This graph stinks . . . but I fixed it: A lesson on data representation

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Abstract: *This article shares a lesson on data representation. Designed for high school mathematics classrooms, the lesson begins by exploring data from the 2020 presidential election. Then, students have the opportunity to find other misleading graphs, analyze how/why they are misleading, and create a revised version of the graph that more accurately shares the data. Throughout the article, we share details of how the lesson was implemented with a group of preservice mathematics teachers. We also share student reactions/work and recommendations for teaching this lesson within a high school setting.*

Keywords: *mathematics education, high school mathematics, data representation, statistics*

1 Introduction

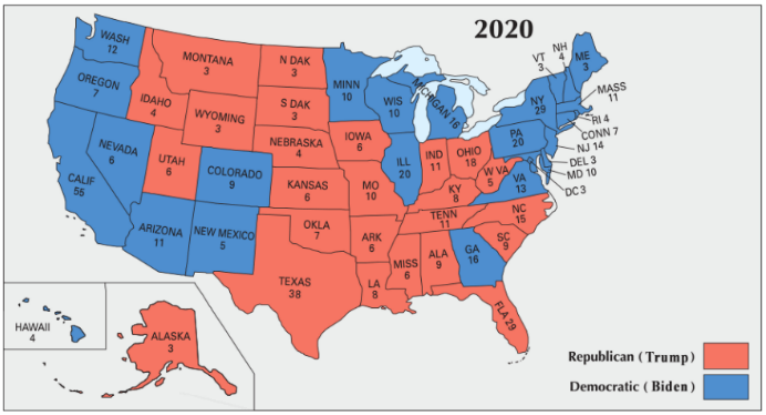
How often is the mathematics we teach void of context? Because mathematics is a powerful tool for understanding the world (Izard, 2018; Gutstein & Peterson, 2006), we believe we are doing our students a disservice if we do not provide them with opportunities to "identify, interpret, evaluate, and critique the mathematics embedded in social, commercial and political systems and claims, from advertisements, such as in the financial sector, to government and interest-group pronouncements" (Ernest, 2015, p. 191; as underscored by NCTM (2018)). Students are constantly surrounded by mathematical information, and mathematics classrooms are the perfect place to help them learn how to process that information.

In this article, we present a high school mathematics lesson that brings these ideas to light. This lesson is intended for a high school statistics or data analysis unit, but we implemented it with a class of preservice mathematics teachers at a public university in the northeastern United States. Throughout this article, we share details of the lesson, student reactions/work, and recommendations for implementing it within a high school setting.

2 A Meaningful Context for Mathematical Exploration: Nevada Election Results

The journey to developing this lesson began with our experiences with the 2020 presidential election. On nearly all news outlets, election results were shown with simplistic red and blue colored maps with little context (see Figure 1).

Figure 1: *US Map of 2020 Presidential Election (GISGeography, 2022).*

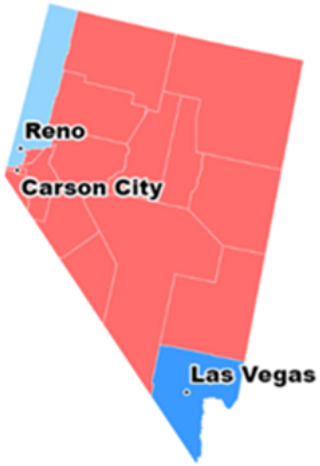


Without an explanation of what the graphs were portraying, we can see how many Americans would see "more red" across the map and feel confused about how Biden won the election. This was the motivation for our lesson which we discuss in more detail in the paragraphs that follow.

2.1 Initial Analysis of Nevada Election Results

We began the lesson by showing students the map results from Nevada (see Figure 2). We wanted to discuss Nevada because the map results seem to indicate that Donald Trump won the state, but Joe Biden was the winner.

Figure 2: *Map of the 2020 Nevada Presidential Election Results (CNN, 2020).*



Note that the map shows counties within the state. The counties that are shaded red had more votes for Donald Trump, while blue counties had more votes for Joe Biden. After this brief description, we asked students, "*Based only on this map, who won?*," and gave them 2-3 minutes to discuss with a neighbor. When coming back together as a whole class, students made statements like, "there is a lot of red on the map," and "it looks like Trump won," but they were skeptical that this was true. This group of students remembered that the 2020 presidential election was close in many different states, and they were trying to recall what happened in the state of Nevada. They were eager to look this up, but we told them to hold off and transitioned into the next part of the activity.

2.2 Introducing the "Oh Nevada" Student Handout

Next, we gave students the following handout to guide their mathematical investigation of the Nevada election.

The table below shows the number of votes that Joe Biden and Donald Trump received in each county of Nevada as well as the totals for the state.

1. In the table, highlight the counties that Trump won with one color and the counties that Biden won with a different color.

County	# of votes for Biden	# of votes for Trump
Washoe	128,128	116,760
Humboldt	1,689	5,877
Elko	4,557	16,741
Pershing	547	1,731
Lander	496	2,198
Eureka	105	895
White Pine	859	3,403
Storey	902	1,908
Lyon	8,473	20,914
Douglas	11,571	21,630
Churchill	3,051	9,372
Mineral	829	1,423
Nye	7,288	17,528
Lincoln	330	2,067
Esmeralda	74	400
Carson City	12,735	16,113
Clark	521,852	430,930
Total	703,486	669,890

2. What do you notice about the counties that Trump won compared to the counties that Biden won?
3. Compare the map of the Nevada results with the table. Do these two representations of the data convey the same message?
4. How are they similar and how are they different? Why might the map not be the most ideal way to present this data?
5. Brainstorm some alternate ways that you might present this data.

The handout asks students to review a table of the number of votes that each candidate received in each county along with the totals in the state. After reviewing this table, students were asked to discuss what they noticed about the counties that Trump won compared to the counties that Biden won, and how the map and table conveyed (or did not convey) the same message.

2.3 Student Insights

After students worked in groups, we discussed their responses as a whole class. In the discussion, students noticed that "While Trump won more of the counties, those counties had smaller numbers." In other words, they noticed that the counties that Trump won tended to have smaller populations. One student explained that the map looked very red because Trump won more counties but that those counties were not as populated, ultimately concluding, "certain counties have more space but fewer people."

Students seemed amused because they could see how the map was misleading at first glance, but they realized that once you dig into the numbers, you see something different. Another student explained that the map was showing land and counties, rather than the actual population. This student emphasized this by saying "Land doesn't count as a vote," meaning that just because the map showed a lot of red, that did not mean there were more republican votes. Rather, that was a rural county with a lot of land and not as many people.

When discussing whether the map and table conveyed the same message, one student explained, "The map and table do show the same thing–that Biden won–but, in the map, you can't see how many people are in each county. You can't tell who won." Another student echoed this by saying, "What you're looking at [the map] doesn't tell the full story." When students were asked to consider other ways of representing the election results (rather than through the map in Figure 2), they had a few ideas. One group suggested a map that uses shades of red/blue to indicate the density of votes in that county. The counties that were more populated could be darker and the counties that were not as populated could be lighter. They acknowledged that this still would not clearly show that Biden won the state, but that it would at least provide a better understanding of the election results.

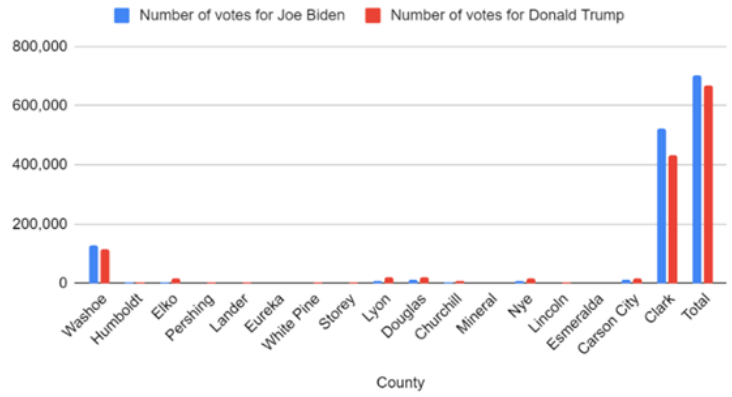
Another group suggested that the map in Figure 2 could still be used, but with one addition: Dots that help to show the population of each county could be added throughout the map. The size of each dot would be scaled to represent the population; for a denser population, a bigger dot would be used and for a more rural population, a smaller dot would be used. Again, someone viewing this map might not know that Biden won the state, but they would have a better understanding of the results.

Other groups suggested tables and graphs to show the results, rather than a map. One group thought that a table like the one in the "Oh Nevada!" handout could be used, but that the rows could be shaded with red or blue to indicate who won the state. They thought this would provide a thorough summary of what happened. Another group thought a pie chart could be created that avoided the use of counties. They did not see value in focusing on how each county voted, instead they thought the pie chart could simply illustrate the number and percentage of the totals in the state. While there were interesting ideas from other groups too, the general theme of this discussion was that we should reconsider the ways in which election results are shared so that viewers of this data can have a better sense of what occurred in the state.

2.4 Extension Ideas

If you are looking to extend this part of the lesson, you could have your students create alternative data representations that better reflect the election results, such as the one provided in Figure 3, and share their work and discussing findings in a gallery walk.

Figure 3: *Bar Graph of 2020 Presidential Election Results in the State of Nevada.*

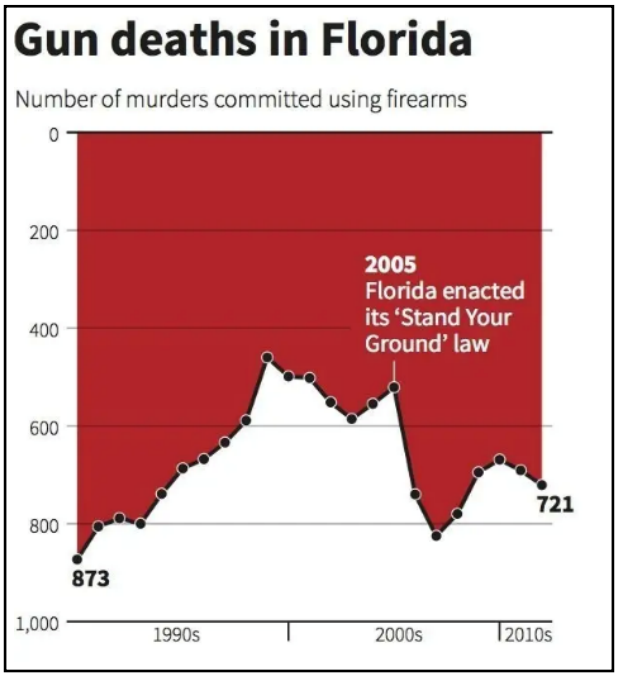


3 A Meaningful Context for Mathematical Exploration: Gun Deaths

3.1 Introducing Gun Death Data

Next, we explained that we were going to consider another graph. We showed students Figure 4 and asked them to interpret its message.

Figure 4: *Gun Deaths in Florida (Source: Christine Chan/Florida Department of Law Enforcement).*



3.2 Initial Student Reactions to the Data Display

Initially, our students made comments such as the following: "the murder rates increased from the 90s to the 2000s," "gun deaths went down after stand your ground," and "gun deaths before that were very high," but then they noticed something else about the graph: "Wait, oh my god!" one student exclaimed. "The graph is flipped!" yelled another student. They noticed "the y-axis is going down instead of up," and "the graph is flipped and shaded on the opposite side." As instructors, we simply stood there, letting the students react while they became curious about the "Stand Your Ground Law," looking it up and making comments about the graph like, "that makes no sense" and "who does that?" One student explained, "It looks like the graph is telling you one thing about the law [that gun deaths decreased] when it actually did the exact opposite." Once students had expressed their surprise and disappointment in the graph, we acknowledged that the graph was misleading and underscored the importance of reviewing and analyzing data representations. As consumers of data, it is an important life skill to pause, interpret, and critique the data being presented to us. This conversation led us into the larger task of the lesson.

3.3 Introducing the Data Representation Task

Next, we placed students in groups to find another misleading image or graph to analyze and recreate. We provided them with the following handout to guide their exploration.

This is a group task. In your group, please designate a role for each person. The roles are as follows.

Manager: Keeps the group on track, making sure everyone is participating.
Scribe: Takes the lead on creating the final group product.
Material Manager: Collects and returns any materials.

1. Find (on the internet) an image or graph that your group feels is misleading.
2. Analyze the image and make note of:
   (a) What you think the image is trying to convey.
   (b) How/why it is misleading.
   (c) If you think it is intentionally or accidentally misleading.
3. Create a new graphic or image that more clearly represents the data.
4. Your final product will be a poster that you share with the class. The poster must contain:
   (a) The original image.
   (b) Your analysis of the original image.
   (c) Your new graphic or image.
   (d) An explanation of how your new image better represents the data/message.

Students found all sorts of interesting and misleading data representations using Google images. We share several examples in the following paragraphs.

3.3.1 Group 1 (Unemployment Rate)

One group found the graph below (Figure 5), published by Fox News in 2011. In their poster, they indicated, "The points are inaccurately graphed. If you look at how they graphed the point for March compared to November, you see November is higher than March, where 8.6 should be lower than 8.8." This group created a new graph (Figure 6) to show what this should have looked like.

Figure 5: *Unemployment Rate during Obama's Term (As Presented by Fox News).*

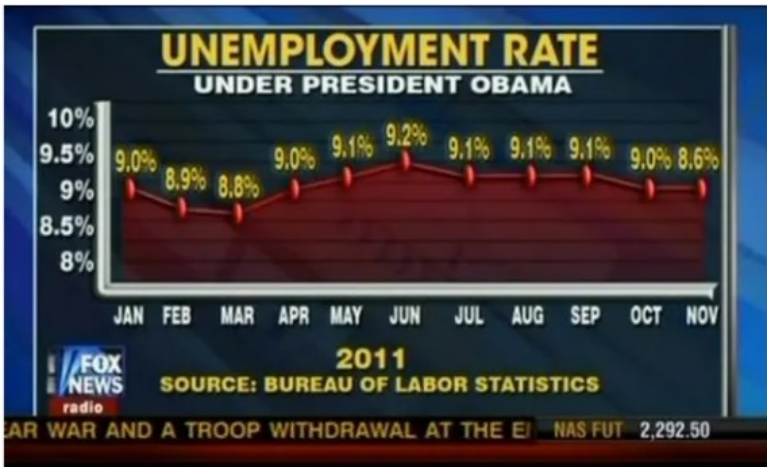
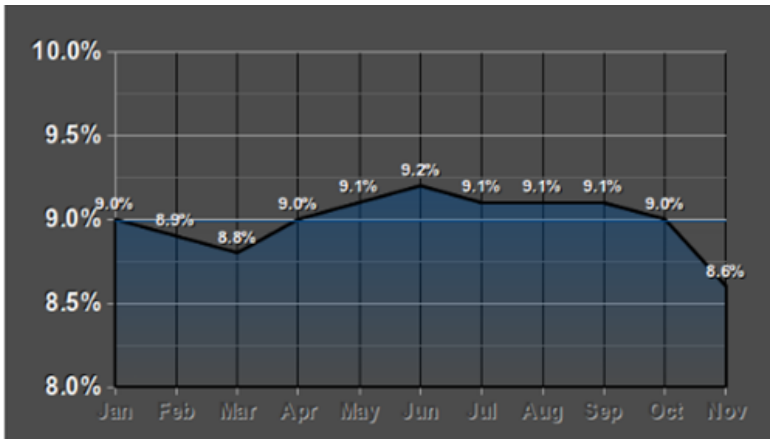


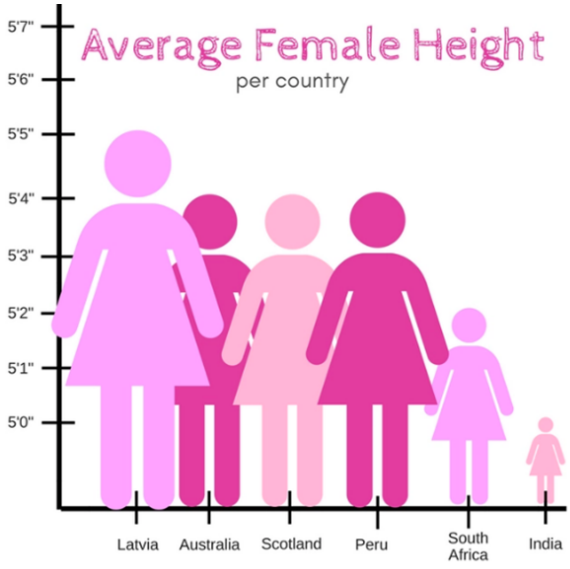
Figure 6: *Alternative Data Display of Same Data (Generated by Students).*



3.3.2 Group 2 (Average Female Height)

Another group shared a data display showing the average female height per country (Figure 7).

Figure 7: *Alternative Data Display of Same Data (Source: https://venngage.com/blog/bad-infographics/).*

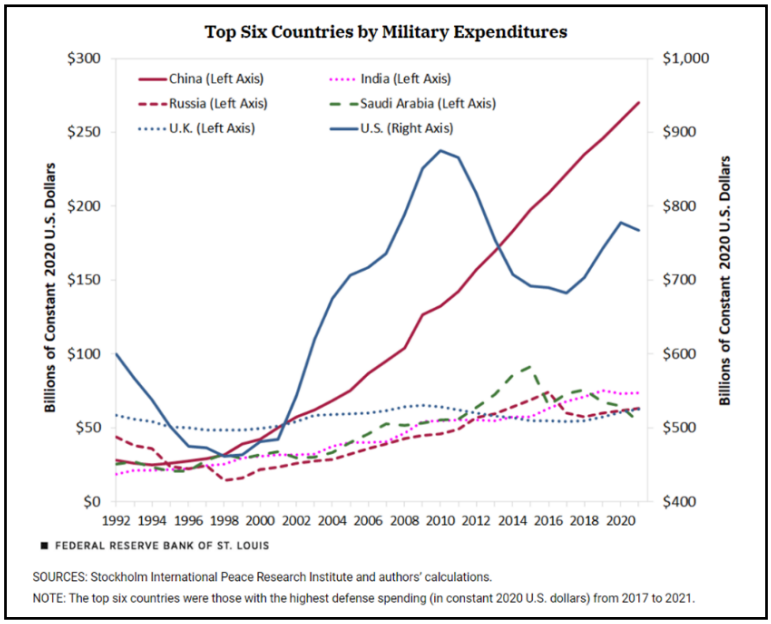


While this group indicated that they did not think this graph was intentionally misleading, they found it comical because it makes Indian women look incredibly small compared to women from other countries. They explained that because the y-axis started at 5 feet and then increased by a scale of 1 inch, this made just a 4-inch height difference take up about twice as much space as the distance between 0 and 5 feet. Because of this, a 5'5" woman looked much bigger than a 5'0" woman. This group did not finish putting together a new graph for this data, but their plan was to adjust the y-axis to start at 0.

3.3.3 Group 3 (Military Expenditures)

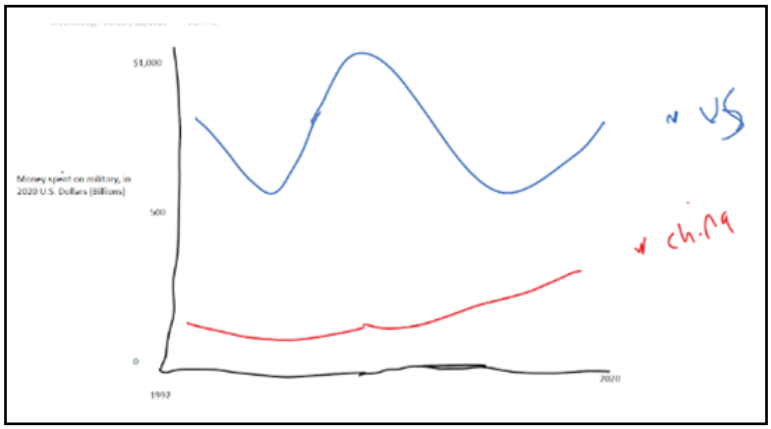
A third group found a graph (see Figure 8) showing the military expenses of the 6 nations who spend the most in the world on defense spending.

Figure 8: *Top Six Countries by Military Expenditures (as depicted by the Federal Reserve Bank of St. Louis).*



Our students noticed that the data for every country, except for the United States, used the y-axis on the left. The United States data referenced the y-axis on the right—an axis with larger numbers and larger step size. Students felt as though this graph was intentionally misleading, suggesting that China had surpassed United States' military expenditures. They created a sketch of a new graph showing the military spending of both countries on the same axis, shown in Figure 9.

Figure 9: *Revised Student Graph of Military Expenditures of China and United States.*



3.4 Implementation Ideas

3.4.1 Gallery Walk

When we implemented this lesson with undergraduates, we provided a digital space (Google Jamboard) for students to display their graphs. If we were to implement the Data Representation Task in a high school classroom, we would make this a two-day lesson. The first day would include our launch and introduction to the Data Representation Task. Day 1 would conclude with students working in small groups to find misleading graphs for further exploration. In Day 2, students would create a

poster that includes the original representation and their revised data display. Splitting this lesson into two days would allow the teacher to print copies of the student graphs for their posters, and it would also give students more time to analyze the graph and create new representations.Ultimately, students share their posters in a gallery walk. We suggest having students place sticky notes on each other's posters with their reactions, thoughts, and critiques. Then, students can return to their posters to read the notes they received.

3.4.2 Lesson Closure

As a closure to this lesson, we provided students with the prompts in Table 1 and conducted a think, pair, share to reflect on the lesson. Because we were working with preservice teachers, we asked them to put their "student hat" or "teacher hat" on for certain questions.

Table 1: *Lesson Closure Questions*

Student hat	Why do we need to be able to analyze data representations? What other takeaways do you have from this lesson?
Teacher hat	Why do you think this might be an important lesson for your students? What do you take away from this lesson as a teacher?

In other words, they were asked to reflect on the lesson as a student who had participated in the lesson and as a teacher who might incorporate a lesson like this into their classroom. If we were teaching this lesson in a high school classroom, we would only use the "student hat" questions. Overall, students discussed the importance of incorporating lessons like this into their future teaching. They expressed that it was important to have these conversations with students because of the "danger of misinformation" and "how quickly people tend to latch onto ideas, especially young people." They indicated that "graphs can be misleading," and it's important to teach students to "look at all of the details" to fully understand the data being shared. They appreciated that this lesson brought "relevant real-world content into the classroom" that was "more engaging than hypotheticals" and "very applicable to real life." They also expressed that "students need to be informed citizens" who can "read data to make informed decisions" and that this lesson was one example of bringing these ideas to life.

Perhaps because we were teaching this lesson at a somewhat liberal institution in the northeastern part of the country, we did not experience students who were hostile to this lesson or the ideas presented. Rather, they were eager to engage in the discussion and they expressed value in incorporating this type of work into their future teaching. However, they also shared a nervousness, especially as soon-to-be new teachers, to teach this specific lesson. Students made comments like, "the political aspect of the lesson can be hard to talk about with high school students because it is so polarizing," and to "beware of parents complaining about "woke education" with this lesson." They suggested to "maybe make this lesson less political," because "bringing in politics has the potential to get messy and distract from lesson." This last comment highlights the sentiment that bringing politics and social justice issues into mathematics classrooms can distract from the other content being taught, which is a common argument occurring right now within society. In the current climate in which "35 states have introduced 137 bills limiting what schools can teach with regard to race, American history, politics, sexual orientation and gender identity." (Gross, 2022), it is understandable that teachers are hesitant to broach these kinds of issues in their classroom. While we do not have answers here, we have some suggestions that might support teachers in this work, which we will address in our final section.

4 Final Thoughts

Overall, this was a very successful lesson with preservice teachers that we believe has potential in high school classrooms. Our students found value in the ideas presented; however, there were concerns raised, which we would like to address in this section.

One concern was that this lesson might not fit into the traditional standards. While we understand this concern, we believe this lesson demonstrates the calls from NCTM (2018) for students to leave high school with the skills necessary to make sense of quantitative information "in order to inform the decisions they must make now and in the future" (p. 56). As said by NCTM, "Stated simply, it is necessary that every high school graduate have the statistical reasoning skills to judge statistical information" (p. 56). This lesson addresses these important skills, and we believe it could fit into a unit on data analysis or statistics. If it does not align with a specific standard within that unit, then it could be used as an introduction to the unit, to help students see the importance of carefully analyzing any data they are presented with.

Further, to acknowledge our students' concerns about the political nature of this lesson, we would like to share ideas that might support teachers in this work:

- Remember that while these issues can be controversial or polarizing, students are living through them. Rather than ignore these topics, we should provide students with opportunities to discuss them in an educational setting with the support of facts and data. If we do not provide these spaces, then we are setting students up to only be informed by social media, their peers, or other misrepresented sources.
- Also remember that because students have lived through these experiences, all students have a doorway into the content. While these issues could be seen as controversial, we encourage educators to see them through a different lens: as inclusive because all students can access the material.
- Reach out to your colleagues and/or administrators to get their support and input. Specifically, collaborating with a history or government teacher could be helpful as they are typically well versed in teaching these types of lessons. Further, if this was a joint project across a couple of classes, then both teachers could support each other in the event of any push back.
- Finally, and perhaps most importantly, consider starting the lesson by setting up expectations for how to discuss controversial issues. The University of Michigan Center for Research on Learning and Teaching suggests that teachers should work with their students to establish ground rules for discussing controversial and emotionally-charged issues. Suggested guidelines for these discussions include the following (University of Michigan Center for Research on Learning and Teaching, n.d.):
  - Listen respectfully, without interrupting.
  - Listen actively and with an ear to understanding others' views. (Don't just think about what you are going to say while someone else is talking.)
  - Criticize ideas, not individuals.
  - Commit to learning, not debating. Comment in order to share information, not to persuade.
  - Avoid blame, speculation, and inflammatory language.
  - Allow everyone the chance to speak.
  - Avoid assumptions about any member of the class or generalizations about social groups. Do not ask individuals to speak for their (perceived) social group.

We hope the ideas above will support teachers in bringing these types of important lessons into their classrooms. Within this lesson specifically, we believe that students will see the value and importance of statistical literacy. They will see that graphs can convey powerful messages to viewers, and that the message of the graph should be consistent with the message in the data. Students are always asking the question "why do we need to know this," and this lesson is a very real space for students to clearly see "why."

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