Design Thinking in Motion

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Abstract

Incorporating design thinking into classroom culture fosters mathematical agency and deepens skills that support mathematical modeling. Designing animations in Dynamic Geometry Software can demonstrate the value of iterative thinking and increase students' tolerance for risk and problem-solving.
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1 Introduction

Since the publication of *Building Thinking Classrooms* (Liljedahl, 2021), teachers have been reimagining what mathematics classrooms can look like. Students are engaging in problem solving, revision, and collaboration in ways that prioritize shared thinking and mathematical connections. This shift requires letting go of direct instruction and other practices that standardize learning. This shift is not easy. However, the challenge is not a shortage of good tasks or activities, but rather that the success of those activities depends on the curiosity and mathematical thinking of students, not the expertise of the teacher. This kind of classroom, one where students are persevering, reasoning, and communicating mathematically, is precisely what the eight mathematical practices (National Governors Association Center for Best Practices & Council of Chief State School Officers [NGA Center & CCSSO], 2010) call for. Being able to create such an environment for learning is the primary focus of this paper.

We can learn a lot from the design world about cultivating curiosity and taking risks. In a culture of design, students move beyond merely solving problems to becoming architects of their own mathematical thinking. They create, iterate, and make strategic decisions driven by their own objectives and ideas. Integrating a design thinking framework into mathematics instruction empowers students to define the criteria and scope of their work, effectively shifting classroom norms away from compliance toward mathematical agency and creativity. When the principles of design thinking are translated into the mathematics classroom, they can help bridge the gap between teacher-led instruction and genuine student ownership of learning. This paper explores what that can look like in practice.

2 Mathematical Design Thinking

Design thinking principles suggest a fundamental shift in how we enact mathematical tasks. Rather than guiding all students toward a predetermined solution, we create environments that enable students to design for themselves, tolerate productive failure, and exercise agency over their own mathematical boundaries. This approach allows students to explore and connect mathematical representations.

Kolko (2015) lays out five principles of design thinking. These principles are:

1. Focus on users' experiences, especially their emotional ones
2. Tolerate failure
3. Use prototypes to explore potential solutions
4. Create models to examine complex problems
5. Exhibit thoughtful restraint

Interpreting these design principles for a mathematics classroom provides insight into how to structure a classroom built around student thinking.

2.1 Principle 1: Focusing on Users' Experiences

At the heart of design culture is the user's experience. This means understanding the intended audience, what the user is feeling, and how the user engages with design. While we do consider our students' experience with this task, here we are speaking of the ways we ask students to consider their audience when designing mathematics. Due to the nature of the classroom setting, students are typically limited to designing for themselves, classmates, and teachers. In our experience, students typically begin working with the expectation that they will have something to demonstrate to others (the teacher in particular) rather than for their own understanding. It is at this point, when students are deeply thinking mathematically and independently, that we, the teachers, move into the role of an advisor during the design process, listening for students' mathematical thinking.

As there is a need to document, communicate, and verify mathematical design in the classroom, mathematical empathy (Harper et al., 2024) is important. Empathy entails the search to comprehend another person through their frame of reference. Mathematical empathy requires that sort of comprehension, but instead of emotions and feelings, it is focused on mathematical ideas. Thus, mathematical empathy refers to "the willingness to cede mathematical expertise to others and see them as someone from whom to learn" (p. 1062).

2.2 Principle 2: Tolerance of Failure

For a design culture to thrive, students and teachers alike must be tolerant of failure. Students conditioned to value a polished, singular solution over the process of construction may find their creativity limited by the risk of failure. Who isn't frustrated when hard work and ambition don't work out in the end? However, intellectual risks are an inherent part of engaging with complex problems, those requiring conceptual exploration and iterative modeling without a clear solution path. These challenging situations are essential, as they give students opportunities to explore solution spaces rather than simply find solutions. To support mathematical design thinking, the emphasis or learning goals cannot be placed solely on the product and must be inclusive of the process; engaging in the design process itself is a valuable experience, regardless of the ultimate success of the design. We have all heard the power of the word 'yet' in fostering a growth mindset.

2.3 Principle 3: Use Prototypes to Explore Potential Solutions

A prototype is not a finished product. It is a low-stakes, exploratory artifact that allows designers to test ideas before committing to them fully. In a mathematics classroom, prototyping might look like a rough sketch of a graph or an initial equation that captures only part of a solution. This rough draft thinking (Jansen, 2023) encourages revision, refinement and deeper mathematical reasoning. Rather than waiting until ideas are fully formed, students are encouraged to begin working with incomplete solutions, using the process of testing and adjusting to move their thinking forward. Just

as a designer returns to a prototype again and again, students are invited to revisit and rework their mathematical ideas. Each iteration brings them closer to a solution that is both personally meaningful and mathematically sound.

2.4 Principle 4: Create Models to Examine Complex Problems

Complex problems are rarely solved in a single sitting. Designers break problems apart and look at them from multiple perspectives. In a mathematics classroom, this principle invites students to examine the structure of a problem before rushing toward a solution. Rather than seeking an immediate answer, students are encouraged to carefully understand the problem. This shifts the focus from answer-getting to sense-making, positioning students as curious thinkers who explore problems rather than consumers who simply execute procedures.

2.5 Principle 5: Exhibiting Thoughtful Restraint

For students to have true agency and ownership, they must be allowed to decide for themselves what their models cannot and should not accomplish. This requires the freedom to exhibit thoughtful restraint. In the context of mathematical tasks, this is interpreted as being intentional about goals, strategies, and determining when we have reached a final solution. This grants students choice over both ambition and risk, allowing them to determine the extent and limits of their personal mathematical boundaries. As we explore the Animation Task with you, we will give some insight into the ways in which we encourage students to attempt ambitious ideas, but also allow for imperfection, imprecision, and assumption.

3 Mathematical Design Thinking in Action: The Animation Task

One of the tasks that we use to inspire creative thinking and mathematical design is the Animation Task. This task asks students to explore a complex phenomenon through modeling with Dynamic Geometry Software (DGS).

The Animation Task

1. Find a real-life object or scene that you can model using DGS. Describe it here. You may want to come up with two or three ideas so that you can choose between them later.
2. How would you expect your object or scene to act if it were to "come alive"?
3. Create an animated sketch using DGS that brings your object to life. Carefully consider how this object moves or changes in the real world and choose appropriate and reasonable mathematics to model its behavior. A slider is required, but you may use it in whatever way supports your thinking.
4. You are free to make your model as artistic/creative/real as you wish. You will be graded on the validity of your animation as well as how well the real-life phenomenon is connected to and represented by technology. You should select an ambitious project and be prepared to demonstrate that you learned something new.

This task supports interdisciplinary learning as students consider the question of how to use mathematics to model scientific phenomena, including cultural events. It was ultimately designed to support the practice of Modeling with Mathematics (MP 4), though we have seen this task support the development of all eight mathematical practices (MP), while the connections to MP 1, 2, 5, and 6 are the most common and visible (NGA Center & CCSSO, 2010).

3.1 Setting the Stage for Thinking

Prior to the assignment of the Animation Task, we are careful to make sure that we have set the stage accordingly. We have used this task with students from varied backgrounds in mathematics. A pre-algebra course provides helpful prerequisite knowledge. Mathematically, this task would be appropriate for middle or secondary school students who have been introduced to the coordinate plane (CCSSM 6.G.A.3) along with representing quantities and change using algebraic expressions (CCSSM 6.EE.C.9), though proficiency is not required. Experience with transforming geometric objects in the coordinate plane using reflections, rotations, and translations (CCSSM 8.G.A.3) would also add excitement and depth to the mathematics. However, if students have not studied these formally, this task might be an opportunity for informal play with those mathematical tools.

Technologically, students must have worked within a DGS before. In our classroom, we use GeoGebra (International GeoGebra Institute, 2026). A key feature students need to be familiar with is the use of sliders. In GeoGebra, a slider is an interactive control element that appears on screen as a horizontal or vertical bar with a movable point. The slider represents a variable, typically a number or angle, whose value changes continuously as the user drags the point along the bar, or as the value is incremented automatically through GeoGebra's animation function. Crucially, this variable can be linked to mathematical objects such as coordinate pairs, lines, or geometric transformations, so that as the slider value changes, those objects update dynamically in response. For example, a slider representing the parameter t might control the position of a point along a path, the angle of a rotation, or the magnitude of a translation vector. It is important that students have had some experience with creating sliders and linking them to such objects, since this is the mechanism through which animation is controlled in the Animation Task. Students do not need to be proficient with these skills, as the task will motivate additional exploration.

3.2 Supporting Student Exploration on the Task

The principles of design thinking help us identify ways to support students as they move through the task. First, we attend to user experience and encourage our students to do the same. Thinking about their peers or friends as an audience for this work might help generate ideas and motivate mathematical learning. We also consider our students' experience in the lesson design; students need time to interpret the task and envision what they want to build. Even if students have had experiences with linking sliders to other mathematical objects, they still need time to consider what other phenomena might be well-suited for animation.

In the ideation stage and as we launch the task, we ask students to collectively brainstorm objects and scenes that they might be able to represent in the DGS environment. These responses may be static suggestions such as "a basketball" or "a garden". After ideas have been shared, we select one and think aloud together about which DGS tools would help make the design. You can use student responses to highlight specific objects and tools such as polygons, perpendicular and parallel lines or even transformations. From there, we quickly move on to discussing how these images or scenes might "come alive" for our audiences. In other words, how would you expect these objects or scenes to move or change?

We have had students choose familiar contexts such as projectile motion, mechanisms and machines (Figure 1), natural phenomena (Figure 3), and the movement of objects through space (Figure 2). Jordan's Slip 'N Slide in Figure 4 is a compelling example of coordinate geometry and polygons, where the central problem is to move a human body in a recognizable way through space.

Figure 1

Cory's working clock with hour, minute and second hands; slider linked to angle of rotation. Explore the sketch at https://www.geogebra.org/m/tzkms23z.

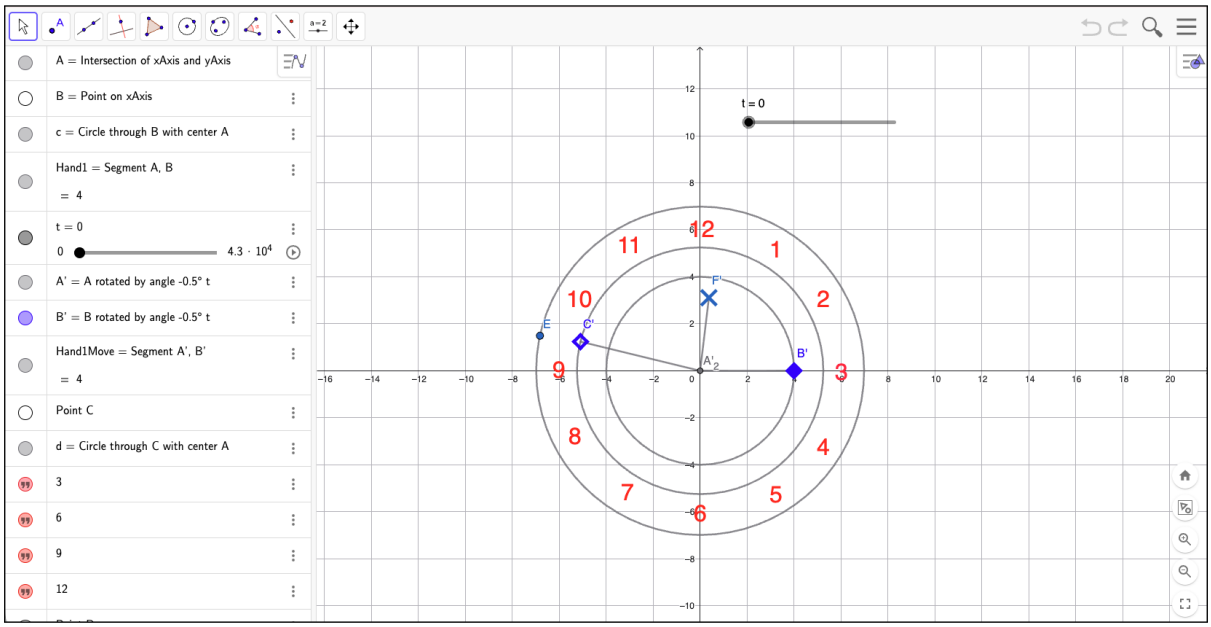


Figure 2

Sasha's animated toys; slider linked to rotational movement of toys. Explore the sketch at https://www.geogebra.org/m/kpjajnqg.

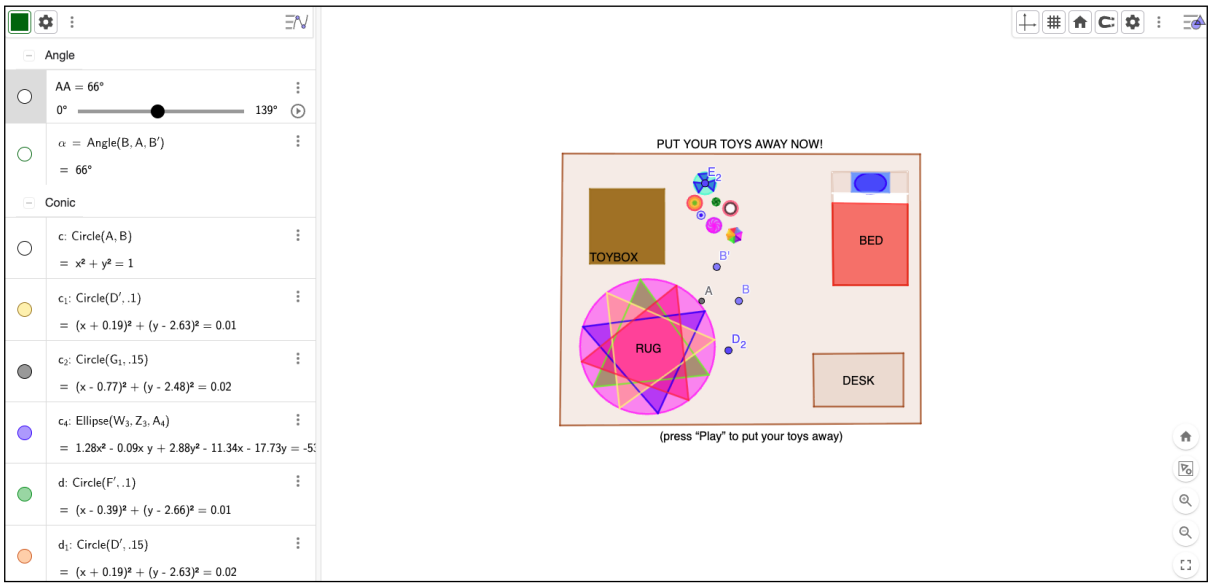
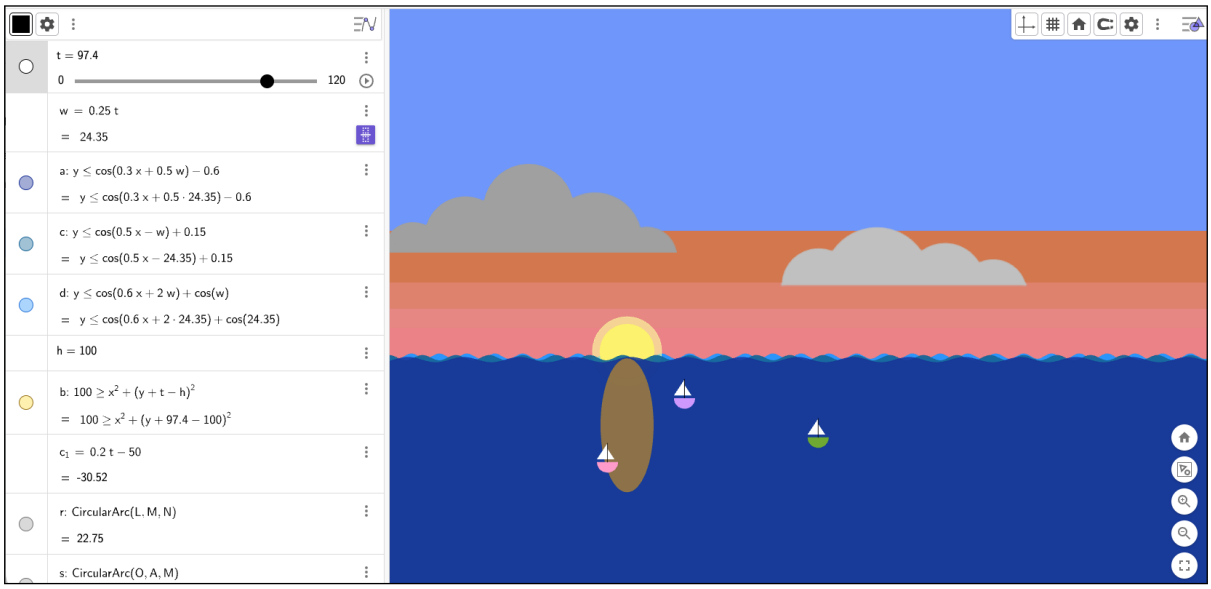


Figure 3
Kei's animated sunset on the ocean; slider linked to a timeline for sunset and boat motion. Explore the sketch at https://www.geogebra.org/m/tutwkj4m.



Eventually, students select a phenomenon and search for the specific technological and mathematical tools needed to model it. We typically provide one class period of parallel exploration where each student experiments with their own ideas, but works alongside a few other classmates in case they want to think out loud or ask for advice. Here, students learn to tolerate failure. Since students identified multiple ideas in the ideation stage, they feel comfortable with abandoning ideas that prove too basic, too challenging, or too boring to pursue further. The expectation is that some ideas will fail, and it's ok when they do.

Building prototypes is a helpful way to identify which designs seem advantageous or interesting for the students. Naming these initial builds as prototypes helps them feel more temporary and invites revision. Some students are satisfied with their first and only idea, while others generate possibilities to sift through, changing tack when inspiration hits, or when problems arise for which they do not (yet) have technological or mathematical solutions.

Once a prototype emerges as interesting, ambitious, and do-able, students are encouraged to keep working independently to build their models and study the mathematical structure of the complex phenomenon they have chosen. Each animation is built upon a slider; however, identifying the underlying driver behind a phenomenon is a mathematical challenge. For many students, the driver is time, and the animation becomes a series of time-dependent variables (e.g., Figures 3 & 4). Other students build motion on different foundations, such as angular rotation (e.g., Figures 1 & 2). Students need time to work through these challenges creatively. We expect this stage to take place over the course of about a week's time with no requirement to check in or update the teacher until the end.

Prior to submission, we provide time in class for each student to demonstrate rough drafts to small groups of peers. This isn't intended to be a summative assessment; rather, a formative period where students can receive advice and show off their accomplishments. It's helpful to have thought partners in the work. Sometimes, students are struggling with their own expectations and support means helping them adjust to time or resource constraints. More often, students are concerned about our expectations as teachers and ask questions about whether they have done enough. We embrace thoughtful restraint at this stage of the task, recognizing that a simple animation with clear mathematical thinking is as valuable as a complex one with sophisticated coding. Our role is to

help students focus on and articulate the mathematical process, whether they have used informal approaches (like trial-and-error to find the right coefficients) or formal strategies (like deriving equations from properties of the phenomenon).

When students present designs that don't work quite right, we treat these as opportunities for mathematical sense-making rather than failures. By asking "What would need to change mathematically to fix that?", we help students see imperfect work as part of the iterative design process and deepen their understanding of the mathematical relationships at play. The careful discussion of sufficiency and ambition is important because it validates diverse entry points while maintaining mathematical rigor.

While these principles supported the design of good mathematics, they also supported students in developing mathematical practices (NGA Center & CCSSO, 2010). Cory's (Figure 1) work illustrates both attending to precision (MP 6) and persevering in solving the problem (MP 1). Specifically, Cory works to ensure that the hands of his animated clock (linked by angular rotation to the slider's speed) would be as precise and accurate as possible. Sasha's (Figure 2) design highlights the use of appropriate tools strategically (MP 5). Faced with the challenge of moving the toys into the toy box, Sasha considered a variety of mathematical approaches before choosing to show a reflected image of the toys through point (B′), whose rotational motion was controlled by the slider *AA*. Kei's work (Figure 3) illustrates reasoning abstractly and quantitatively (MP 2). Kei considered what functions would best represent the motion of sailboats and the way the spherical sun appears when its light is reflected on the surface of the water, demonstrating how slider-based animation can prompt sophisticated mathematical reasoning. We will be exploring Jordan's (Figure 4) thinking in more detail in the next section.

Figure 4

Jordan's animated Slip 'N Slide; slider linked to a timeline for water hose and human motion. Explore the sketch at https://www.geogebra.org/m/yekcyupx.

4 Encouraging Self-Reflection and Assessment

You may choose to have students formally present their animations to one another, but this isn't always necessary. The important part is that they share the work with their peers and engage in self-reflection; the reflection and presentation process deepens their mathematical learning. While

designing animations develops mathematical thinking, the acts of sharing work publicly and reflecting deeply on mathematical choices are equally essential to the design process. On this assignment, we want to set the expectation that students design for themselves and their peers, not only for us as their teachers. This is an important distinction that has implications for the way that work is submitted, discussed, and assessed. To support this peer-centered design culture, we structure both the logistics of sharing and the reflection prompts intentionally. Logistically, we share links to our animations through a Google document so that everyone has direct viewer (not editor) access to all of the work. We also ask students to publicly share their thoughts on four reflection prompts:

1. What was your intention? What were you trying to model or represent?
2. What did you need to investigate/learn in order to make your design work?
3. Are there any special features you'd like to talk about?
4. What more would you like to accomplish if time were infinite?

The first question is important for two reasons. First, it may not be clear from the animation itself what the student is representing. Second, since we gave students agency to exhibit thoughtful restraint, students get to choose the degree of aesthetic refinement in their designs, deciding for themselves how to balance visual appeal with mathematical representation.

We assess our students on the validity of their animation, as well as the degree to which they tackled something ambitious. The validity of the animation can be gauged with a simple question – does the animation reflect the expected or intended motion? We would expect a football to travel a parabolic path as it scores a field goal; we would expect the reflection of the sun on the water to lengthen as the sun approaches the horizon; and we would expect the hands of a clock to rotate at a specific speed. We can look to the student's responses to reflection questions 1 and 4 to help us decide the degree to which we want to hold students accountable for realistic motion. To assess ambition, we can look to the student's response to questions 2 and 3. This will indicate the degree to which the student stretched to reach the goal and will also indicate areas of pride where students feel that something important or interesting was accomplished.

In this paper, we have curated a collection of more aesthetic designs because it is easier for readers to grasp the mathematical purpose and complexity when the context is more apparent. However, those reading should not assume that this is a requirement of the assignment. Thus, the contextual cues about the phenomenon may be quite vague or even missing. In one recent case, a student submitted a bare-bones animation of a small polygon moving in a parabolic path across the coordinate plane toward a rudimentary field goal consisting of just three segments. The context was all but missing, but it was clear that the football was moving as you'd expect once it had been kicked.

On the other hand, as in the case of the Slip 'N Slide, the mathematical design may be camouflaged under the aesthetics. Visit https://bit.ly/slipnslide_movie to watch a movie of Jordan's GeoGebra file. As Jordan writes,

> This is a Slip'n Slide! I used multiple sliders to control different motion and objects. For instance the hose is controlled by its own slider called "Hose" and the person is controlled in the horizontal direction by slider "j" and in the vertical direction by sliders "k" and "l." I had to research GeoGebra scripting on how to get the sliders to depend on each other and be triggered by buttons.

The aesthetics of Jordan's animation hide the mathematical nuances required to get it to look and move the way that it does. Encouraging her to reflect on the process in writing helps us see the hidden mathematics and understand what she has learned (including scripting, which was not previously

taught in class). Hidden behind the image are two timelines. Turning on the hose sets in motion a slider called "Hose", which is linked to the x-coordinate of some of the points within the light blue polygon, extending the length to appear as water flooding the path. Another timeline drives the jump. The human is a set of polygons, beige and pink, whose vertices are linked to a series of sliders (j, k, l) which control the horizontal position of the human (j) which moves exclusively to the left, and the vertical position of the human (k), which moves both down and up as the person bounces along the path. In order to make the bounce look realistic, Jordan dampened the oscillation of slider k with another slider, l, with the following script: `If(6.35<=k<=6.85, SetValue(k,l))`. In building this animation, Jordan connected her understanding of coordinate geometry with composite functions between sliders, demonstrating how geometric motion can be controlled through algebraic dependencies.

Questions 2 and 3 provide space for students to talk more about their accomplishments and what they learned about the technology, the phenomenon, or the mathematics. Question 4 gives students space to discuss thoughtful restraint and to manage audience expectations about what is possible or desirable given the constraints of time and knowledge. It also encourages students to recognize the value of incomplete or imperfect work; design is an iterative process.

As students present their animations to the class, it is remarkable to witness the pride with which students share their technological and mathematical learning. While variations in aesthetic refinement, technological precision, and mathematics are inevitable and visible, the focus on design principles creates space for genuine admiration across this diversity of products. Students typically walk away feeling connected to others and even inspired by their classmates' efforts. The knowledge is contagious.

Presenting their animations is not the conclusion for many. During the presentation, it is common for classmates to offer suggestions for redesign or provide answers to raised questions. Many students ask if they can do "one more draft," either to incorporate feedback or after watching a classmate present a tool that could resolve a remaining challenge. These reflection prompts honor each student's unique mathematical journey while building a classroom community that values collaboration, diverse approaches, and partial solutions.

5 Conclusion

Building Thinking Classrooms (Liljedahl, 2021) requires more than simply offering open-ended tasks and expecting creativity, problem solving, revision and justification. This transformation demands a fundamental shift in classroom norms. Using the Animation Task as an example, we have shown how design thinking principles can be enacted to give students genuine ownership over their mathematical work, from selecting phenomena to determining when their designs authentically represent their thinking.

Giving students the agency to select ambitious goals and set their own threshold for "enough", we allow all students to engage meaningfully with sophisticated mathematics. Whether through simple animations grounded in clear mathematical relationships or through complex designs involving advanced coding and multiple representations, each student works through individualized mathematical and technological challenges. By structuring reflection prompts that ask students to articulate their intentions, learning, and constraints, we build a classroom community where diverse approaches are not just tolerated but celebrated.

We invite you to consider how design thinking principles might transform tasks in your own classroom. What would it look like to give students agency over not just the solution path, but the scope and criteria of the solution itself? How might your role shift from evaluator to advisor when students design for themselves and their peers? The adventure of mathematical design awaits you and your students!

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