# Proof Without Words: $a^2 - b^2 = (a+b)(a-b)$ via Isosceles Triangle and Right Trapezoid

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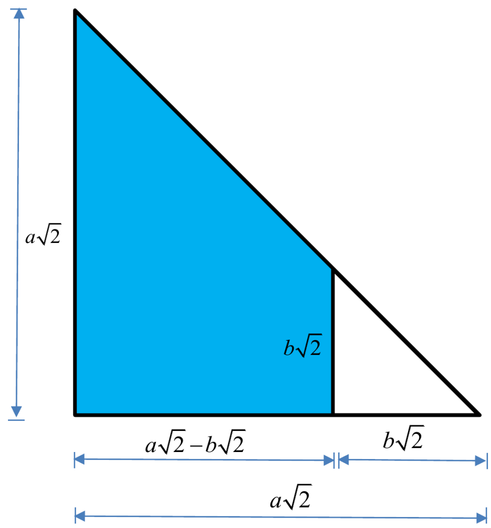
**Summary**

The difference in area between two similar isosceles triangles produces a right trapezoid whose area is $a^2 - b^2$, which in turn equals $(a+b)(a-b)$, illustrating the difference-of-two-squares identity. While retaining the two terms $a\sqrt{2}$ and $b\sqrt{2}$ of Subramaniam and Thomas (2025), a much simpler geometric insight is offered here, requiring only the area of a right trapezoid (see Appendix) and the well-known formula for the area of an isosceles triangle.

**Keywords:** Isosceles Triangle, Right Trapezoid, Visual Reasoning




*difference in area between two similar isosceles triangles = area of right trapezoid*

$$\Rightarrow \frac{(a\sqrt{2})(a\sqrt{2})}{2} - \frac{(b\sqrt{2})(b\sqrt{2})}{2} = \frac{(a\sqrt{2} + b\sqrt{2})}{2}(a\sqrt{2} - b\sqrt{2})$$
$$\Rightarrow a^2 - b^2 = (a+b)(a-b).$$

**Remark 1:** In Subramaniam and Thomas (2025), the spatial reasoning requires the application of the Pythagorean Theorem to arrive at the requisite terms $a\sqrt{2}$, $b\sqrt{2}$, and $a - b$, in addition to the area-calculation knowledge of a trapezoid, whereas the area of the right trapezoid presented above may be readily visualized in terms of the commonly known area calculations of a rectangle and a triangle (see Appendix). For an alternative pedagogical interpretation of the difference of two squares, the reader is referred to Mahmood (2013).

**Remark 2:** Teachers could easily introduce this proof in help sessions or tutorials as a small-group exploration. The proof would also serve well as a visual aid to complement an algebraic proof in a lecture. It would further benefit students if a couple of minutes are spent on it in a regular classroom discussion. Initially the teacher can ask orienting questions, such as labelling the right trapezoid and recalling its area formula, then guide students to connect the figure back to $a^2 - b^2$ via the isosceles-triangle area formula given in the Appendix. Finally, Remark 1 can be raised to initiate a comparative study with Subramaniam and Thomas (2025) and Mahmood (2013).

### Acknowledgment

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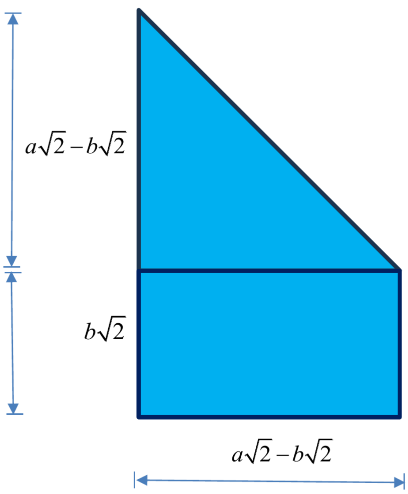
## References

Subramaniam, K. B., & Thomas, A. (2025). Proof without words: $a^2 - b^2 = (a+b)(a-b)$. *Ohio Journal of School Mathematics*, 99.

Mahmood, M. (2013). Proof without words: A characterization of the difference of two squares. *The Mathematical Gazette*, 97(539), 334.

## Appendix




$$\textit{area of isosceles triangle} + \textit{area of rectangle} = \textit{area of right trapezoid}$$

$$\Rightarrow \frac{(a\sqrt{2} - b\sqrt{2})(a\sqrt{2} - b\sqrt{2})}{2} + (b\sqrt{2})(a\sqrt{2} - b\sqrt{2}) = (a\sqrt{2} - b\sqrt{2})\left(\frac{a\sqrt{2} - b\sqrt{2}}{2} + b\sqrt{2}\right)$$

$$= (a\sqrt{2} - b\sqrt{2})\left(\frac{a\sqrt{2} - b\sqrt{2} + 2b\sqrt{2}}{2}\right) = \frac{(a\sqrt{2} + b\sqrt{2})}{2}(a\sqrt{2} - b\sqrt{2}).$$