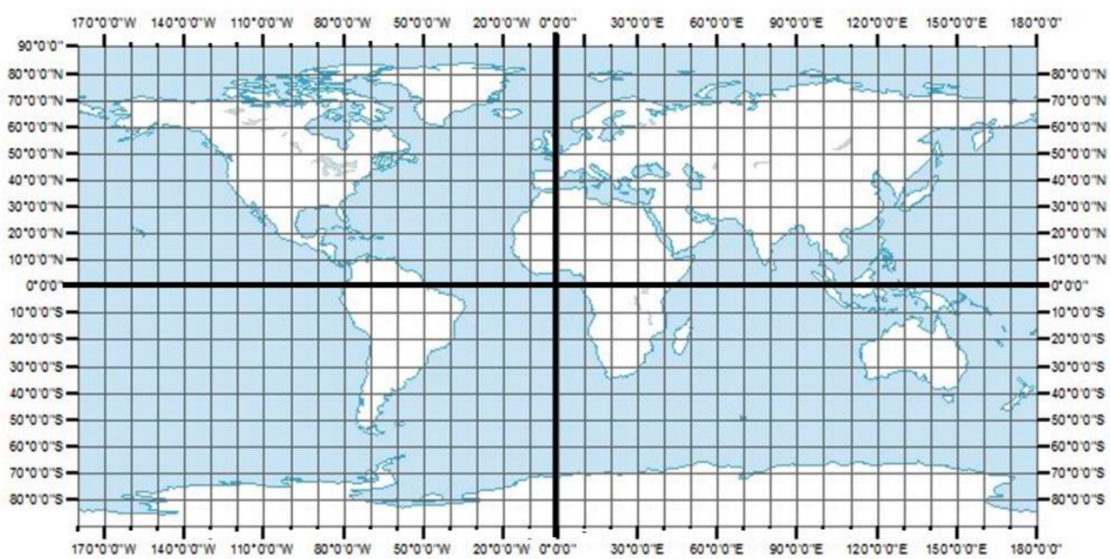
3 Global Positioning System (GPS), Latitude/Longitude, & Ordered Pairs

Every location on Earth can be identified by GPS coordinates which involve geographical coordinates given in terms of latitude and longitude. While latitude lines are horizontal lines, North or South, from the Equator, longitude lines are vertical lines, East or West, from the Prime Meridian. As such, connections can be made between GPS involving the geographical coordinate system and the Cartesian coordinate Plane involving the Cartesian coordinate system, where the Equator represents the x-axis, and the Prime Meridian represents the y-axis.

3.1 Problem 3

Consider the following map, then answer the questions that follow.

Figure 2: *Earth projected onto a 2D map (*Ellipsoid/Spheroid – Our Oblate Spheroid Planet Earth, *2022).*



A. If a GPS location is identified as $50°N\ 120°W$ in the geographical coordinate system, what is the location as an ordered pair identified in the Cartesian coordinate system?

   Answer: $(-120, 50)$

B. If a location is identified as $(160, -30)$ in the Cartesian coordinate system, what is the GPS location identified in the geographical coordinate system?

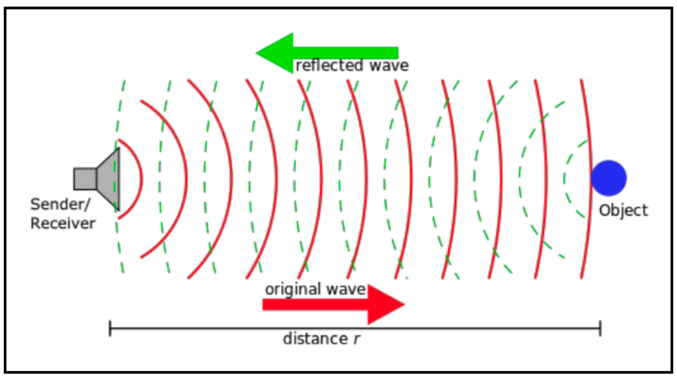
   Answer: $30°S\ 160°E$

4 SONAR (for underwater positioning), Time, & Solving Equations

Although aircraft, vehicles, and surface vessels can use GPS to navigate and identify location in the air, on land, and on the surface of the sea, a different system is needed for underwater positioning since signals transmitted by GPS satellites cannot work in water (*Underwater GPS*, n.d.). With sound waves able to travel through water faster than through air, SONAR (**SO**und **N**avigation **A**nd **R**anging) is utilized under water to navigate and to identify location of underwater objects (U.S. Fleet Forces Command, n.d.)).

There are two types of SONAR, passive and active. With passive SONAR, the Navy will use underwater microphones called hydrophones to listen for sounds. As every object under the sea will create different sounds as well as produce different sound waves with different frequencies as illustrated with spectrograms found at https://www.fisheries.noaa.gov/national/science-data/sounds-ocean, sonar operators must be able to decipher sounds and detect potential submarines or other underwater objects. Once a sound is detected using passive SONAR, active SONAR can be used by a vessel or underwater vehicle by emitting a sound pulse, or ping, through the water that travels to an object and bounces back as an echo as shown in Figure 3 at a speed that is impacted by temperature, salinity, & depth (U.S. Fleet Forces Command, n.d.).

Figure 3: *Visual representation of SONAR (Wiora, 2005).*



4.1 Problem 7

Answer the following:

A. Listen to the active sonar found at https://dosits.org/galleries/audio-gallery/anthropogenic-sounds/sonar/?vimeography_gallery=85&vimeography_video=227206889 that is tracking an underwater object, then determine the time, to the nearest hundredth of a second, it took the pulse to reach the object.

   Answer: While the time between an initial pulse and echo indicates the total time it took for the sound to travel round trip, the amount of time it takes for the pulse to reach the underwater object can be found by dividing the total trip time by 2. Thus, when using the stopwatch on my phone, the total time between the pulse and echo is 3.62 seconds, but to find the time it took the pulse to reach the object, I must divide the total time by 2 making the time to reach the object 1.81 seconds.

B. Understanding that distance = rate $\times$ time and knowing that sound in $20°C$ $(68°F)$ seawater travels approximately 1522 meters per second (*Water – Speed of Sound vs Temperature*, n.d.), determine the distance to the object.

   Answer: Distance = $1522 \times 1.81 = 2754.82$ meters

5 SONAR from the Sky, Sonobuoys, & GPS

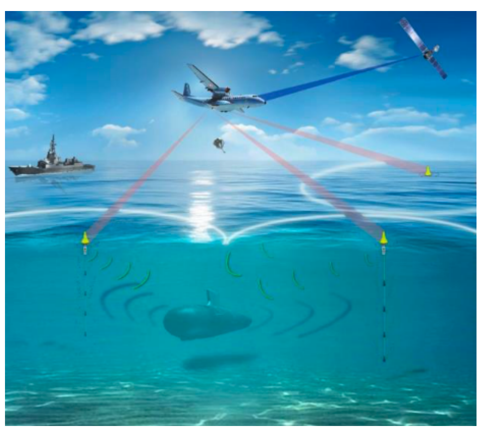
While surface vessels, submarines, and other underwater objects can use SONAR directly, aircraft can also use SONAR indirectly by deploying dipping sonar systems or sonar buoys. Helicopters, for instance, can lower a dipping sonar system via cable into water from a specific GPS point, and hover over that point (Figure 4) giving SONAR technicians time to listen for sounds and/or emit a ping.

Figure 4: *SH-60F Seahawk Dipping Sonar.*

After gathering and analyzing information, the dipping system can then be removed from the water, then flown to and dropped at another GPS location if needed. Additionally, helicopters and other fixed wing aircraft can drop sonobuoys within canisters containing parachutes. When the canister hits the water, the buoy remains at the surface to identify GPS coordinates as the transducer travels to a certain depth to collect acoustical data. Watch https://youtu.be/eidMDdMK38s for a video of a sonobuoy deployment.

Using trilateration involving at least three sonobuoys with three identifiable GPS locations, information on the distance an object is from the transducer can determine the precise location of an underwater object, as shown in Figure 5.

Figure 5: *Sonobuoys and trilateration (*SAES Solutions Defence Security*, 2018).*



6 The Significance of Dr. Gladys West

While various individuals contributed to the creation of GPS, Albert Einstein's theory of relativity and Dr. Gladys West's work involving mathematical modeling of the Earth are what made GPS possible (Siegel, 2021). After graduating from Virginia State College, a Historically Black College now known as Virginia State University, and after being a math teacher for a few years, Dr. West was hired in 1956 as a civilian mathematician by the Navy as only the 2nd black woman ever to be hired (Matthias, 2021). Working at the Naval Surface Warfare Center (formerly called the Naval Proving Ground) in Dalgren, Virginia, Dr. West was known for her ability to accurately calculate complex equations by hand. However, after computers were introduced, Dr. West pivoted her work and mastered an understanding of programming and coding (U.S. Navy, 2018). With Dr. West knowledge and utilization of computers, she was able to calculate the orbits of satellites, primarily over the oceans, laying the foundation for GPS (U.S. Navy, 2018).

7 Summary

The importance of teaching and learning math concepts can be strengthened when highlighting STEM connections in our military. By integrating examples and discussion of topics in the news or opportunities beyond high school related to STEM in our Military, students may begin to see the importance or relevance of what they are learning in the classroom. The STEM highlighted in this article that pertains to GPS and SONAR are just a few topics that highlight opportunities to connect, build, reinforce, or extend math concepts found in middle school classes pertaining to basic concepts and concepts found within algebra curriculum. As all students have different interests, experiences, families, and future pathways, teachers are encouraged to consider connecting math concepts to various real-world topics, such as STEM in the military, within lessons or special exploratory units to make math interesting, meaningful, and relevant in what is happening in the world around them.

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