Weaving Mathematics and Culture into Danish Hearts

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Abstract: *Woven from simple paper shapes, Danish Christmas hearts provide teachers with opportunities to engage their students in ethnomathematical inquiry. After providing cultural background and explaining how to create the hearts, the authors discuss an activity where students explore the symmetries present in the hearts and provide extensions applicable to students in grades 1-12+.*

Keywords: *Geometry, symmetry, measurement, paper craft, ethnomathematics, holiday activities*

1 Introduction

Do you ever sense that students find your lessons boring? Do your students view mathematics as an unpleasant academic requirement, isolated from the "real" world? Do they struggle to make connections or find meaning in what they learn? The field of ethnomathematics suggests that these outcomes result from separating mathematics from its natural context in human culture. According to D'Ambrosio (2001), "The term ethnomathematics is used to describe the relationship between culture and mathematics" (p. 308). By engaging students in mathematics that occurs naturally in the lives of groups of people, we put math back in context. When we guide students to view mathematics as a purposeful human activity, we make connections with other disciplines and cultures, and we create more enriched lessons. In the following article, we discuss a mathematics lesson that engages students in history, culture, and crafting through the exploration of Danish Christmas hearts (see Figure 1). Students learn history and culture as they analyze the mathematics within their own woven baskets.

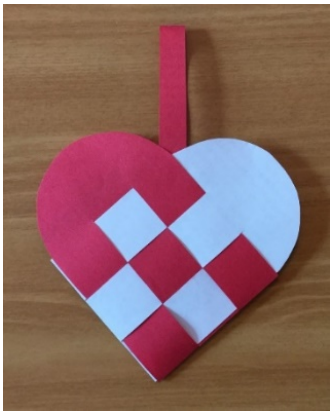

Fig. 1: *Danish Christmas heart basket.*

2 History

Hans Christian Andersen, the notable Danish fairy-tale author, created the first known woven heart in 1860 (Stehling, 2012). Although reasons for their initial popularity at Christmastime are unknown, Danish hearts (*julehjerter*, literally "Christmas hearts") soon became a Christmas tradition in Denmark and other Scandinavian countries. Children and families have a "cut and paste" day on which they create crafts from paper to decorate their trees and homes. They fill the baskets with candies and nuts and hang them on the Christmas tree or give them as gifts. Often, Danish hearts are made in red and white, the colors of the Danish national flag, and are an item of national pride.

According to the Museum of Danish America, Danes began migrating to the United States around 1820. At that time, farming was the main industry in Denmark, but land was scarce and expensive. By tradition, the eldest son inherited the family property, and other siblings had to buy their own land. Meanwhile, in the United States, the Homestead Act (1862) gave 160 acres to anyone who promised to live on it for five years. Tens of thousands of Danish immigrants arrived in the United States each decade between 1860 and 1930, with approximately 88,000 arriving between 1881 and 1890 alone. These immigrants primarily settled in the Midwestern states and brought their Christmas traditions, which included making Danish hearts. A book for North Dakota school teachers, published in 1924, included instructions for making the hearts (Bassett & Smith, 1924).

Today, paper crafters around the world create intricate and unique variations of Danish hearts, which require mathematical practices in their design. While children often learn to make the hearts in elementary school, the mathematics involved is appropriate for secondary students. Teachers can share historical background as students construct the hearts for a full ethnomathematical experience. While the construction of Danish hearts is a worthwhile project any time of the school year as a review of symmetry, teachers may wish to engage students in this activity before a holiday break or for Valentine's Day.

3 Directions for Making Danish Hearts

Creating and discussing Danish hearts form the basis of an interdisciplinary project involving art, history, and mathematics. First, students weave the Danish hearts using readily-available online directions. We recommend the article "Send a Heartfelt Hello" by Susan Stehling (2012). Students will need at least two colors of paper, a ruler, and scissors. We suggest printer paper for ease of weaving, as this paper is more malleable than construction paper.

Although measuring and cutting paper shapes provide ample opportunities to reinforce practical skills, we created a template (see Appendix A) so that teachers with limited time can simplify the process for students and help them avoid many common errors (see next section). After teachers copy the template onto two colors of paper, students cut out two shapes from different colors, fold on the dotted line, and cut the marked slits. Then, students "weave" the shapes together to create the basket (see Appendix B for instructions with illustrations). Figure 2 shows an example of a finished Danish heart basket.

3.1 Classroom Experience

When creating the hearts, we found it advantageous to get the entire class's attention, demonstrating one step at a time with a model and then providing hands-on help for individuals who needed it. With this lesson, students' success depends on their ability to follow directions. When first exploring Danish hearts with our students, we did not use a template. Unfortunately, some of

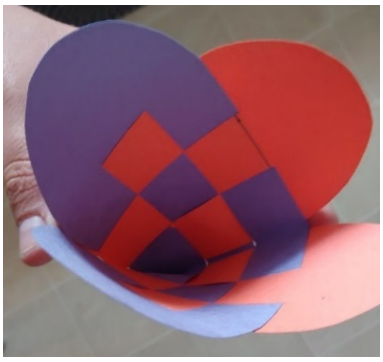

Fig. 2: *The basket can be opened up to reveal a checkerboard pattern on the inside as well.*

our high school students had difficulty measuring a 3-inch by 9-inch rectangle. Others measured from the wrong side of their folded rectangle and thus cut it apart, or did not space slits evenly. Many students found that slits needed to be a fraction of an inch longer for ease of weaving. To address these challenges, we created a template (Appendix A) to better support struggling students. Fortunately, many others reasoned successfully about the measurement process:

STUDENT 1 : *Pondering the 9-inch by 3-inch size* I wonder if the dimensions really matter.

STUDENT 2 : It has to be a 3 by 1 ratio.

STUDENT 3 : *Contemplating the placement of the slits* So they're 1 inch apart from each other?

Furthermore, the weaving process (Appendix B) is different from the over-under pattern students may have learned previously, and so it is confusing for some. After working with several students as they wove strips traditionally, we learned to emphasize that the strips go through each other, not over or under. It is also important that students maneuver the paper strips gently to avoid creasing or tearing.

With patience, assistance, and a little tape to fix mistakes, all of our students wove baskets successfully. Figure 3 highlights several samples from our high school students. The left center example (blue and white) illustrates a basket with uneven slits.

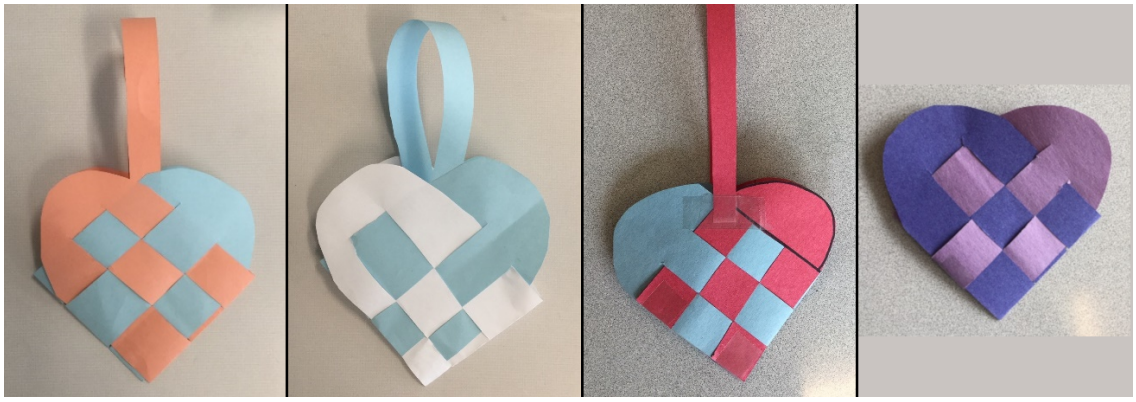

Fig. 3: *Student work samples.*

4 Discovering Mathematics in Danish Hearts

Once our students created their own Danish hearts, we encouraged them to discover the mathematics hidden within their designs. For instance, we asked students to make a list of the symmetries they saw in the square checkerboard portion of the heart, ignoring the curved top portion. Knowing students' background knowledge, we encouraged them to look for various types of symmetries. Students quickly uncovered reflections and then rotational symmetry in the hearts. We suggest that you and your students look for reflections, rotations, translations, and glide reflections. After our students listed all the symmetries they saw, we made a list as a class. As Figure 4 suggests, students found four reflections through the center of a square, translations in two directions, three degrees of rotation, and several glide reflections. As we discussed these observations, students verified that their shapes lined up after each transformation. A colored cutout is useful for demonstrating the combination of reflecting and translating associated with glide reflections.

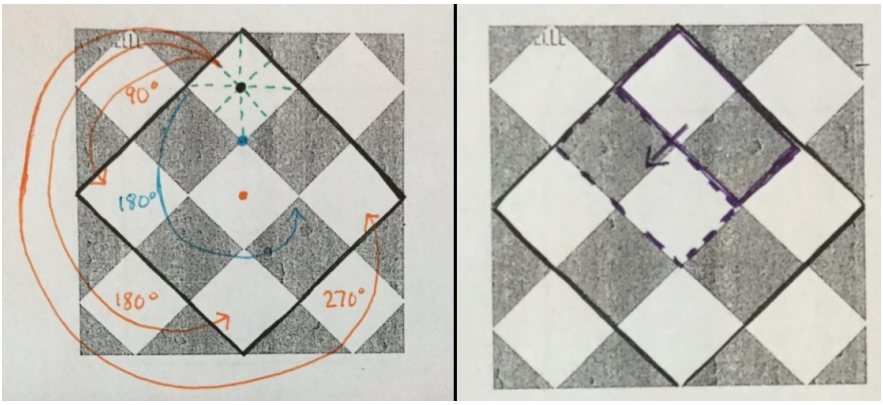

Fig. 4: *(Left) Reflections and rotations identified; (Right) One possible glide reflection.*

Looking for rotations sparked some particularly engaging conversations. First, students debated how many unique rotations there were. One student saw clockwise and counterclockwise rotations but did not recognize them as two different rotations. His classmates saw them as distinct and tried to convince him of this. Eventually, the class identified three distinct rotations: 90-, 180-, and 270-degrees counterclockwise. At this point, we pressed students for more details about the rotations they saw.

TEACHER : Where is the specific point that you are rotating around?
STUDENT 1 : In the middle of the square.
TEACHER : Could it be anywhere else?
STUDENT 2 : You could also use the corner of a square.
TEACHER : Do all of the rotations you found work at both of those points: the center and the vertex of a square? *Students ponder the new idea.*
STUDENT 3 : The 90 [degree rotation] wouldn't work for the vertices. The white square would overlap the red square...the 90 or 270 wouldn't work.

Ultimately, the class decided that $90°$, $180°$, and $270°$ rotations about the center of a square worked, while only the $180°$ rotation about the vertex was a symmetry.

4.1 Alignment to the Standards

The Common Core State Standards for Mathematics for grades 7–12 related to creating the Danish heart basket and finding its symmetries include the following (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010):

- CC.2.3.7.A.2 Visualize and represent geometric figures and describe the relationships between them
- CC.2.3.8.A.2 Understand and apply congruence, similarity, and geometric transformations using various tools
- CC.2.3.HS.A.1 Use geometric figures and their properties to represent transformations in the plane
- CC.2.3.HS.A.2 Apply rigid transformations to determine and explain congruence
- CC.2.3.HS.A.14 Apply geometric concepts to model and solve real world problems.

5 Extensions and Other Ideas for Danish Hearts

Although we found very little existing work describing the mathematics specific to Danish hearts, we see a multitude of possible connections. Some of our favorites for younger students include composite shapes (CC.2.3.1.A.1), finding area and perimeter (CC.2.4.3.A.5, CC.2.3.7.A.1), array models of multiplication (CC.2.2.2.A.3), fractions of a shape (CC.2.3.3.A.2), perfect squares, and lines of symmetry (CC.2.3.4.A.3). Students can find the dimensions of other rectangles in a $3:1$ ratio, and they can calculate the width of each slit for a certain size rectangle and a certain number of slits (CC.2.1.7.D.1) (National Governors Association Center for Best Practices & Council of Chief State School Officers, 2010). With older or more advanced students, the discussion of symmetries can be extended to introduce wallpaper patterns. For example, the checkerboard pattern in the center of the traditional heart basket is a p4m pattern. For more information about this and other wallpaper patterns, see Williams (n.d.).

Students of all ages enjoy seeing variations on the classic Danish heart pattern. Images and directions for many are readily available online by searching for "Danish woven heart variations." For an extra challenge, we showed students two such patterns and asked them to draw the template that they thought would be cut out to create the woven designs. Figure 5 illustrates example woven hearts that we provided, and Figure 6 shows the corresponding templates our students envisioned. Imagining possible templates required a high level of critical thinking and visualization. For further discussion and reasoning, students can compare templates, choose the best ones, and check answers by physically weaving hearts from them.

Fig. 5: *Pattern variations we showed students.*

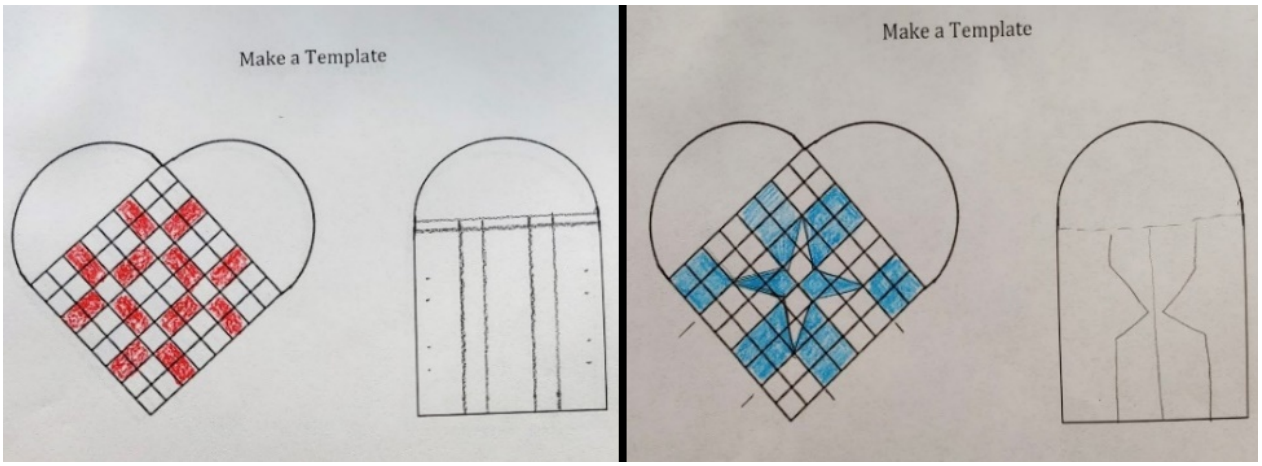

Fig. 6: *Student-generated diagrams and templates.*

Finally, we love the website by Brodal (2012), where students can design a virtual Danish heart with their name or other brief message on it. Constructing these with actual paper requires a craft knife and extreme patience! See Figure 7 for our example.

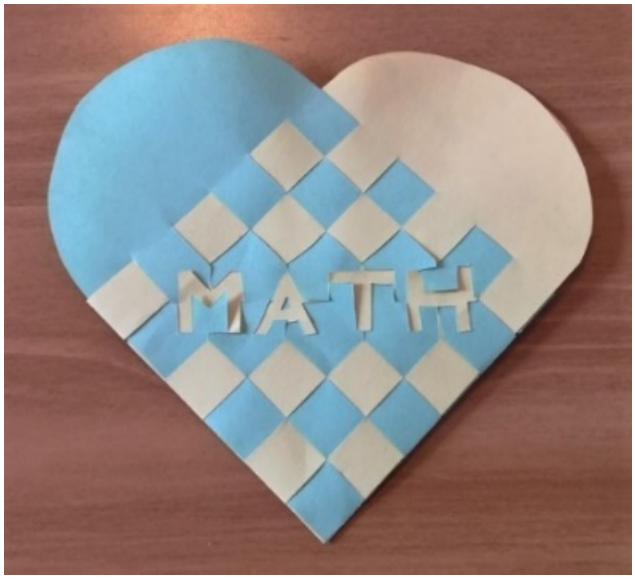

Fig. 7: *Heart created from Brodal (2012) website.*

6 Conclusion

While incorporating cross-curricular lessons can be challenging, teaching with an ethnomathematical perspective provides meaningful learning for students. We believe that creating and discovering the math in Danish hearts can be a fun, thought-provoking, and well-rounded lesson including history and culture. We note that the lesson can be completed over several days if needed. Students could begin by exploring the history and culture of Denmark, then engage in mathematics through the construction of Danish hearts. As an added bonus, the Danish hearts that students weave make a great addition to classroom decor that highlights student learning.

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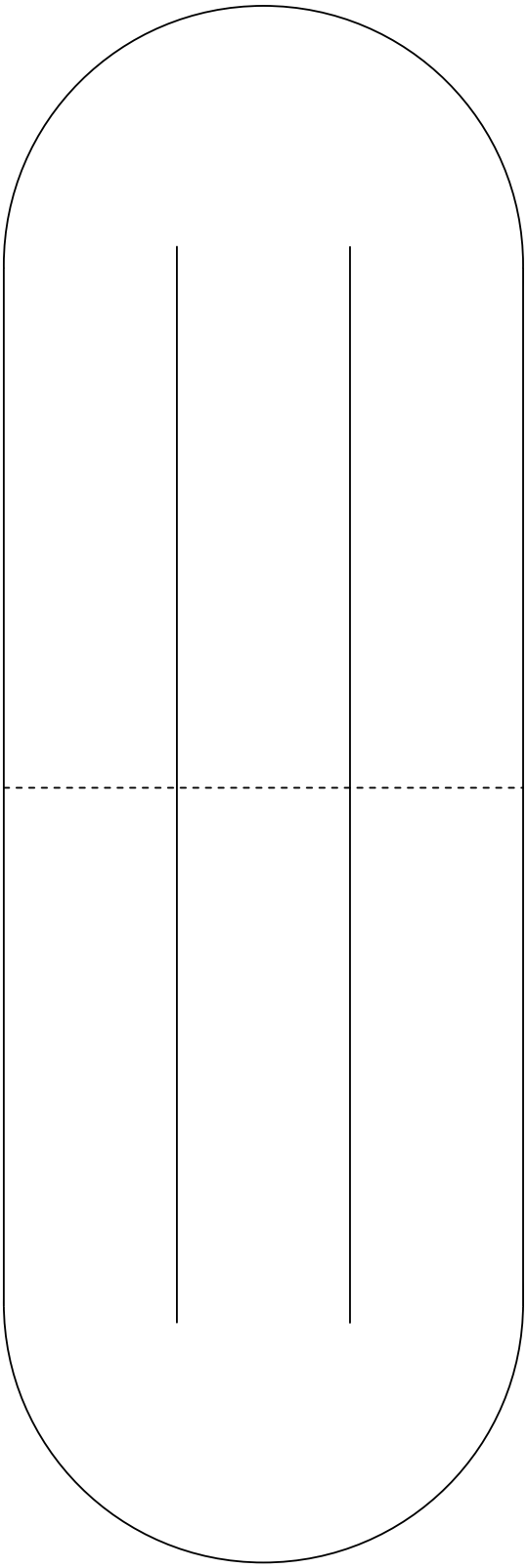
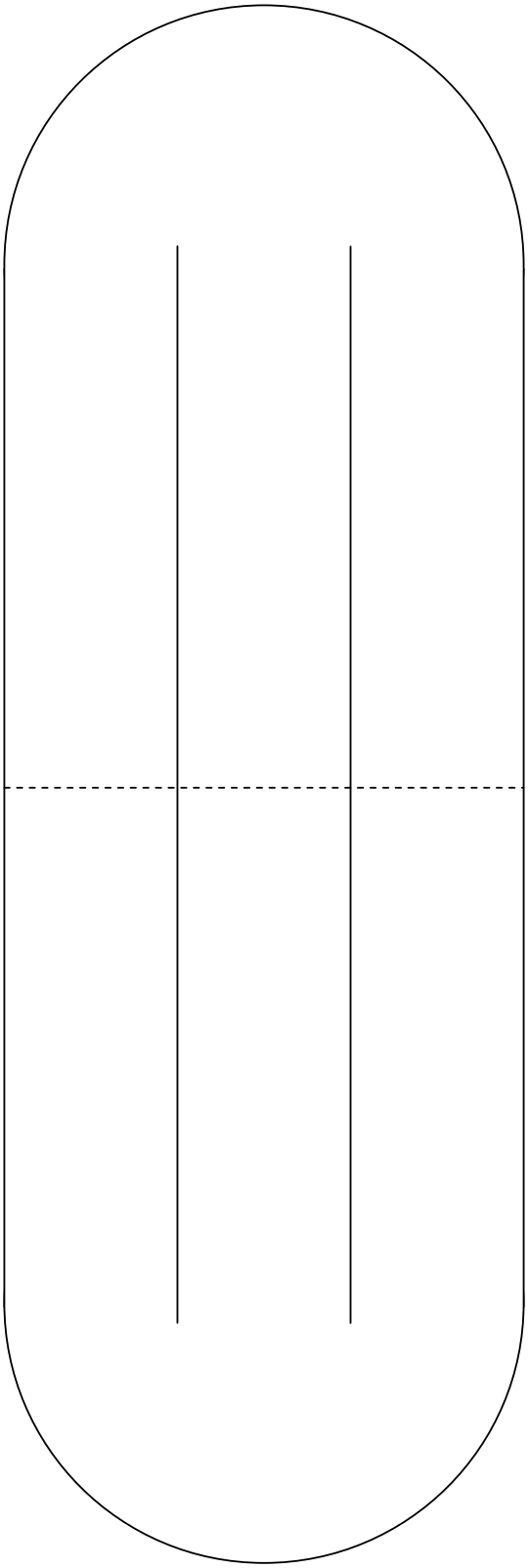
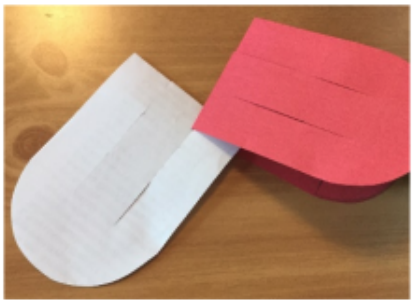
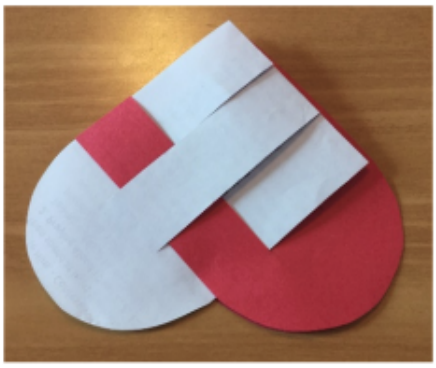
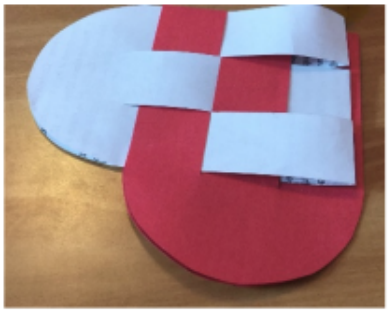
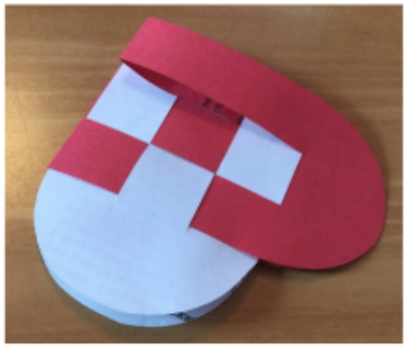
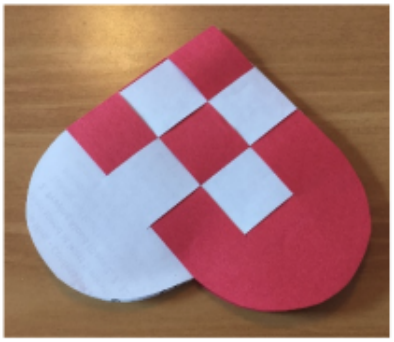
Julie Killian is a mathematics teacher at Middletown Area High School in Middletown, Pennsylvania, where she enjoys designing lessons that make mathematics come alive for her students. She holds a degree in Mathematics with Secondary Teaching Certification from Messiah College and is completing her M.Ed. at Millersville University. In addition to crafts, Julie enjoys reading, playing the clarinet, and traveling with family.

APPENDIX A: Template for creating the Danish hearts





APPENDIX B: Directions for weaving the Danish hearts

1. Put the first white strip through the first red strip. 	2. Put the red strip through the next white strip. 	3. Put the third white strip through the red strip. Slide the red strip toward the curved side. This completes the first row. 
5. Put the second red strip through the first white strip. 	6. Put the second white strip through the red strip. 	7. Put the red strip through the third white strip. This completes the second row. 
8. Put the first white strip through the third red strip. 	9. Put the red strip through the second white strip. Then put the third white strip through the red strip. This completes the heart. 	9. Optionally, cut a thin rectangle and tape or glue it to form a handle.